

Maxwell's equations, radiative transfer, and coherent backscattering: A general perspective

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Abstract

This tutorial paper provides a general overview of the hierarchy of problems involving electromagnetic scattering by particles and clarifies the place of the radiative transfer theory and the theory of coherent backscattering in the context of classical electromagnetics. The self-consistent microphysical approach to radiative transfer is compared with the traditional phenomenological treatment.

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1. Introduction

Natural and man-made environments provide countless examples of scattering media composed of small particles. The varying complexity of these media suggests multiple ways of using electromagnetic scattering for particle characterization and gives rise to a distinctive hierarchy of theoretical models that can be invoked to analyze specific remote-sensing or laboratory measurements. Hence the objective of this overview is to provide a classification of scattering problems involving small particles and briefly outline general solution approaches.

Given the broad scope of this overview, we do not try to provide a comprehensive and detailed reference list. Instead, the preference is given to relevant reviews and books where further references can be found.

2. Electromagnetic scattering by a fixed finite object

A parallel monochromatic beam of light propagates in a vacuum without a change in its intensity or polarization state. However, inserting an object into the beam (see Fig. 1) causes several distinct effects. First, the object extracts some of the incident energy and spreads it in all directions at the frequency of the incident beam. This phenomenon is called elastic scattering and, in general, gives rise to light with polarization state different from that of the incident beam. Second, the object may convert some of the energy contained in the beam into other forms of energy such as heat. This phenomenon is called absorption. The energy contained in the incident beam is accordingly reduced by the amount equal to the sum of the scattered and absorbed

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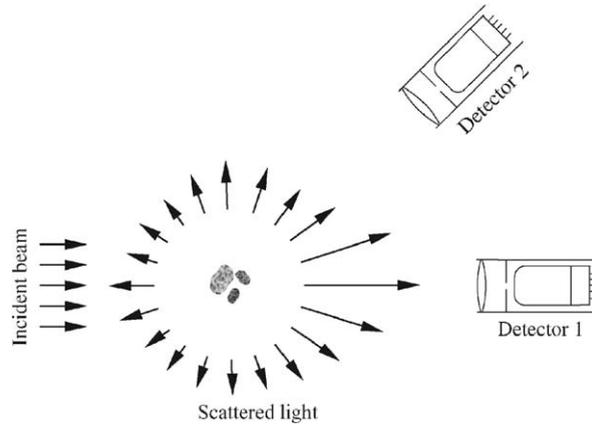


Fig. 1. Scattering by a fixed object. In this case the object consists of three disjoint, heterogeneous, stationary bodies.

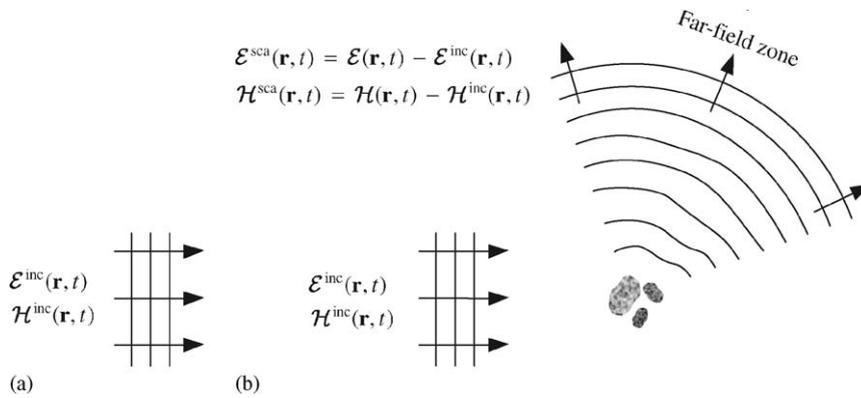


Fig. 2. Schematic representation of the electromagnetic scattering problem.

energy. This reduction is called extinction. The extinction rates for different polarization components of the incident beam can be different, which is called dichroism.

In electromagnetic terms, the parallel monochromatic beam of light is represented by a harmonically oscillating plane electromagnetic wave. The latter propagates in a vacuum without a change in its intensity or polarization state (see Fig. 2a). However, the presence of a finite object, as illustrated in Fig. 2b, changes both the electric, \mathcal{E} , and the magnetic, \mathcal{H} , field that would otherwise exist in an unbounded homogeneous space. The difference between the total fields in the presence of the object, $\mathcal{E}(\mathbf{r}, t)$ and $\mathcal{H}(\mathbf{r}, t)$, and the original fields that would exist in the absence of the object, $\mathcal{E}^{\text{inc}}(\mathbf{r}, t)$ and $\mathcal{H}^{\text{inc}}(\mathbf{r}, t)$, can be thought of as the fields scattered by the object, $\mathcal{E}^{\text{sca}}(\mathbf{r}, t)$ and $\mathcal{H}^{\text{sca}}(\mathbf{r}, t)$, where \mathbf{r} is the position vector and t is time. In other words, the total electric and magnetic fields in the presence of the object are equal to vector sums of the respective incident (original) and scattered fields:

$$\mathcal{E}(\mathbf{r}, t) = \mathcal{E}^{\text{inc}}(\mathbf{r}, t) + \mathcal{E}^{\text{sca}}(\mathbf{r}, t), \tag{1}$$

$$\mathcal{H}(\mathbf{r}, t) = \mathcal{H}^{\text{inc}}(\mathbf{r}, t) + \mathcal{H}^{\text{sca}}(\mathbf{r}, t). \tag{2}$$

The origin of the scattered electromagnetic field can be understood by recalling that in terms of microscopic electrodynamics, the object is an aggregation of a large number of discrete elementary electric charges. The oscillating electromagnetic field of the incident wave excites the charges to vibrate with the same frequency and

thereby radiate secondary electromagnetic waves. The superposition of all the secondary waves gives the total elastically scattered field. If the charges do not oscillate exactly in phase or exactly in anti-phase with the incident field then there is dissipation of electromagnetic energy into the object. This means that the object is absorbing and scatters less total energy than it extracts from the incident wave.

Electromagnetic scattering is an exceedingly complex phenomenon because a secondary wave generated by a vibrating charge also stimulates vibrations of all other charges forming the object and thus modifies their respective secondary waves. As a result, all the secondary waves become interdependent. Furthermore, the computation of the total scattered field by superposing the secondary waves must take account of their phase differences, which change every time the incidence and/or scattering direction is changed. Therefore, the total scattered field depends on the way the charges are arranged to form the object with respect to the incidence and scattering directions.

Since the number of elementary charges forming an object can be extremely large, solving the scattering problem directly by computing and superposing all secondary waves is impracticable even with the aid of modern computers. Fortunately, the scattering problem can be also solved using the concepts of macroscopic electromagnetics, which treat the large collection of charges as one or several macroscopic bodies with a specific distribution of the refractive index. As a consequence, the scattered field can be computed by solving the Maxwell equations for the macroscopic electromagnetic field subject to appropriate boundary conditions. It is this approach that forms the basis of the modern theory of electromagnetic scattering by particles.

