THE AMPLITUDE OF THE OPPOSITION EFFECT DUE TO WEAK LOCALIZATION OF PHOTONS IN DISCRETE DISORDERED MEDIA

(Letter to the Editor)

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Abstract. Weak localization of photons in discrete disordered media is considered as a possible physical mechanism of the opposition effect of some atmosphereless bodies. The amplitude of the opposition effect is calculated by using the rigorous *vector* multiple-scattering theory and the *scalar* approximation. It is shown that the scalar approximation can significantly overestimate the amplitude of the opposition effect. Thus, this approximation should not be used in interpreting the observational data, and some previous results obtained with this approximation may require substantial revision.

Photometric phase curves of many atmosphereless bodies in the solar system exhibit a sharp peak when the phase angle approaches zero. A number of physical mechanisms have been proposed to explain this strong opposition effect which results from the interaction of light with porous, powder-like surface layers or rough surfaces of the atmosphereless bodies (see, e.g., Lumme and Bowell, 1981; Hapke, 1986; and reviews by McGurn, 1990; and Muinonen, 1990). Possible relevance of the so-called weak localization of photons in discrete disordered media (or coherent back-scatter mechanism) to the opposition effect was first mentioned by Kuga and Ishimaru (1984). Analogous assumption was then made by Shkuratov (1988, 1991) and Muinonen (1989). Shkuratov's work is based on the *scalar* theory of multiple light scattering in a powder-like medium of independently scattering, randomly positioned discrete scatterers. An important rigorous result of this scalar theory is that in exactly the back-scattering direction (i.e., at zero-phase angle), the following identity holds (e.g., Ishimaru and Tsang, 1988) as

$$\gamma = \gamma^1 + \gamma^L + \gamma^C \equiv \gamma^1 + 2\gamma^L , \tag{1}$$

where γ is the total back-scattering coefficient in exactly the back-scattering direction, γ^1 is the contribution of the first-order scattering, γ^L is the contribution of all the other ladder diagrams, and γ^C is the contribution of all the cyclical (or maximally crossed) diagrams (for terminology, see Frisch, 1968; and Tsang *et al.*, 1985). Diffusely scattered intensity (or incoherent background intensity) comes from the sum $\gamma^1 + \gamma^L$, while the opposition peak comes from the coherent contribution γ^C. Thus, we have for the

amplitude of the opposition effect

$$\tilde{\zeta} = [\gamma^1 + \gamma^L + \gamma^C]/[\gamma^1 + \gamma^L] \equiv [\gamma^1 + 2\gamma^L]/[\gamma^1 + \gamma^L], \tag{2}$$

where $\tilde{\zeta}$ is the *scalar* back-scattering enhancement factor defined as the ratio of the total back-scattered intensity and the incoherent background intensity.

In the *vector* case (i.e., when the vector nature of light is fully taken into account), Equations (1) and (2) substantially alter. Let $\mathbf{S}$ be the total Stokes's back-scattering matrix in the common (I, Q, U, V)-representation of polarized light. We write

$$\mathbf{S} = \mathbf{S}^1 + \mathbf{S}^L + \mathbf{S}^C, \tag{3}$$

where (as earlier) 1 denotes the contribution of the first-order scattering, L denotes the contribution of all the other ladder diagrams, and C denotes the contribution of all the cyclical diagrams. If we assume that the scattering medium is composed of randomly positioned, independent discrete scatterers and using Saxon's (1955) reciprocity relation for the single-scattering amplitude matrix, one can express the elements of the matrix $\mathbf{S}^C$ in the elements of the matrix $\mathbf{S}^L$ (cf. Mishchenko, 1991) as

$$\mathbf{S}^C = \begin{bmatrix} S_{11}^C & S_{12}^L & 0 & 0 \\ S_{12}^L & S_{22}^C & 0 & 0 \\ 0 & 0 & S_{33}^C & S_{34}^L \\ 0 & 0 & -S_{34}^L & S_{44}^C \end{bmatrix}, \tag{4}$$

