

Impulse-response functions and anthropogenic CO₂

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Abstract. Non-linearities in the carbon cycle make the response to atmospheric CO₂ perturbations dependent on emission history. We show that even when linear representations of the carbon cycle are used, the calculation of time scales characterizing the removal of excess CO₂ depends on past emissions.

Introduction

Several authors have defined single time-scales to characterize the relaxation of the atmosphere to steady-state after a CO₂ perturbation. Lashof and Ahuja [1990] defined the atmospheric "effective residence time" of a CO₂ pulse emission as the average time needed by the ocean to completely absorb it. Working with the Maier-Reimer and Hasselmann (MRH) [1987] impulse-response function, they found a value of 230 years, and used it to calculate relative global warming potentials (GWPs) for various greenhouse gases. Their calculations were later adopted, with some modifications, by the IPCC panel on climate change [Houghton, 1990]. However, their results were dependent upon an additional, arbitrary time-constant of 1000 years, introduced into the original response function to eliminate problems of integration. Alternatively, Rodhe [1990] defined the "decay time" of atmospheric CO₂ as the time needed to reduce a given perturbation by a factor 1/e of its initial value. Calculations with the ocean model by Siegenthaler and Oeschger [1987] gave a value of 120 years. Recently, Moore and Braswell [1994] defined the "half-life" of atmospheric excess CO₂ as the time needed to reduce any given excess CO₂ perturbation by a factor of 1/2, once emissions have ceased. Using impulse-response functions derived from coupled ocean-biosphere models and the historical CO₂ emission data, these authors found "half life" values between 20 and 90 years, with a best estimate of about 30 years.

The fact that different time scales have been used to describe the removal of excess atmospheric CO₂ is linked to the very nature of the carbon cycle, characterized by several sinks acting together with different time constants [Houghton, 1990]. Insufficient knowledge about a missing sink [Siegenthaler and Sarmiento, 1992] contributes to widen the range of calculations. We recently showed that CO₂ removal can be described by a set of fundamental time scales which depend on emission history [O'Neill et al., 1994]. Caldeira [1994] had found this depend-

ence for a non-linear model of CO₂ oceanic uptake. In this work, we show that even when the simplest, linear representations of the carbon cycle are used, calculations of CO₂ removal depend on past emissions.

Response to a perturbation: adjustment time

An anthropogenic perturbation to a specified atmospheric CO₂ steady-state level may be defined as any excess concentration above that level, $\Delta C(t)$. Once emissions cease, the atmosphere will necessarily relax to a new steady-state. We propose to use the (mass-weighted) average time needed to restore steady-state atmospheric conditions as a time scale characterizing CO₂ removal, in addition to those previously described. We thus define the *adjustment time* as:

$$\tau_{\text{adj}} \equiv \frac{\int_{t_0}^{\infty} (\Delta C - a_0 I_{\text{tot}}) dt}{\Delta C_0 - a_0 I_{\text{tot}}}, \quad (1)$$

where ΔC is the time-dependent excess CO₂ concentration in the atmosphere; $t=t_0$ is the emissions cut-off time; ΔC_0 is the excess CO₂ present in the atmosphere at time $t=t_0$; a_0 is an asymptotic airborne fraction, with value between zero and unity; and I_{tot} is the total CO₂ emitted over the entire emission history, i.e.: from time $t=0$ up to time $t=t_0$. The term in the denominator represents total excess CO₂ removed by all sinks after emissions are cut off.

The adjustment time is introduced here in an attempt to generalize and unify the various definitions previously described. First, as it will be shown below, it is a generalization of the c-folding time to any decay function, thus extending the decay time of Rodhe [1990]. Second, it coincides with the residence time defined by Lashof and Ahuja [1990] in the case of zero airborne fractions. Third, the adjustment time is an average over all the time scales controlling relaxation to steady-state. Conversely, the "half-life" weights CO₂ removal on the shorter time scales in the system more than it does on the longer ones [Moore and Braswell, 1994].

Calculations of the adjustment time and of any of the other time scales obviously depend on the particular response function chosen to approximate the removal process. However, once such a choice has been made, calculations still depend on the emission scenario considered.

