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CRITERION FOR CONVECTION IN AN INHOMOGENEOUS STAR

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ABSTRACT

To resolve the question of whether the Schwarzschild criterion or the Ledoux criterion should be used to test for convective instability in a star, a well-observed cluster of chemically inhomogeneous massive stars, in which the choice of the criterion for convection makes a crucial and easily observable difference, is required. NGC 330, a metal-poor cluster in the Small Magellanic Cloud, is ideal for this test. Its large evolved stellar population contains both blue and red supergiants, of which its many red supergiants should be absent if a gradient of mean molecular weight did not choke off rapid convective motions in the inhomogeneous region connecting the envelope and core. Thus the Ledoux criterion for convection is strongly indicated as being correct.

Subject headings: convection — stars: interiors

A long-standing and fundamental problem in the theory of thermal convection is how to test for the outbreak of mass motions in inhomogeneous fluids. Should one use the Schwarzschild criterion based on the temperature gradient or the Ledoux criterion based on the density gradient? This problem becomes especially well posed in the interior of ordinary stars, which consist of ideal gases heated by one or more centrally concentrated energy sources. Related astrophysical situations, however, may arise in Earth's outer atmosphere (Spiegel 1969) and in the interiors of the giant planets (Stevenson & Salpeter 1977).

The classical criterion for stability against convective motions, which is derived by considering a small vertical displacement of a fluid element from its equilibrium position in a fluid of local density ρ, is written

$$-\frac{d\rho}{dr} > -\left(\frac{d\rho}{dr}\right)_{\rm ad}, \tag{1}$$

where $(d\rho/dr)_{\rm ad}$ is the adiabatic density gradient and r is distance from the origin. If the chemical composition of the fluid is uniform and pressure balance is maintained, this condition can be expressed in terms of the temperature T as

$$-\frac{dT}{dr} < -\left(\frac{dT}{dr}\right)_{\rm ad}. \tag{2}$$

This is the original criterion derived by K. Schwarzschild (1906). In a chemically inhomogeneous fluid, condition (1) becomes

$$-\frac{dT}{dr} < -\left(\frac{dT}{dr}\right)_{\rm ad} - Q\,\frac{T}{\mu}\frac{d\mu}{dr}, \tag{3}$$

where μ is the mean molecular weight and Q assumes the value unity for an ideal gas. This version of the criterion was introduced by Ledoux (1947). It is seen that an outward decrease of μ favors stability. In an ordinary nondegenerate star, dT/dr is simply the radiative temperature gradient.

A long debate has been carried on as to whether condition (2) or (3) should be used as the correct criterion for convection in a region of a star containing a strong gradient of mean molecular weight. The question arises because overstable oscillatory modes, which may lead to mixing, can exist even if the normal convective modes are neutrally stable according to criterion (3). Detailed considerations based on both local and global linearized stability analyses have variously favored the Schwarzschild criterion (Schwarzschild & Härm 1958; Kato 1966; Gabriel 1970; Shibahashi & Osaki 1976; Langer, Sugimoto, & Fricke 1983) or the Ledoux criterion (Sakashita & Hayashi 1959, 1961; Gabriel 1969; Auré 1971; Mimura & Suda 1971; Stevenson 1977). Laboratory and oceanographic experiments with salinity gradients in water, originally adduced by Spiegel (1969) as being relevant to the stellar case, are probably not applicable for a variety of physical reasons (Gabriel 1970; Stevenson 1977). Stellar evolution calculations themselves have led to ambiguous conclusions, chiefly because the stars that were studied contain a normal metal abundance, which suppresses large differences between the alternative modes of evolution up to all but the highest stellar masses, where a number of still unravellable problems (notably mass loss) arise (Stothers 1975, 1991; Stothers & Chin 1976; Fitzpatrick & Garmany 1990; Tuchman & Wheeler 1990).

A wider astrophysical window of opportunity opens when the initial stellar metal abundance, Z_e, is low. NGC 330, a cluster in the Small Magellanic Cloud, contains evolved supergiants of 9–14 $M_\odot$, whose envelope metallicity is 10 times smaller than solar (Carney, Janes, & Flower 1985; Reitermann et al. 1990; Barbuy et al. 1991; Spite, Richtler, & Spite 1991), thus implying $Z_e = 0.002$. Their envelope helium abundance is also known to be close to $Y_e = 0.24$ (Reitermann et al. 1990; Russell & Dopita 1990). At these rather moderate stellar masses, surface wind mass loss is evolutionarily unimportant (de Jager, Nieuwenhuijzen, & van der Hucht 1988), and convective core overshooting has recently been shown to be negligible (Stothers 1991; Stothers & Chin 1991). New radiative opacities (Rogers & Iglesias 1992) are now available that have led to a close agreement between calculated theoretical stellar models and critical observations of main-sequence stars (Stothers & Chin 1991) and classical Cepheids (Moskalik, Buchler, & Marom 1992) in the Galaxy. The time is therefore ripe for making a clear-cut test of the criterion for convection by use of the evolved massive supergiant stars in NGC 330.

Since close binary systems appear to be absent among the brightest stars in this cluster (Feast & Black 1980), the evolutionary histories of its supergiants are also uniquely uncomplicated by mass loss due to binary interaction (Roche lobe overflow).