where

$$S_{11}^C = \tfrac{1}{2}[S_{11}^L + S_{22}^L - S_{33}^L + S_{44}^L], \qquad S_{22}^C = \tfrac{1}{2}[S_{11}^L + S_{22}^L + S_{33}^L - S_{44}^L],$$

$$S_{33}^C = \tfrac{1}{2}[-S_{11}^L + S_{22}^L + S_{33}^L + S_{44}^L], \qquad S_{44}^C = \tfrac{1}{2}[S_{11}^L - S_{22}^L + S_{33}^L + S_{44}^L]. \tag{5}$$

Equations (4) and (5) hold for randomly-oriented particles of any size, shape, and refractive index. From these equations, we have for the *vector* back-scattering enhancement factor

$$\zeta = [S_{11}^1 + S_{11}^L + S_{11}^C]/[S_{11}^1 + S_{11}^L] =$$

$$= [S_{11}^1 + S_{11}^L + \tfrac{1}{2}(S_{11}^L + S_{22}^L - S_{33}^L + S_{44}^L)]/[S_{11}^1 + S_{11}^L]. \tag{6}$$

It is well known (see, e.g., Prishivalko *et al.*, 1984, and references quoted therein) that the Bethe–Salpeter equation under the latter approximation of independent discrete scatterers results in the common vector radiative transfer equation (Chandrasekhar, 1950; Ishimaru and Yeh, 1984). Therefore, by solving this radiative transfer equation numerically, we can calculate the contributions $\mathbf{S}^1$ and $\mathbf{S}^L$ and then determine the back-scattering enhancement factor ζ for particular scattering models. Results of such calculations are given in Table I. The calculations have been performed for semi-infinite homogeneous slabs composed of spherical particles whose sizes are specified by the

TABLE I

Single-scattering albedos w, scalar back-scattering enhancement factors $\tilde{\zeta}$, and vector back-scattering enhancement factors ζ for homogeneous semi-infinite slabs composed of polydisperse spherical particles with $v_{\text{eff}} = 0.04$, $\text{Re}N = 1.6$, and $\lambda = 0.6$ μm

r_{eff}	$\text{Im}N = 0$			$\text{Im}N = 0.01$			$\text{Im}N = 0.03$		
	w	$\tilde{\zeta}$	ζ	w	$\tilde{\zeta}$	ζ	w	$\tilde{\zeta}$	ζ
0.1	1.0	1.93	1.58	0.925	1.83	1.64	0.804	1.71	1.63
0.7	1.0	1.72	1.45	0.848	1.40	1.35	0.676	1.32	1.31

standard gamma distribution

$$n(r) = \text{constant}\, r^{(1-3v_{\text{eff}})/v_{\text{eff}}} \exp[-r/(r_{\text{eff}} v_{\text{eff}})]\,, \tag{7}$$

with $v_{\text{eff}} = 0.04$ and $r_{\text{eff}} = 0.1$ and 0.7 μm. The free-space wavelength is $\lambda = 0.6$ μm and the real part of the refractive index is $\text{Re}N = 1.6$. The back-scattering enhancement factors ζ are given for the case of normal incidence of light on the boundary of the medium. Scalar enhancement factors $\tilde{\zeta}$ are given as well for the sake of comparison. Also, single-scattering albedos w of the particles are indicated. In the computations, we used the computational procedures that have been proposed and extensively described by Dlugach and Yanovitskii (1974), Domke (1975), de Rooij (1985), and de Rooij and van der Stap (1984).

From Table I, one sees that the scalar theory significantly overestimates the amplitude of the opposition effect for small and (or) non-absorbing or slightly absorbing particles. Therefore, this theory should not be used in interpreting the observational data, and the results of Shkuratov (1988, 1991) may require substantial revision.

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