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Turbulent convection: is 2D a good proxy of 3D?

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Abstract. Several authors have recently carried out 2D simulations of turbulent convection for both solar and massive stars. Fitting the 2D results with the MLT, they obtain that $\alpha_{MLT} > 1$ specifically, $1.4 \leq \alpha_{MLT} \leq 1.8$. The authors further suggest that this methodology could be used to calibrate the MLT used in stellar evolutionary codes. We suggest the opposite viewpoint: *the 2D results show that MLT is internally inconsistent because the resulting $\alpha_{MLT} > 1$ violates the MLT basic assumption that $\alpha_{MLT} < 1$.* When the 2D results are fitted with the CM model, $\alpha_{MLT} < 1$, in accord with the basic tenet of the model.

On the other hand, since both MLT and CM are local models, they should be replaced by the next generation of non-local, time dependent turbulence models which we discuss in some detail.

Key words: convection – stars: interiors

1. 2D and 3D turbulence

It is known that turbulence is a 3D, not a 2D phenomenon. The latter is an interesting conceptual model that has challenged our understanding of the mechanism of non-linear interactions but physically, 3D turbulence is fundamentally different from the 2D counterpart. In 3D turbulence, energy is conserved while in 2D both energy and enstrophy are conserved. In the 3D case, energy in the largest scales dribbles down to increasingly smaller scales under the vortex stretching phenomenon. The "cascade process" does not stop until it reaches scales where molecular viscosity halts any further cascading, rather, it dissipates energy into heat. In the 2D case, the opposite occurs: energy climbs from the smallest to the largest scales piling up most of the turbulent kinetic energy in just a few large scales (the so-called anti Robin Hood effect, from the poor to the rich, while enstrophy dribbles down toward smaller scales). In the 2D case, the bulk of energy resides in a few very large scales, quite a different situation from 3D.

On the basis of these general arguments, one infers that a 2D model would overestimate the extent of overshooting: in fact, the piling of most of the energy into the largest scales, which govern a diffusive process like overshooting, also overestimates the extent to which they travel. Recent work on two different fronts lends support to these general ideas. On the astrophysical side, Schlattl & Weiss (1999), generalizing work of Blocker et al. (1998), used the 2D prescription for the extent of overshooting below the solar convective zone, but were unable to simultaneously reproduce the solar sound speed profile provided by helio-seismology as well as to account for the observed Li depletion.

From the numerical simulation point of view, Kupka & Muthsam (2000) have recently studied the case of an optically thick fluid with a prescribed radiative conductivity. The 2D simulations systematically overestimate the extent of overshooting vis a' vis the 3D result as well as the kinetic and potential energy, especially near the boundaries of the convective regions and in the overshooting region.

Thus, if 2D turbulence is not the correct physical template of 3D, why use it at all? The reason is practicality. We recall that in a 3D case the number of relevant dynamical scales is quite large: if L is the largest scale and ℓ is where viscous dissipation begins, one has the well known relation:

$$\frac{L}{\ell} \sim Re^{3/4}. \tag{1}$$

In the sun, for example, where conservatively $Re \sim 10^{12}$, one obtains

$$\frac{L}{\ell} \sim 10^{9}. \tag{2}$$

The first dissipation scale is a billion times smaller than the largest scale. The number N of grid points that a numerical simulation must resolve is given by the cube of (1) and thus

$$N \sim Re^{9/4} \sim 10^{27}, \tag{3}$$

which is some 18 orders of magnitude larger than

$$N \sim 10^{9}, \tag{4}$$

which is the best computers can do today. Translated in more physical terms, this means that large eddy simulations (LES) resolve numerically not even 1% of all the scales, leaving the bulk of them to be accounted for by a subgrid scale model (SGS), a misnomer borrowed from engineering turbulence where the much lower Re allows to numerically resolve eddies well inside the Kolmogorov region, something not easily achieved in stellar LES. Thus, LES must model more than 90% of the unresolved scales and no present SGS model is physically complete. For example, all present models assume that the SGS are purely dissipative while it can be shown quite generally (Canuto 2000) that they are dissipative, advective (stirring) and diffusive (mixing). Particularly deficient are the so-called hyperviscosity models which have been shown to have a "skill index" of barely 10%, meaning that once the LES results are compared with an eddy resolving model, the LES captures 10% of the real values (Gille & Davis 1999). Thus, both the inability of 3D LES to catch most of the scales and the difficulties associated with the SGS have made the 2D case very attractive since one can resolve many more scales thus alleviating considerably the burden of the SGS model to capture the unresolved scales, not to mention the concomitant saving of computer time.