To simplify the solution of the scattering problem, the following four restrictions are usually adopted [1–3]:

- (1) It is assumed that the unbounded host medium surrounding the scattering object is homogeneous, linear, isotropic and nonabsorbing.
- (2) It is assumed that the scattering object is illuminated by either
 - (a) a time-harmonic plane electromagnetic wave given, in the complex-field representation, by

$$\left. \begin{aligned} \mathbf{E}(\mathbf{r}, t) &= \mathbf{E}_0 \exp(i\mathbf{k} \cdot \mathbf{r} - i\omega t), \\ \mathbf{H}(\mathbf{r}, t) &= \mathbf{H}_0 \exp(i\mathbf{k} \cdot \mathbf{r} - i\omega t), \end{aligned} \right\} \mathbf{r} \in \mathfrak{R}^3, \quad (3)$$

with constant amplitudes \mathbf{E}_0 and \mathbf{H}_0 , where ω is the angular frequency, \mathbf{k} is the real-valued wave vector, $i = (-1)^{1/2}$, and \mathfrak{R}^3 denotes the entire three-dimensional space, or

- (b) a quasi-monochromatic parallel beam of light given by

$$\left. \begin{aligned} \mathbf{E}(\mathbf{r}, t) &= \mathbf{E}_0(t) \exp(i\mathbf{k} \cdot \mathbf{r} - i\omega t), \\ \mathbf{H}(\mathbf{r}, t) &= \mathbf{H}_0(t) \exp(i\mathbf{k} \cdot \mathbf{r} - i\omega t), \end{aligned} \right\} \mathbf{r} \in \mathfrak{R}^3, \quad (4)$$

where fluctuations in time of the complex amplitudes of the electric and magnetic fields, $\mathbf{E}_0(t)$ and $\mathbf{H}_0(t)$, around their respective mean values occur much more slowly than the harmonic oscillations of the time factor $\exp(-i\omega t)$.

- (3) Nonlinear optics effects are usually excluded by assuming that the conductivity, permeability, and electric susceptibility of both the scattering object and the surrounding medium are independent of the electric and magnetic fields.
- (4) It is assumed that electromagnetic scattering occurs without frequency redistribution, i.e., the scattered light has the same frequency as the incident light. This restriction excludes inelastic scattering phenomena such as Raman and Brillouin scattering and fluorescence.

The theoretical and numerical techniques for computing the electromagnetic field elastically scattered by a finite fixed object composed of one or several physical bodies are many and are reviewed in detail in Refs. [2–4]. Since all of these techniques have certain limitations in terms of the object morphology and object size relative to the incident wavelength, a practitioner should carefully analyze the relative strengths and weaknesses of the available solution techniques before attempting to address the specific problem in hand.

3. Actual observables

Traditional optical instruments cannot measure the electric and magnetic fields associated with the incident and scattered waves. Rather they can measure quantities having the dimension of energy flux. Thus, in order to make the theory applicable to analyses of actual optical observations, the scattering process must be characterized in terms of derivative quantities that can be measured directly. This explains why the concept of an actual *observable* is central to the discipline of light scattering by particles.

Although one can always define the magnitude and the direction of the electromagnetic energy flux at any point in space in terms of the Poynting vector, the latter carries no information about the polarization state of the incident and scattered fields. The conventional approach to ameliorate this problem dates back to Sir George Gabriel Stokes. He proposed to use four real-valued quantities which have the dimension of monochromatic energy flux and fully characterize a *transverse* electromagnetic wave inasmuch as it is subject to practical optical analysis. These quantities are called the Stokes parameters, form the so-called four-component Stokes column vector, and carry information about both the intensity and the polarization state of the wave [1,3,5].

In the so-called far-field zone of a fixed object, the propagation of the scattered electromagnetic wave is away from the particle (Fig. 2b). Furthermore, the electric and magnetic field vectors vibrate in the plane perpendicular to the propagation direction and their amplitudes decay inversely with distance from the object. This allows one to define the Stokes parameters of both the incident plane wave and the scattered spherical wave and to describe the response of a well-collimated polarization-sensitive detector of light in terms of the 4×4 so-called phase and extinction matrices. Specifically, detector 2 in Fig. 1 collects only the scattered light, and its response is fully characterized by the product of the phase matrix, which specifies the transformation of the Stokes parameters of the incident wave into the Stokes parameters of the scattered wave, and the Stokes column vector of the incident wave. The response of detector 1 consists of three parts:

- (1) the one due to the incident light;
- (2) the one due to the forward-scattered light; and
- (3) the one due to the interference of the incident wave and the wave scattered by the object in the exact forward direction.

The third part is described by minus the product of the extinction matrix and the Stokes vector of the incident wave and accounts for both the total attenuation of the detector signal due to extinction of light by the object and the effect of dichroism.

The phase and extinction matrices depend on particle characteristics such as size, shape, refractive index, and orientation and can be readily computed provided that the scattered field is known from the solution of the Maxwell equations [3].

The main convenience of the far-field approximation is that it allows one to treat the object essentially as a point source of scattered radiation. However, the criteria defining the far-field zone of an object are rather stringent and are often violated in practice. A good example is remote sensing of water clouds in the terrestrial atmosphere using detectors of electromagnetic radiation mounted on aircraft or satellite platforms. Such detectors typically measure radiation coming from a small part of a cloud and do not “perceive” the entire cloud as a single point-like scatterer. Furthermore, the notion of the far-field zone of the cloud becomes completely meaningless if a detector is placed inside the cloud. It is thus clear that to characterize the response of such “near-field” detectors one must define quantities other than the Stokes parameters and the extinction and phase matrices. Still the actual observables must be defined in such a way that they can be measured by an optical device ultimately recording the flux of electromagnetic energy.

4. Foldy–Lax equations

Many theoretical techniques based on directly solving the differential Maxwell equations or their integral counterparts are applicable to an arbitrary fixed finite object, be it a single physical body or a cluster consisting of several distinct components, either touching or spatially separated. These techniques treat the object as a

single scatterer and compute the total scattered electromagnetic field. However, if the object is a multi-particle cluster then it is often useful to represent the total scattered field as a vector superposition of the partial fields scattered by the individual cluster components. This means, for example, that the total electric field at a point \mathbf{r} is written as follows:

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}^{\text{inc}}(\mathbf{r}, t) + \sum_{i=1}^N \mathbf{E}_i^{\text{sca}}(\mathbf{r}, t), \quad \mathbf{r} \in \mathfrak{R}^3, \quad (5)$$

where N is the number of particles in the cluster and $\mathbf{E}_i^{\text{sca}}(\mathbf{r}, t)$ is the i th partial scattered electric field. The total magnetic field is given by a similar expression. The partial scattered fields can be found by solving vector so-called Foldy–Lax equations which follow directly from the volume integral equation counterpart of the Maxwell equations and are exact [6,7]. By iterating the Foldy–Lax equations, one can derive an order-of-scattering expansion of the scattered field which, in combination with statistical averaging, forms the basis of the modern theory of multiple scattering by random particle ensembles.