The 24 supergiants in NGC 330 lie well away from the main sequence of stars burning core hydrogen on the Hertzsprung-Russell diagram (Carney et al. 1985), and are doubtless burning core helium, as the later phases of evolution are much too rapid to be readily observable. The evolutionary history of supergiants like these depends critically on whether or not a large fully convective zone (FCZ) has developed in the intermediate layers containing initially an unmixed or semi-convectively mixed gradient of mean molecular weight (Stothers & Chin 1968; Chiosi & Summa 1970). In massive stars, local stability of these layers is generally lowered by the prevailing high radiation pressure, which reduces both $(-dT/dr)_{\rm ad}$ and Q in inequalities (2) and (3). However, nonlocal effects eventually become important if convection actually breaks out, because the size of the FCZ depends on the available amount of helium-rich material that can be mixed upward; the total hydrogen and helium content in nonburning regions must be conserved, and semiconvection (slow mixing) may be too inefficient to preserve a marginal state of convective neutrality in all of the unstable layers. For the supergiants in NGC 330, we find that a large FCZ is always forced to develop shortly after the end of central hydrogen burning when the Schwarzschild criterion is adopted, but that no FCZ is able to originate at any time as a result of the more stringent Ledoux criterion.

After a very rapid convective mixing and chemical homogenization of material in the FCZ (if it is formed), a broad hydrogen plateau becomes permanently established over a large fraction of the inner stellar envelope just above the thin hydrogen-burning shell. This important structural modification inhibits the normal expansion of the surface layers. The star remains a hot (blue) supergiant and depletes core helium in that configuration. On the other hand, if an FCZ fails to form, the star evolves directly into a cool (red) supergiant, although it later executes a blue loop on the Hertzsprung-Russell diagram into the domain of blue supergiants. Tracks for two representative stellar masses illustrate the alternative possible evolutionary histories in Figure 1; the tracks run from the initial main-sequence stage to a stage near the end of core helium burning, and the heavy portions of the plotted lines represent the slow (readily observable) stages. The chosen evolutionary sequences turn out to be satisfactorily robust against changes in the input parameters; qualitatively the same results emerge even when we allow for the maximum estimated uncertainties in Z_e, Y_e, ^{12}C(α, γ)^{16}O reaction rate, mass-loss rate, and convective mixing length in the outer convection zone. Details of these sequences will be published elsewhere (Stothers & Chin 1992).

The stars in NGC 330 may be rotating rapidly, but even an extremely rapid rotation would not be expected to change the results qualitatively. For evolutionary sequences without FCZs this is known to be generally true (Kippenhahn, Meyer-Hoffmeister, & Thomas 1970; Meyer-Hofsmeister 1972). Since meridional circulation could only help mixing, this is probably also true a fortiori for the evolutionary sequences with FCZs, although rotation does slightly lower the luminosity and hence the convective instability. But the mass of a convectively equivalent nonrotating star is estimated to be lower by only 3%.

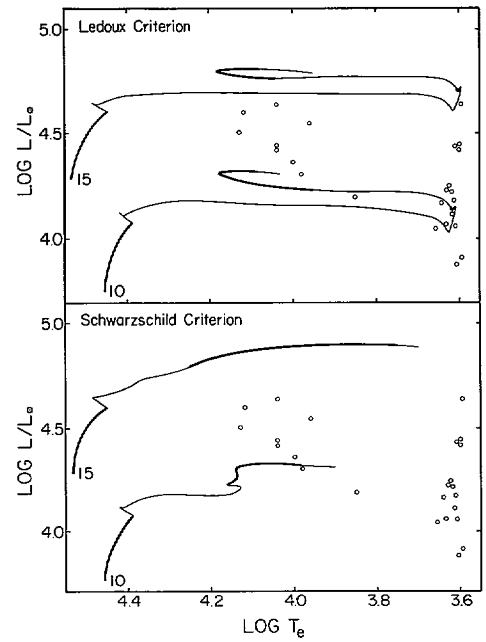

FIG. 1.—Hertzsprung-Russell diagram showing luminosity vs. effective temperature. Evolutionary tracks are plotted for stellar masses of 10 and 15 $M_\odot$, separately for the two different criteria for convection. (In the outer convection zone, the ratio of mixing length to local pressure scale height is taken to be $\alpha_P = 1.4$.) Slow phases of core hydrogen burning (in the main-sequence stars) and core helium burning (in the supergiant stars) are indicated by heavy segments of the lines. Supergiant members of NGC 330 are represented by circles.

Locations of the 24 observed supergiants in NGC 330 are also plotted in Figure 1. Observational data and transformation relations have been taken from Carney et al. (1985) (and references therein) except for the transformation of the blue supergiant data, for which the relations given by Fitzpatrick & Garmany (1990) have been used. The estimated uncertainty of the effective temperatures is less than 10%.

Notice that there are nine blue and 15 red supergiants in NGC 330. This overwhelming presence of red supergiants constitutes very strong evidence in favor of the Ledoux criterion and against the Schwarzschild criterion. This conclusion is supported further by the observed range of the effective temperatures of the blue supergiants. (However, it is not at present possible to predict, for the Ledoux criterion, the precise relative numbers of blue and red supergiants, because the fraction of time spent on the red supergiant branch is rather sensitive to the choices of the free input parameters; it is sufficient merely to note that many red supergiants are in fact predicted.) A curiosity of the present red supergiant models is that none of them has an outer convection zone that penetrates down into the nuclear-processed layers and thereby alters the original ratio of the surface abundances of nitrogen and carbon. This specific prediction, which is a consequence of the low metallicity in these supergiants, agrees with the "normal" N/C ratios detected by Barbuy et al. (1991) in two red supergiants and one blue supergiant in NGC 330.

Finally, we should point out that our conclusion about the correct criterion for convection seemingly implies that the Ledoux criterion ought to be used as the condition of convective neutrality in semiconvective regions of the star. Although this inference may be an oversimplification of a complex problem (Auré 1971; Stevenson 1977; Umezu & Nakakita 1988; Spruit 1992), much work has already shown that the altered μ gradient that is set up by semiconvection does not, by itself, lead to evolutionarily important structural changes in a massive star.

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