The bad news is that 2D is at best a doubtful substitute for a 3D case.

2. The 2D solution

Ludwig et al. (1999; LFS) and Asida (2000) have carried out 2D simulations of turbulent convection and calibrated the mixing length for the sun and red giant envelopes. As one observes from Fig. 5 of LFS, the values of α_{MLT} derived from the 2D code are all larger than unity, the maximum being around 1.8. Asida (2000) also finds that $\alpha > 1$, specifically, 1.4 for a red giant of 1.2 solar masses. These results imply that the condition of validity of the Boussinesq approximation upon which the MLT model is based (Spiegel & Veronis 1960)

$$\alpha \equiv \frac{\ell}{H_P} < 1 \tag{5}$$

is violated. Stated differently, in order to reproduce the 2D data, the MLT must violate the basic condition for its existence, Eq. (5). It may be of interest to recall that in the case of the very convective earth's boundary layer where $\ell = 1km$, $H_P = 10km$, condition (5) is satisfied, thus justifying the use of the Boussinesq approximation.

Next, consider the CM model. It was constructed with the specific purpose of restoring at the very least some semblance with the real 3D convection. Because of the arguments presented earlier, this meant that one had to account for the large family of eddies that span the range given by (2). The specific turbulence model employed is immaterial since the same result was obtained using three different turbulence models. Since the CM is still a largely local model while by definition convection is non-local, a further attempt was made to introduce some non-locality via the relation

$$\ell = z + \alpha_{CMT} H_P, \tag{6}$$

which is non-local in the sense that what happens at a given z in a star depends on what is between that point and the "wall" where convection dies. Thus, far away points can influence local points. The left-over non-locality is parameterized with the second term in (6) which should satisfy (5). This is indeed the case, as Fig. 6 of LFS shows.

3. Different interpretation of the 2D results

We now offer our interpretation of the 2D results. The relevance of the work of Ludwig et al. (1999) and of Asida (2000) is that they show for the first time that in order to reproduce the 2D simulation data with the MLT, one has to chose an α_{MLT} that violates the premises of the MLT itself. We view this as another proof that the MLT is internally inconsistent and thus not a viable model while the previous authors interpret their $\alpha_{MLT} > 1$ as a way to calibrate the MLT for stellar codes. Since $\alpha_{MLT} > 1$ had already been used before, one could be tempted to interpret the numerical 2D results as an a posteriori justification. But, in our opinion, two wrongs do not make one right. Accepting $\alpha_{MLT} > 1$ is tantamount to sweeping under the rug an inconsistency under the claim that the model "fits" the data. Inconsistencies should be resolved, not overlooked or much less accepted. By contrast, the CM model faces the 3D problem quite directly and accounts for all the eddies. It is internally consistent and does not violate (5).

4. Life after local models

Since a value of $\alpha_{MLT} > 1$ had already been arrived at by empirically fitting the MLT to stellar data, one could have concluded long ago that such an α violates the basic tenet of MLT, $\alpha_{MLT} < 1$. The case was never made because it was repeatedly alleged that such an empirical "α" covered uncertainties other than those of convection. The 2D calculations of Ludwig et al. (1999) and of Asida (2000) make that excuse no longer tenable. They provide for the first time a one-to-one correspondence between models of convection and the 2D results, other astrophysical uncertainties being subtracted out. It is also important to stress that both the sun and red giants yield the same result, $\alpha_{MLT} > 1$. Thus, the MLT fits the data only with an α that is internally inconsistent while the CM model is internally consistent.