5. Dynamic and static scattering by random particle groups

Solving the Maxwell equations yields the field scattered by a fixed object. This approach can be used directly in analyses of microwave analog measurements [8], in which the scattering object is held fixed relative to the source of electromagnetic radiation during the measurement cycle, but is inapplicable in the majority of laboratory and remote-sensing observations. Indeed, even if the scattering object is a single microparticle trapped inside an electrostatic or optical levitator, it rapidly changes its position and orientation during the time necessary to take a measurement. Furthermore, one often encounters situations in which light is scattered by a very large group of particles forming a constantly varying spatial configuration. A typical example is a cloud of water droplets or ice crystals in which the particles are constantly moving, spinning, and even changing their shapes and sizes due to oscillations of the droplet surface, evaporation, condensation, sublimation and melting. Although such a particle collection can be treated at each given moment as a fixed cluster, a typical measurement of light scattering takes a finite amount of time over which the spatial configuration of the component particles and their sizes, orientations, and/or shapes continuously and randomly change. Therefore, the registered signal is in effect an average over a large number of different clusters.

When a fixed group of particles is illuminated by a monochromatic, spatially coherent plane wave (e.g., laser light), the light scattered by the group onto a distant screen generates a characteristic speckle pattern consisting of randomly located bright spots of various size and shape (see Fig. 3a modified after Ref. [9]). This pattern is the result of random constructive and destructive interference of the partial waves scattered by different particles towards a point on the screen. When the particles move, the phase relations between the partial waves constantly change, thereby leading to rapid fluctuations of the speckle pattern. Accumulating the

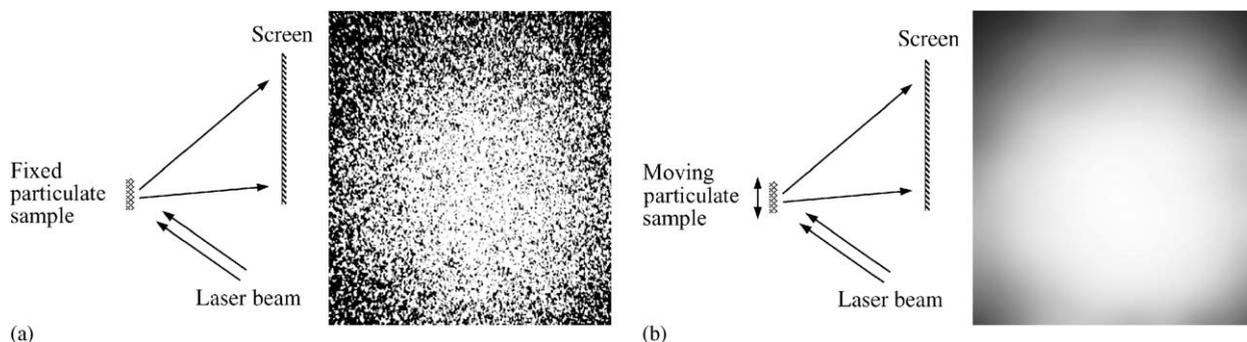


Fig. 3. (a) Speckle pattern produced by laser light reflected by a fixed particulate sample. (b) Moving the scattering sample during the measurement averages the speckle pattern out.

signal over a sufficiently long period of time averages the speckle pattern out and results in a rather smooth “incoherent” distribution of the scattered intensity (Fig. 3b).

It has been shown that measurements of the temporal and/or spatial fluctuations of the speckle pattern contain useful information about the particles, in particular about their motion. Statistical analyses of light scattered by dilute and dense particle suspensions, respectively, are the subject of the disciplines called photon correlation spectroscopy (PCS) and diffusing wave spectroscopy (DWS) and form the basis of many well established experimental techniques for the measurement of various particle characteristics such as velocity, size and dispersity [10,11]. The recent extension of the PCS to account for particles changing the polarization state of the incident coherent beam, the so-called polarization fluctuation spectroscopy, enables the morphology in addition to the size of particles to be sensed [12,13].

The PCS and DWS study *dynamic* aspects of light scattering by groups of randomly moving particles. Alternatively, one can assume that the effect of temporal fluctuations is eliminated by averaging the speckle pattern over a period of time much longer than the typical period of the fluctuations and deal with the average, static component of the scattering pattern. This assumption is at the core of the discipline called *static* light scattering.

6. Ergodicity

Quantitative analyses of static scattering measurements require the use of a theoretical averaging procedure. Let us consider, e.g., the measurement of a scattering characteristic A of a cloud of spherical water droplets. This characteristic depends on time implicitly by being a function of time-dependent physical parameters of the cloud such as the coordinates and sizes of all the constituent particles. The full set of particle positions and sizes will be denoted collectively by ψ and determines the state of the entire cloud at a moment in time. In order to interpret the measurement of $A[\psi(t)]$ accumulated over a period of time extending from $t = t_0$ to $t = t_0 + T$, one needs a way of predicting theoretically the average value

$$\bar{A} = \frac{1}{T} \int_{t_0}^{t_0+T} dt A[\psi(t)]. \quad (6)$$

Quite often the temporal evolution of a complex scattering object such as the cloud of water droplets is controlled by several physical processes and is described by an intricate system of equations. To incorporate the solution of this system of equations for each moment of time into the theoretical averaging procedure of Eq. (6) can be a formidable task and is rarely, if ever, done. Instead, averaging over time is replaced by ensemble averaging based on the following rationale.