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Impulse-response functions

Perturbation experiments with three different ocean models [Maier-Reimer and Hasselmann, 1987; Sarmiento and Orr, 1992; Caldeira, 1994] suggest that within certain ranges of

emissions, absorption of anthropogenic CO₂ by the ocean—a non-linear process—may be approximated by linear impulse-response functions. As simplified models of oceanic uptake, these functions have thus been used by several authors to investigate the climatic impacts of CO₂ emissions [Harvey, 1989; Wigley, 1990; Lashof and Ahuja, 1990]. Although as CO₂ rises and climate warms they will not correctly predict future atmospheric concentrations [Caldeira, 1994], impulse-response functions might still be used to calculate GWPs, because non-linear effects controlling climatic forcing and oceanic uptake could to a first approximation cancel each other out [Caldeira and Kasting, 1993]. Impulse-response functions may be written as:

$$g(t) = a_0 + \sum_{k=1}^n a_k e^{-k_i t}, \quad (2)$$

where: $\sum_{k=0}^n a_k = 1$, and $\tau_i = 1/k_i$ are characteristic time-constants

[Lasaga, 1989]. The coefficient a_0 is the asymptotic airborne fraction. Indeed, non-zero airborne fractions are necessary to model CO₂ removal on the time scales of ocean circulation (several hundred to a few thousand years [Maier-Reimer and Hasselmann, 1987]). Further removal of excess CO₂ by carbonate dissolution of ocean sediments and silicate weathering (a process of a few thousand to a few hundred thousand years [Broecker and Peng, 1982; Berner et al., 1983]), is usually not considered in these linear models. Thus, the adjustment time calculated with such functions will describe CO₂ removal up to a few thousand years.

Impulse-response functions can be used inside a convolution integral to calculate any anthropogenic perturbation to steady-state, $\Delta C(t)$, caused by an emission function, $I(t)$ [Harvey, 1989; Wigley, 1991]:

$$\Delta C(t) = \int_0^t g(t-t') I(t') dt', \quad (3)$$

where $g(t)$ is the impulse-response function defined in (2). Because the system considered is linear, calculations do not depend on the pre-perturbed steady-state concentration.

We will use equations (2) and (3) to calculate the adjustment time of the atmosphere to both an impulse perturbation and historical CO₂ emissions. In our simplified approach, the atmosphere-ocean system is thus considered linear, while no potential effects of terrestrial sinks are included.

Steady-state response

First consider the system at steady-state. A pulse emission at time $t=t_0$ can be written as: $I(t) = I_0 \delta(t-t_0)$, where: $\delta(t-t_0)$ is Dirac's delta function, defined to be zero everywhere but for $t=t_0$. By using equations (1), (2) and (3), the steady-state adjustment time is found to have the following form :

$$\tau_{\text{adj, steady-state}} = \frac{\sum_{k=1}^n a_k \tau_k}{1 - a_0}. \quad (4)$$

Its value is given by a simple combination of coefficients in the impulse-response function $g(t)$. It does not depend on either the specified equilibrium level or the size of the pulse emission, however, because the system considered is linear.

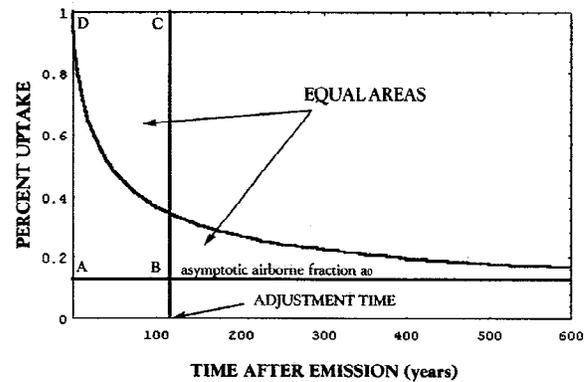


Figure 1. Steady-state response to a pulse-emission as simulated by the MRH impulse-response function. The steady-state adjustment time is obtained by calculating the mass-weighted integral of CO₂ removed with respect to time, according to (3). It defines the point in time where the two areas shown in the figure are equal. Thus the area between the decay curve and the asymptote corresponds to that of the rectangle ABCD. This geometric equivalence, rather than numerical reduction by a factor 1/e, is the physical definition of e-folding time. The adjustment time thus extends the concept of single exponential e-folding time to impulse-response functions by preserving its physical meaning.