The next step is to abandon all local models in favor of non-local models which avoid altogether the introduction of parameters like α. What are the available choices? A 3D simulation is simply too time consuming to be routinely used in stellar structure calculations, notwithstanding the still unresolved problem of how well the subgrid scales have been represented thus far (Canuto 2000); a 2D simulation is not a reliable template of a 3D case, as the previous theoretical, numerical and astrophysical arguments have indicated. A key feature of any model that attempts to describe turbulent convection (astrophysical or otherwise) is non-locality. Since positive buoyancy overpowers gravity, large eddies can and do exist and are diffusive and advective, in contrast to small eddies that are mostly dissipative. As a counter example, we can cite the case of shear driven turbulence in the absence of convection: on average, the eddies are smaller and thus non-locality is less important (Kaimal & Finnigan 1994). A typical example of non-locality is the equation for the turbulent kinetic energy K:

$$\frac{\partial K}{\partial t} + \frac{\partial}{\partial z} F_{\kappa e} = P - \epsilon, \tag{7}$$

where

$$F_{\kappa e} = \frac{1}{2}\overline{q^2 w}, \qquad K = \frac{1}{2}\overline{q^2}. \tag{8}$$

Here, the second term on the left is the non-locality represented by the divergence of the flux of turbulent kinetic energy $F_{\kappa e}$, a third-order moment (TOM). The *local limit* corresponds to taking $P = \epsilon$, that is, production (P) equals dissipation (ϵ): *turbulence is dissipated where it is produced.* Both MLT and the CM model are based on the $P = \epsilon$ assumption. The non-local term, which may act as a source and/or as a sink of turbulent kinetic energy, represents a new dynamical feature. Once a non-local model is constructed, it is expected to reproduce key features of turbulent convection. Specifically:

1) the up/downdrafts (first discussed in geophysical not astrophysical flows, Haugen 1973),

2) Petrovay (1990) first pointed out that even quite general formulations of convection can reproduce the most general topological features of the up/down drafts,

3) geophysical studies that predate astrophysical studies have provided general rules to study the filling factors of the up/down drafts. The relevance of these studies is to exhibit the key role of the "skewness" of the velocity field:

$$S_w = \overline{w^3}/(\overline{w^2})^{\frac{3}{2}} \tag{9}$$

a third-order moment that governs the topological filling factor, namely the area σ occupied by the updrafts (or 1-σ for the downdrafts)

$$\sigma = \frac{1}{2}\left[1 - S_w(4 + S_w^2)^{-\frac{1}{2}}\right]. \tag{10}$$

Thus, a key challenge of any model is to compute S_w. However, since there are several TOM's

$$\overline{w^2\theta}, \quad \overline{w\theta^2}, \quad \overline{\theta^3}, \quad \overline{q^2 w}, \tag{11}$$

which are related to S_w, one must model all of them. The problem of constructing a reliable model for the TOMs has a long history in geophysical flows dealing with strong convection. Suffices to say that at present, there is only one analytical model capable of reproducing the LES data (Canuto et al. 1994; Zilitinkevich et al. 1999).

4) numerical simulations of turbulent convection exhibit several interesting features (for a clear presentation, see Cattaneo et al. 1991, especially Fig. 14c,d): the strong downflows transport heat upward at nearly the same rate that they transport kinetic energy downward, without actually contributing to the net energy transport. We must note, however, that Rieutord & Zahn (1995) have pointed that this may be due to the relatively low Re used in the numerical simulations. If confirmed by large eddy simulations, the main transport process to carry heat is the updrafts which is precisely the mechanism studied by all turbulent models for many decades. Turbulence thus provides well tested models to quantify the heat transfer by the updrafts.

A new non-local, time dependent model which underwent extensive testing on several types of turbulent flows (Canuto & Dubovikov 1998; CD98) has recently also been used to study astrophysical convection (Kupka 1999a,b). The CD98 model was shown to reproduce well the major features of convection (e.g., fluxes and filling factors) in a fraction of the time required by the numerical simulations thus opening the possibility of a hook-up with stellar codes. A full non-local, as well as compressible, model also exists (Canuto 1997) and it will next be solved and compared with numerical simulations results. More recently, Kupka & Muthsam (2000) have shown that depending on the specific problem studied, the results of the 2D simulations are worse or comparable to that of the CD98 model with the added advantage that the time required is a small fraction of the 3D/2D calculations.

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