Although the coordinates and sizes of water droplets in the cloud change with time in a specific way, the range of instantaneous states of the cloud captured by the detector during the measurement becomes representative of that captured over an infinite period of time provided that T is sufficiently large. We thus have

$$\bar{A} \approx \lim_{\tau \rightarrow \infty} \frac{1}{\tau} \int_{t_0}^{t_0+\tau} dt A[\psi(t)] = \langle A \rangle_t. \quad (7)$$

Notice now that the infinite integral in Eq. (7) can be expected to “sample” every physically realizable state ψ of the cloud. Furthermore, this sampling is statistically representative in that the number of times each state is sampled is large and tends to infinity in the limit $\tau \rightarrow \infty$. Most importantly, the cumulative contribution of a cloud state ψ to $\langle A \rangle_t$ is independent of the specific moments of time when this state actually occurred in the process of the temporal evolution of the cloud but rather depends on how many times this state was sampled. Therefore, this cumulative contribution can be thought of as being proportional to the probability of occurrence of the state ψ at *any* moment of time. This means that instead of specifying the state of the cloud at each moment t and integrating over all t one can introduce an appropriate time-independent probability distribution function $p(\psi)$ and integrate over the entire physically realizable range of cloud states:

$$\langle A \rangle_t \approx \int d\psi p(\psi) A(\psi) = \langle A \rangle_\psi, \quad (8)$$

where

$$\int d\psi p(\psi) = 1. \quad (9)$$

The assumption that averaging over time for a “sufficiently random” object can be replaced by ensemble averaging is called the ergodic hypothesis. Although it has not been possible to establish mathematically the full ergodicity of real dynamical systems, more restricted versions of the ergodic theorem have been proven. Physical processes such as Brownian motion and turbulence often help to establish a significant degree of randomness of particle positions and orientations, which seems to explain why many theoretical predictions based on the ergodic hypothesis have agreed very well with experimental data (see, e.g., Ref. [10]). Therefore, it is commonly assumed that the scattering system in question is ergodic and, thus, Eq. (8) is applicable.

7. Single scattering by random particles

The simplest stochastic scattering object is a single particle undergoing random changes of position, orientation, size and/or shape during the measurement. A good example is a solid or liquid particle trapped inside an electrostatic or optical levitator. In this case particle positions are confined to a small volume with diameter often much smaller than the distance from the volume center to the detector (Fig. 4). It is then rather straightforward to show that the detector signal accumulated over a period of time is independent of particle positions and can be described in terms of phase and extinction matrices averaged over appropriate ranges of particle orientations, sizes and shapes. The formalism remains largely the same as in the case of far-field scattering by a fixed object.

A more difficult case is the scattering by a small random group of particles (Fig. 5). Still most of the far-field-scattering formalism can be preserved if the group is observed from a large distance and is sufficiently tenuous. Specifically, if the number of particles is sufficiently small and the separation between them is sufficiently large then one can neglect the response of each particle to the fields scattered by all other particles and assume that each particle is excited only by the external field. This is the essence of the so-called single-scattering approximation, which leads to a significant simplification of the Foldy–Lax equations. Another assumption is that particle positions are uncorrelated and sufficiently random and are independent of particle states (i.e., combinations of particle sizes, refractive indices, shapes and orientations). One can then show that the signal accumulated by a distant detector over a period of time can be directly described in terms of single-particle phase and extinction matrices averaged over the states (but not the positions!) of all the particles and multiplied by the number of particles [14].

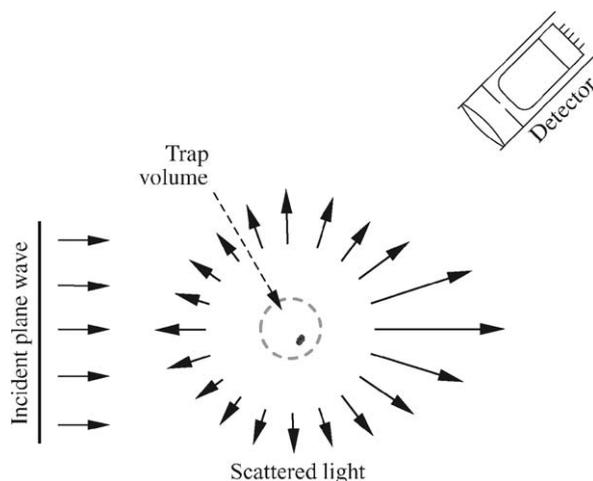


Fig. 4. Scattering by a single random particle.

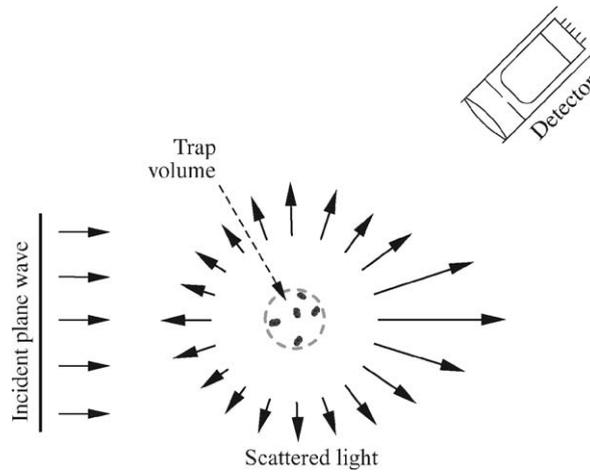


Fig. 5. Scattering by a small random particle group.

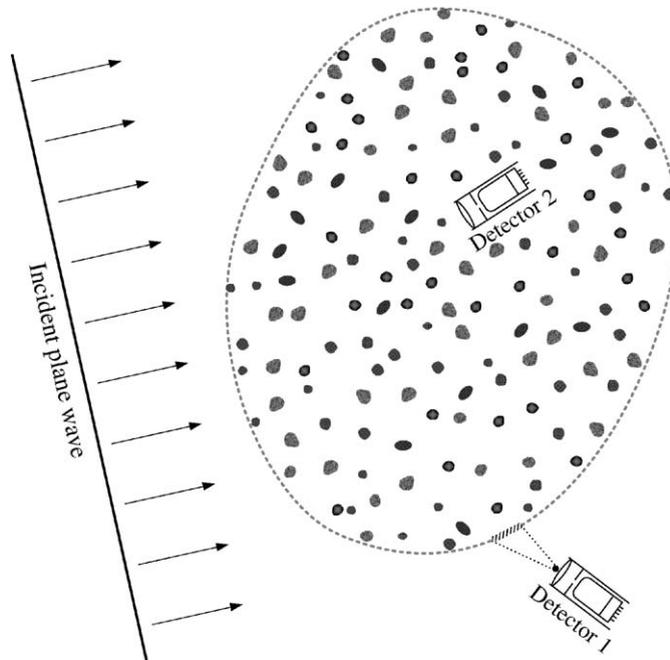


Fig. 6. Scattering by a large random particle group.

8. Multiple scattering by random particles: radiative transfer

The problem of utmost complexity is electromagnetic scattering by a very large group of particles occupying a large volume of space (Fig. 6). The far-field-scattering formalism becomes totally inapplicable since the angular aperture of an external detector may subtend only a small fraction of the scattering volume (detector 1) or, worse, the detector may be placed inside the scattering medium (detector 2). Furthermore, the field created by a particle in response to the fields scattered by all the other particles forming the medium can be comparable to or even greater than that created in response to the incident field, which means that the single-scattering approximation is no longer valid.