Equation (4) generalizes the atmospheric residence time defined by Lashof and Ahuja [1990] to the case of asymptotic airborne fractions different from zero. Unlike the atmospheric residence time of Lashof and Ahuja [1990], its calculation does not require the introduction of an arbitrary time constant. In addition, Fig. 1 shows that the adjustment time properly extends the concept of e-folding time, thus generalizing the decay time defined by Rodhe [1990]. As an example, three different impulse-response functions were used to calculate and compare the adjustment time, the decay time of Rodhe [1990], and the half-life of Moore and Braswell [1994]. Results are shown in Table 1. We used the Maier-Reimer and Hasselmann [1987] (MRH), Sarmiento and Orr [1992] (SAR), and Caldeira [1994] (CAL) impulse-response functions, corresponding to pulse experiments of .25 times an initial atmospheric CO₂ equilibrium level (265ppm for MRH and CAL, 280ppm for SAR). The values of the adjustment time and the decay time were found to be very similar for the three functions used, showing the close relationship between the two time scales at steady-state. The value of the half-life was about 70% smaller than the other two time scales.

Table 1. Steady-state Adjustment Time, Decay Time, and Half-life for Three Impulse-response Functions.

MODEL	adjustment time	decay time	half-life
MRH	116.3	99.3	31.3
SAR	130.0	132.1	30.8
CAL	96.0	94.3	25.3

Response to an emission scenario

We next considered a continuous emission scenario, $I(t)$, satisfying the condition: $I(t)=0$ for time $t>t_0$. Again using (1), (2) and (3) we calculated the adjustment time to the emission scenario $I(t)$, or the time required to reach a new steady-state after emissions have ceased:

$$\tau_{\text{adj}} = \frac{\sum_{k=1}^n a_i \tau_i I_i e^{-k_i t_0}}{\sum_{k=1}^n a_i I_i e^{-k_i t_0}}, \quad (5)$$

$$\text{where: } I_i = \int_0^{t_0} e^{k_i t'} I(t') dt'.$$

Equation (5) clearly shows that the relaxation to steady-state of excess CO_2 in the atmosphere depends on the emission history prior to cut-off. It follows that the adjustment time to a continuous emission scenario does not coincide with the response to a single pulse. More specifically, equation (5) implies that:

$$\tau_{\text{adj, steady-state}} < \tau_{\text{adj}} < \tau_n, \quad (6)$$

where τ_n is the largest time constant in the impulse-response function. Equation (6) shows that linear atmosphere-ocean systems can absorb excess CO_2 most efficiently (i.e.: with the shortest adjustment time) when emissions are injected into the steady-state atmosphere in one single spike. The perturbation uptake efficiency decreases when total emissions are distributed over time, as CO_2 absorption becomes dominated by the longer time scales in the model. The largest time scale in the response function sets an upper limit for the CO_2 relaxation process.

An example: anthropogenic emissions

The adjustment time, the half-life, and the decay time were then calculated using the historical anthropogenic emissions and three impulse-response functions [Maier-Reimer and Hasselmann, 1987; Sarmiento and Orr, 1992; Caldeira, 1994], assuming emissions ceased in 1989. The atmosphere was assumed to be in equilibrium at 280ppm in 1850 [Watson et al. 1990]. Estimates by Keeling [1991] and Marland [1991] for fossil fuel emissions, and Houghton's [1991] estimates for land use were used. Calculations were only meant to be qualitative, showing the dependence of all time scales on the emission history prior to cut-off. As shown in Table 2, decay time and adjustment time calculations were found to be very different away from steady-state. Indeed, the decay time, as defined by Rodhe [1990], does not account for non-zero airborne fractions, thus overestimating

Table 2. Adjustment Time, Decay time, and Half-life, Calculated for Anthropogenic Emissions.