To deal with this problem, one has to make several critical assumptions [6]. The first of them is to assume that each particle is located in the far-field zones of all the other particles and that the observation point is also located in the far-field zones of all the particles forming the scattering medium. This assumption results in a dramatic simplification of the Foldy–Lax equations wherein the latter are converted from a system of volume integral equations into a system of linear algebraic equations. However, it limits the applicability of the final result by requiring the scattering medium to be rather rarefied.

The algebraic system of the far-field Foldy–Lax equations can be cast into an order-of-scattering form, in which the total electric field at a point in space is represented as a sum of contributions arising from light-scattering paths going through all possible particle sequences. The second major assumption, called the Twersky approximation [15], is that all paths going through the same particle more than once can be ignored. It can be demonstrated that doing this is justified provided that the total number of particles in the scattering volume is very large.

The third major assumption is that of full ergodicity, which allows one to replace averaging over time by averaging over particle positions and states.

The fourth major assumption is that (i) the position and state of each particle are statistically independent of each other and of those of all the other particles and (ii) the spatial distribution of the particles throughout the medium is random and statistically uniform. As one might expect, this assumption leads to a major simplification of all analytical derivations.

The next major step is the characterization of the multiply scattered radiation by the coherency dyadic

$$\vec{C}(\mathbf{r}) = \langle \mathbf{E}(\mathbf{r}, t) \otimes \mathbf{E}^*(\mathbf{r}, t) \rangle_t, \quad (10)$$

followed by the angular decomposition

$$\vec{C}(\mathbf{r}) = \int_{4\pi} d\hat{\mathbf{q}} \vec{\Sigma}(\mathbf{r}, \hat{\mathbf{q}}) \quad (11)$$

in terms of the so-called specific coherency dyadic $\vec{\Sigma}(\mathbf{r}, \hat{\mathbf{q}})$, where the integration is performed over all propagation directions as specified by the unit vector $\hat{\mathbf{q}}$. The introduction of these quantities offers three crucial benefits. First, one can sum the so-called ladder diagrams appearing in the diagrammatic representation of the coherency dyadic and show that the specific coherency dyadic satisfies a radiative transfer equation (RTE). Second, the specific coherency dyadic can be used to define the so-called specific intensity column vector which also satisfies an RTE. Third, one can use the integral form of the RTE to show that the specific intensity column vector directly describes the radiometric and polarimetric response of detectors 1 and 2 in Fig. 6 averaged over a period of time [6,16].

The fact that the specific intensity column vector can be both computed theoretically by solving the RTE and measured with a suitable optical device explains the practical usefulness of the radiative transfer theory in countless applications in various branches of science and engineering [17–30]. Furthermore, the microphysical derivation of the RTE outlined above and described in detail in Refs. [6,16] gives the radiative transfer theory the firm footing that it had needed for many decades to refute the severe criticism and lack of trust on the part of physicists.

9. Coherent backscattering

Despite the approximate character of the radiative transfer theory, it provides a powerful and reasonably general prescription for the treatment of the interaction of light with particulate media and is accordingly applicable to a broad range of practical situations. However, owing to some of the basic assumptions in the derivation of the RTE, there are circumstances for which it is not sufficient. An important example is the so-called coherent backscattering of light (otherwise known as weak localization of waves). To explain the physical origin of this phenomenon, let us consider a layer composed of randomly positioned particles and illuminated by a plane electromagnetic wave incident in the direction $\hat{\mathbf{n}}_{\text{ill}}$ (Fig. 7). The (infinitely) distant observer measures the intensity of light reflected by the layer in the direction $\hat{\mathbf{n}}_{\text{obs}}$. The reflected signal is composed of the contributions made by waves scattered along various paths inside the layer involving different combinations of particles. Let us consider the two conjugate scattering paths shown in Fig. 7 by solid

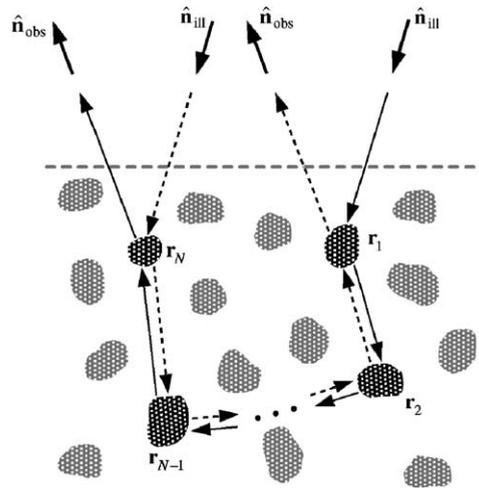


Fig. 7. Schematic explanation of coherent backscattering.

and dashed lines. These paths go through the same group of N particles, denoted by their positions $\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N$, but in opposite directions. The waves scattered along the two conjugate paths interfere, the interference being constructive or destructive depending on the phase difference

$$\Delta = k_1(\mathbf{r}_N - \mathbf{r}_1) \cdot (\hat{\mathbf{n}}_{ill} + \hat{\mathbf{n}}_{obs}), \tag{12}$$

where k_1 is the wave number in the surrounding medium. If the observation direction is far from the exact backscattering direction given by $-\hat{\mathbf{n}}_{ill}$, then the waves scattered along conjugate paths involving different groups of particles interfere in different ways, and the average effect of the interference is zero owing to the randomness of particle positions. Consequently, the observer measures some average, incoherent intensity that is well described by the RTE. However, at exactly the backscattering direction ($\hat{\mathbf{n}}_{obs} = -\hat{\mathbf{n}}_{ill}$), the phase difference between conjugate paths involving any group of particles is identically equal to zero, Eq. (12), and the interference is always constructive, thereby resulting in a coherent intensity peak superposed on the incoherent background [31,32].

The failure of the RTE to reproduce the coherent backscattering peak is explained by the fact that of all kinds of diagrams in the diagrammatic representation of the coherency dyadic it keeps only the ladder diagrams, whereas coherent backscattering is caused by so-called cyclical (or most-crossed) diagrams. The inclusion of the cyclical diagrams makes the computation of the coherency dyadic much more involved and limits the range of problems that can be solved accurately. However, the reciprocal nature of each single-scattering event leads to an interesting exact result: the characteristics of the coherent backscattering effect at the exact backscattering direction can be rigorously expressed in terms of the solution of the RTE [33].

The ladder and cyclical diagrams are the dominant but not the only types of diagrams in the diagrammatic representation of the coherency dyadic. However, to include all the other diagrams in the calculation of multiply scattered radiation is a formidable problem that still awaits its solution.

10. Classification of scattering problems in the framework of classical macroscopic electromagnetics

To develop a comprehensive and universal classification of electromagnetic scattering problems borders on impossible. This paper provides only an outline tailored to the specifics of radiative transfer and coherent backscattering, whereas those working on another aspect of electromagnetic scattering might prefer a modified classification with somewhat different emphases. It is hoped, however, that this outline, summarized graphically in Fig. 8, fulfils its limited objective and explains adequately the place of radiative transfer and coherent backscattering within the broader context of macroscopic electromagnetics.

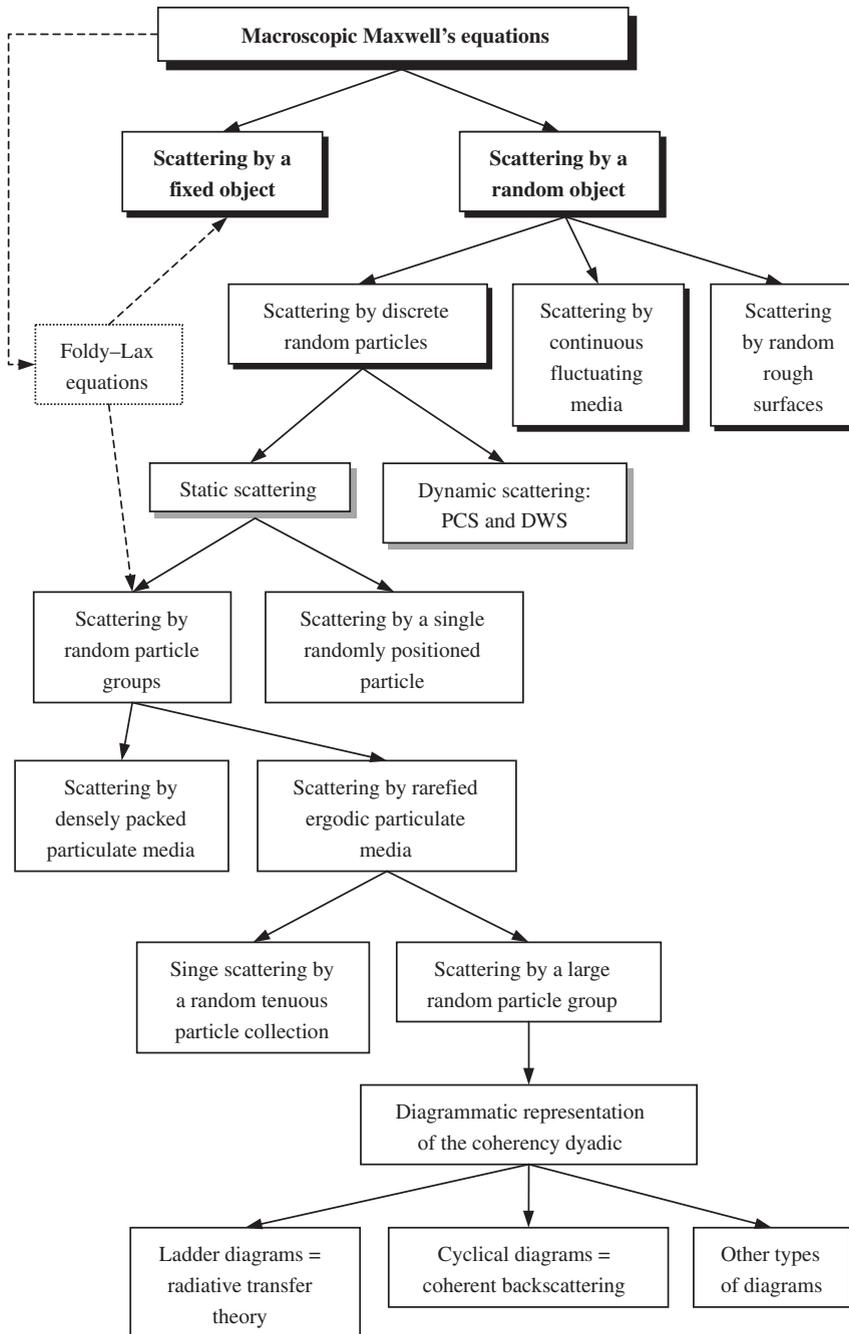


Fig. 8. Classification of electromagnetic scattering problems.

As is obvious from the diagram in Fig. 8, there are two broad classes of problems that we have not touched upon so far. The focus of our discussion has been on multiple scattering by randomly positioned discrete particles with refractive index distinctly different from that of the surrounding medium. However, one can also consider multiple scattering in continuous media with random fluctuations of the refractive index. This class of problems requires special solution approaches described in the monographs [34–36].

Another important problem is electromagnetic scattering by an infinite random rough surface separating two half-spaces with different refractive indices. Although some rough surfaces, such as the ocean surface,

indeed change randomly with time, many rough interfaces between pairs of solid slabs do not change and are deterministic rather than random. However, quite often their position relative to the source of light and/or the detector is not fixed during the measurement and their vertical profile is described by a highly irregular function of lateral coordinates. Even minute displacements of the source of light and/or the detector change phase differences entirely, thereby destroying the speckle pattern. Furthermore, the detector may view different parts of the surface at different moments in time, thereby in effect recording an average over a temporally varying surface profile. These two factors make the concept of a random rough surface a good model for describing the results of many actual static measurements. Detailed information on this subject can be found in the monographs [36–38].

One can also think of more complex problems involving different types of volume and/or surface scattering. A good example is electromagnetic scattering by a layer of continuous fluctuating medium comprising randomly positioned discrete particles and bounded by random rough surfaces. Although problems like this one are important in practice and have been treated using various phenomenological approaches, microphysical treatments based on consistent application of the Maxwell equations have been extremely scarce.

11. Phenomenological approach to radiative transfer

It is instructive to compare the self-consistent microphysical approach to radiative transfer developed in Refs. [6,16] with the traditional phenomenological approach (e.g., [17,39,40]). Since the latter cannot be used to *derive* many facts that appear as corollaries of classical electromagnetics in the framework of the microphysical approach, one has to *postulate* them. For example, it naturally follows from the microphysical derivation that the average (coherent) field inside the discrete random medium is exponentially attenuated and serves to replace the constant-amplitude incident field as the de facto source of multiple scattering. In contrast, the phenomenological approach begins with a *postulate* that the incident parallel beam of light is exponentially attenuated as it propagates through the medium and serves as the initial source of multiple scattering.