MODEL	adjustment time	decay time	half-life
MRH	175.2	274.0	68.5
SAR	247.5	449.1	79.0
CAL	149.2	232.3	79.4

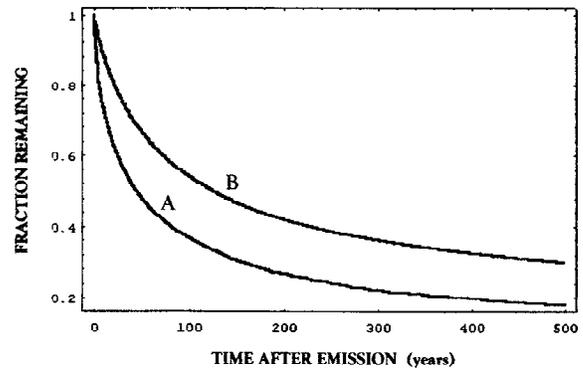


Figure 2. Residual forcing of anthropogenic CO_2 , calculated by using the MRH impulse-response function, and assuming a constant instantaneous forcing, $F=0.015 \text{ W m}^{-2} \text{ ppm}^{-1}$ [Lashof and Ahuja, 1990]. Quantities are normalized to show the different shape of the two curves. A) Forcing associated to any CO_2 emission pulse; B) Forcing of 1989 atmospheric excess CO_2 , calculated by assuming anthropogenic emissions ceased afterwards. The atmosphere was assumed to be in equilibrium at 280ppm in 1850 [Watson et al. 1990]. Estimates by Keeling [1991] and Marland [1991] for fossil fuel emissions, and Houghton's [1991] estimates for land use were used to calculate the various terms in equation (5).

the relaxation process on the time scales of ocean removal as emissions are spread over time. Increases in all time scales suggested that the current atmospheric CO_2 perturbation—about 70 ppm—could be absorbed by the ocean more slowly than predicted by steady-state calculations. This would occur because a significant amount of previously emitted anthropogenic CO_2 has already diffused into the ocean mixed-layer, slowing the net removal of additional CO_2 from the atmosphere. Specifically, as a result of the diminished uptake efficiency of the fastest sinks in the model, the adjustment time was found to increase 50% to 90%, while the half-life increased 120% to over 200%.

Our non-steady state half-life calculations are consistent with values found by Moore and Braswell [1994] when using the box diffusion model of Oeschger et al. [1975] (79 years), and the twelve-box ocean model of Bolin et al. [1983] (81 years). However, our results suggest that these values should not be used as absolute indicators of atmospheric CO_2 removal, as their value depends on the specific emission scenario considered.

Marginal response and forcing of anthropogenic CO_2

The fate of a single pulse emitted in a given emission scenario may be described by the marginal response time [Caldeira and Kasting, 1993]. This time scale is calculated by perturbing the emission function $I(t)$ in equation (3), and by performing a time-average on the corresponding change in $\Delta C(t)$. It can be shown that the marginal response time of linear impulse-response functions always coincides with the steady-state adjustment time defined by equation (4). As it has been previously noted, however, this time scale does not characterize relaxation to steady-state of general atmospheric CO_2 perturbations. It thus follows that, as illustrated by Fig. 2, forcing associated to a unit CO_2 emission and forcing of anthropogenic CO_2 already in the atmosphere are different.

Discussion and conclusions

We have proposed the adjustment time as a well-defined measure of the decay of anthropogenic CO₂ following an emission shut-off. This time scale is more balanced than either the decay time or the half-life, although its value depends on the choice of the asymptotic airborne fraction, as shown in equation (1). Also, the value of all three time scales does not specify how badly the carbon cycle system has been perturbed. In order to do this, the airborne fraction of additional pulses added on the margin of the perturbation should be calculated [Caldeira, 1994]. However, the adjustment time could be used at least qualitatively to assess the state of a perturbed system. This time scale was in fact shown to have a numerically well-defined range of variation, from its steady-state value (system only slightly perturbed), to the largest time constant in the removal function (system extremely perturbed).

We have shown that even in simple, linear models of the carbon cycle, time scales characterizing removal of excess CO₂ strongly depend on the emission scenarios considered. No single number indicating removal of excess CO₂ should thus be used indiscriminately across emission scenarios. Thus, pulse-experiment simulations used in the past to calculate removal time scales of anthropogenic CO₂ and global warming potentials do not necessarily apply to continuous emission scenarios. In turn, values calculated using specific emission data should not be taken to represent intrinsic, invariant properties of the carbon cycle itself.

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