Another postulate of the phenomenological approach is that the diffuse radiation field at each point \mathbf{r} inside the scattering medium and at each moment in time can be represented by a collection of elementary ‘rays’ with a continuous distribution of propagation directions $\hat{\mathbf{q}}$ and can be characterized by the local four-component diffuse specific intensity column vector:

$$\tilde{\mathbf{I}}_d(\mathbf{r}, \hat{\mathbf{q}}) = \begin{bmatrix} \tilde{I}_d(\mathbf{r}, \hat{\mathbf{q}}) \\ \tilde{Q}_d(\mathbf{r}, \hat{\mathbf{q}}) \\ \tilde{U}_d(\mathbf{r}, \hat{\mathbf{q}}) \\ \tilde{V}_d(\mathbf{r}, \hat{\mathbf{q}}) \end{bmatrix}. \quad (13)$$

The elementary rays are postulated to be mutually incoherent and make independent contributions to $\tilde{\mathbf{I}}_d(\mathbf{r}, \hat{\mathbf{q}})$. The elements $\tilde{Q}_d(\mathbf{r}, \hat{\mathbf{q}})$, $\tilde{U}_d(\mathbf{r}, \hat{\mathbf{q}})$ and $\tilde{V}_d(\mathbf{r}, \hat{\mathbf{q}})$ describe the polarization state of the ray propagating in the direction $\hat{\mathbf{q}}$ through the observation point specified by the position vector \mathbf{r} , whereas the product

$$dS dt d\Omega \tilde{I}_d(\mathbf{r}, \hat{\mathbf{q}})$$

gives the amount of electromagnetic energy transported through a surface element dS normal to $\hat{\mathbf{q}}$ and centered at \mathbf{r} in a time interval dt in all directions confined to a solid angle element $d\Omega$ centered at the direction of propagation $\hat{\mathbf{q}}$. All elements of the specific intensity column vector have the dimension of radiance. The direct propagation of the incident parallel beam of light through the medium is described by a ‘monodirectional’ four-component Stokes column vector $\mathbf{I}_c(\mathbf{r})$ having the dimension of intensity.

Thus, there is a fundamental difference between how the phenomenological and microphysical approaches treat the random radiation field. The phenomenological approach begins with a postulate of existence of the diffuse specific intensity column vector and the Stokes column vector of the direct light at each moment in time. In the framework of the microphysical approach these quantities are derived from more fundamental ones and are shown to describe the directional flow of electromagnetic radiation averaged over a sufficiently long period of time.

The phenomenological radiative transfer theory treats the medium filled with a large number of discrete, sparsely and randomly distributed particles as continuous and locally homogeneous and is fundamentally based on the concept of an elementary (or differential) volume element of the scattering medium. Specifically, it replaces the concept of single scattering and absorption by an individual particle with the concept of single scattering and absorption by an elementary volume element. It is assumed that the result of scattering is not the electromagnetic transformation of a plane incident wave into a spherical scattered wave in the far-field zone of the volume element, but rather the transformation of the diffuse specific intensity column vector of the incident light into the diffuse specific intensity column vector of the scattered light. This assumption appears to be especially artificial because the scattering transformation law is then written in the form

$$\tilde{\mathbf{I}}_d(\mathbf{r}, \hat{\mathbf{q}}) \propto \mathbf{Z}_{dV}(\hat{\mathbf{q}}, \hat{\mathbf{q}}') \tilde{\mathbf{I}}_d(\mathbf{r}, \hat{\mathbf{q}}'),$$

and the $\mathbf{Z}_{dV}(\hat{\mathbf{q}}, \hat{\mathbf{q}}')$, called the phase matrix of the elementary volume element, is computed from electromagnetics. Specifically, it is postulated that

$$\mathbf{Z}_{dV}(\hat{\mathbf{q}}, \hat{\mathbf{q}}') = n_0 dV \langle \mathbf{Z}(\hat{\mathbf{q}}, \hat{\mathbf{q}}') \rangle_{\xi},$$

where n_0 is the particle number density, dV is the size of the elementary volume element, $\mathbf{Z}(\hat{\mathbf{q}}, \hat{\mathbf{q}}')$ is the single-particle phase matrix describing the transformation of an incident plane electromagnetic wave into the scattered spherical wave and $\langle \dots \rangle_{\xi}$ denotes an average over all physically realizable particle states.

It is further postulated that the change of the Stokes column vector of direct light $\mathbf{I}_c(\mathbf{r})$ over a differential length ds parallel to the incidence direction $\hat{\mathbf{s}}$ is caused by extinction and dichroism and is described by

$$\frac{d\mathbf{I}_c(\mathbf{r})}{ds} = -n_0 \langle \mathbf{K}(\hat{\mathbf{s}}) \rangle_{\xi} \mathbf{I}_c(\mathbf{r}), \quad (14)$$

in which, again, the single-particle extinction matrix $\mathbf{K}(\hat{\mathbf{s}})$ is computed from Maxwell's electromagnetics.

In addition, it is postulated that the cumulative change of the diffuse specific intensity column vector over the length dq of an elementary cylindrical volume element having bases of an area dA perpendicular to $\hat{\mathbf{q}}$ is caused by:

- (1) the effect of extinction and dichroism;
- (2) the contribution of the diffuse light illuminating the volume element from all directions $\hat{\mathbf{q}}'$ and scattered into the direction $\hat{\mathbf{q}}$; and
- (3) the contribution of the attenuated external beam scattered into the direction $\hat{\mathbf{q}}$.

These three components are described by the first, second, and third terms, respectively, on the right-hand side of the integro-differential RTE:

$$\frac{d\tilde{\mathbf{I}}_d(\mathbf{r}, \hat{\mathbf{q}})}{dq} = -n_0 \langle \mathbf{K}(\hat{\mathbf{q}}) \rangle_{\xi} \tilde{\mathbf{I}}_d(\mathbf{r}, \hat{\mathbf{q}}) + n_0 \int_{4\pi} d\hat{\mathbf{q}}' \langle \mathbf{Z}(\hat{\mathbf{q}}, \hat{\mathbf{q}}') \rangle_{\xi} \tilde{\mathbf{I}}_d(\mathbf{r}, \hat{\mathbf{q}}') + n_0 \langle \mathbf{Z}(\hat{\mathbf{q}}, \hat{\mathbf{s}}) \rangle_{\xi} \mathbf{I}_c(\mathbf{r}). \quad (15)$$

It is thus clear that the phenomenological approach is based on a rather eclectic combination of concepts borrowed from pure radiometry (light rays as geometrical trajectories along which radiant energy is assumed to be propagated, the concept of incoherent radiance) and pure electromagnetism (electromagnetic scattering of plane waves, Stokes parameters, phase and extinction matrices).

The concept of an elementary volume element is implicitly based on the modified uncorrelated single-scattering approximation (MUSSA) discussed in Ref. [14]. A fundamental problem here is that the MUSSA is only valid in the far-field zone of the elementary volume element as a whole and cannot be applied to adjacent volume elements having common boundaries. In particular, it has been demonstrated in Ref. [14] that the far-field zone of a volume element may begin at a distance exceeding the volume element's size by several orders of magnitude.

Another problem is caused by the assumption that the RTE describes the instantaneous state of the radiation field. Indeed, in order to justify the use of the phase and extinction matrices averaged over all particle states in Eqs. (14) and (15), one has to require that all physically realizable particle states (sizes, shapes, orientations, refractive indices, etc.) be well represented in each volume element at any moment in

time. Since this requirement may imply an unrealistically large size of an elementary volume element, it has been concluded that the RTE may need a substantial modification when it is applied to scattering media such as terrestrial water clouds (e.g., Ref. [41]). However, this conclusion does not take into account the following important consequences of the microphysical derivation of the RTE: (i) the concept of an elementary volume element has no actual relevance to the radiative transfer theory and (ii) the RTE describes a time average of the directional flow of electromagnetic radiation rather than its instantaneous pattern. Therefore, the range of applicability of the RTE is significantly wider than what the phenomenological approach may imply.

Of course, there is nothing wrong with the conception of postulating certain basic physical laws. In fact, any advanced physical theory must ultimately be based on a self-consistent set of well-defined axioms and have the formal structure of a mathematical theory. The seemingly self-evident phenomenological concepts of radiative transfer have been taken for granted for more than a century and, with a few exceptions, have been traditionally presented as something that does not need proof. However, postulating phenomenological concepts such as the notion of the diffuse specific intensity or the Bouguer–Beer's extinction law has the adverse effect of implying that the transfer of electromagnetic energy in discrete random media is controlled by fundamental physical laws other than the Maxwell equations.

One might argue that the microphysical derivation of the RTE from the Maxwell equations in Refs. [6,16] is too complicated and, unlike a half-a-page phenomenological 'derivation', requires many pages of formulas and graphs. However, the microphysical approach has several decisive advantages. First, one can make certain that the radiative transfer theory does not need any basic physical postulates other than the Maxwell equations. Second, the exact physical meaning of all participating quantities and their relation to more fundamental physical quantities become clear and unambiguous. Third, the range of applicability of the RTE becomes well characterized.

Another phenomenological way to introduce the RTE is to invoke the Einstein's concept of photons, describe the radiation field in terms of a 'photon gas', and postulate that the photon gas satisfies the Boltzmann kinetic equation (see, e.g., Refs. [26,42]). This approach is based on associating energy transport with the directional flow of localized particles of light, photons, each carrying energy of amount $h\nu$, where h is Planck's constant and ν is frequency. The diffuse specific intensity is then given by

$$\tilde{I}_d(\mathbf{r}, \hat{\mathbf{q}}) = h\nu c f(\mathbf{r}, \hat{\mathbf{q}}),$$

where c is the speed of light and $f(\mathbf{r}, \hat{\mathbf{q}})$ is the photon distribution function such that $dS d\Omega c f(\mathbf{r}, \hat{\mathbf{q}})$ is the number of photons crossing an element of surface area dS normal to $\hat{\mathbf{q}}$ and centered at \mathbf{r} in directions confined to an element of solid angle $d\Omega$ centered around $\hat{\mathbf{q}}$ per unit time.

The concept of a photon as a localized particle of light was proposed by Albert Einstein in his 1905 paper on the photoelectric effect. Specifically, he suggested that the energy of a light ray spreading out from a point source is not continuously distributed over an increasing space but consists of a finite number of energy quanta which are localized at points in space (see Ref. [43]).

However, it is known from quantum electrodynamics that there is no position operator for a photon and that it is impossible to introduce a photon wave function in the coordinate representation (e.g., Section 2.2 of Ref. [44]). In fact, photons are quantum excitations of the normal modes of the electromagnetic field and as such are associated with plane waves of definite wave vector and definite polarization but infinite lateral extent [45]. This means that photons are not localized particles [46]. Thus, the quantum theory of radiation does not allow one to associate the position variable \mathbf{r} with a photon and even to speak about the probability of finding a photon at a particular point in space [47]. It is, therefore, impossible to define $f(\mathbf{r}, \hat{\mathbf{q}})$ as a function of photon coordinates and claim that it satisfies the Boltzmann transport equation.

Another fundamental problem with the 'photonic' approach is that it remains unclear why the phase and extinction matrices entering the RTE are still defined and computed in the framework of classical electrodynamics.

The concept of photons has been thoroughly misused in the phenomenological treatment of radiative transfer. The fact that photons are not localized particles (e.g., Section 4.10 of Ref. [48]; Section 88 of Ref. [49]; Section 5.1 of Ref. [50]) makes the words like "photon position", "photon path" or "local flow of photons" physically meaningless. Although the term "photon" is ubiquitous in quantum electrodynamics and quantum optics, there it means nothing more than a quantum of a single normal mode of the electromagnetic field [45].

Since the normal modes have an infinite lateral extent, they cannot be interpreted as ‘particles’. If the solution of a specific problem does require quantization of the electromagnetic field then the most one can say is that the photons represent a discrete character of light in that specific application but not a ‘particle’ character.

If one is tempted to use the word “photon” to describe a relatively localized ‘packet’ of radiation, it should be remembered that the Fourier analysis requires a wavepacket to consist of a superposition of normal modes. The drawback of this ‘particle’ interpretation is that each source emits its own kind of wave packets, which leaves one with a wide variety of analytical representations of a wave packet or worse, no analytical representation at all [51].

The quest for a photon as a universal localized quantum of light appears to be as hopeless now as it has ever been, as revealed by the October 2003 supplement to *Optics and Photonics News* titled “The Nature of Light: What is a Photon?” [52]. Unfortunately, most undergraduate textbooks on modern physics and even many graduate texts remain profoundly confusing and often misleading on this issue. Their authors keep relishing the so-called “wave–particle duality” of light which was discarded following the development of quantum electrodynamics seven decades ago. Furthermore, they appear not to realize that one does not need the concept of a photon as a particle of light to explain the photoelectric and Compton effects and that this concept is inconsistent with the Planck energy distribution law, the facts established in the 1910s and 1920s (see, e.g., Ref. [53] and references therein). An excellent remedy to these textbooks are the thorough discussions of the concept of a photon and its history in Refs. [46,53]. In Ref. [53], the authors boldly assert that elementary texts would do well to drop the corpuscular photon (except, perhaps, as a historical topic) and switch to the semi-classical treatment as the first approximation to the modern quantum electrodynamics approach.

Unfortunately, the word “photon” is invoked most commonly in circumstances in which the electromagnetic field is classical and has no quantum character whatsoever. The word “photon” then serves as nothing more than a catchy synonym for “light.” This usage of the word “photon” is especially misleading and should be avoided.

It is worth emphasizing again that the detailed microphysical derivation of the RTE in Refs. [6,16] leads quite naturally to the definition of the coherent and diffuse Stokes column vectors, clarifies the physical meaning of all quantities entering Eqs. (14) and (15) and makes unnecessary the multiple controversial assumptions of the phenomenological approach. In particular, it eliminates the need to introduce the troublesome and vague notion of an elementary volume element and avoids completely the use of the misleading ‘photonic’ language.

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