Turbulent Atmospheric Plumes above Line Sources with an Application to Volcanic Fissure Eruptions on the Terrestrial Planets

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ABSTRACT

The theory of turbulent plumes maintained above steady line sources of buoyancy is worked out in detail within the limitations of Taylor's entrainment assumption. It is applied to the structure of a pure plume injected into a stably stratified atmosphere. Volcanic basalt eruptions that develop from long, narrow vents create line source plumes, which rise well above the magmatic fire fountains playing near the ground level. The eruption of Laki in 1783 may provide an example of this style of eruption. Flood basalts are more ancient examples. Evidence of enormous fissure eruptions that occurred in the past on Mars and Venus also exists. Owing to the different properties of the atmospheres on these two planets from those on the Earth, heights of line source plumes are expected to vary in the ratios 1:6:0.6 (Earth:Mars:Venus). It is very unlikely that the observed increase of sulfur dioxide above the Venusian cloud deck in 1978 could have been due to a line source volcanic eruption, even if it had been a flood basalt eruption.

1. Introduction

In a stable atmosphere, a localized source of heat creates a convective plume over the source by drawing ambient air into the incipient plume, warming it, and causing it to rise buoyantly. If the heating rate is sufficiently great, the upward flow will be turbulent. In general, the source emits fluxes of mass and momentum in addition to a flux of buoyancy. Since, however, the initial mass and momentum fluxes are continually being increased by the buoyancy force, the buoyancy-generated fluxes at large heights may dominate the forced convective input at the source (Morton 1959). For many practical purposes, therefore, it is sufficient to analyze the structure of a pure plume, standing well above the jetlike region near the source and maintained by a specified input of buoyancy flux.

Morton et al. (1956) have examined in this way the structure of an axisymmetric turbulent plume maintained above a virtual point source. Their main dynamical approximation was to adopt Taylor's entrainment hypothesis, according to which the mean lateral inflow velocity of the engulfed ambient fluid at a specified radius is simply proportional to the mean upward velocity of the fluid along the plume's central axis. Their very simple, semi-analytic model works surprisingly well for interpreting observations of plumes in laboratory experiments and plumes above outdoor fires (Briggs 1969) and even above large historic central-vent volcanic eruptions (Settle 1978; Wilson et al. 1978). It has also been used to estimate the heights of plumes generated by hypothetical central-vent eruptions on both Mars (Mouginis-Mark et al. 1982; Wilson and Head 1983) and Venus (Esposito 1984; Head and Wilson 1986).

With the confidence thus gained over the years in Taylor's entrainment assumption, it becomes worthwhile to study at the same level of approximation the structure of a plane-symmetric plume maintained above an infinite line source. This problem has previously been solved in the context of a homogeneous environment like the situation prevailing in typical laboratory experiments (Lee and Emmons 1961). Some limited and specialized results for stably stratified incompressible fluids under laboratory and marine conditions have also been published (Koh and Brooks 1975; Fischer et al. 1979). The present paper contains a derivation in detail of the plume solution for an environment consisting of a stably stratified compressible fluid medium, and applies the results to atmospheric plumes in particular.

A number of possible atmospheric applications of the line source plume model come to mind. A series of closely spaced smokestacks in a still atmosphere behaves like a line source. A line-fire in a burning forest provides another example if the atmosphere remains quiet enough. Here we consider volcanic basalt fissure-type eruptions for which the fissure length is large compared to the plume height. Observations show that, if the fissure eruption has an explosive character, it generates an energetic fire fountain up to hundreds of

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meters high that consists of a mixture of pyroclasts, fine ash particles, volcanic gases, and entrained air. Engulfed atmospheric gas rapidly heats up through efficient exchange of energy with the volcanic particles, expands, and rises in a turbulent plume above the fire fountain. Specific applications will be made here to elongate fissure eruptions on Mars and Venus in addition to those on the Earth for which results have already been published (Stothers et al. 1986). The present theory may also be useful for checking and interpreting the considerably more elaborate products of three-dimensional, time-dependent computer calculations (Wilson et al. 1987; Tripoli and Thompson 1988). However, it is not yet clear that a more detailed treatment gives reliably better results than does a simpler theory, as long as the real physics of turbulent entrainment and the actual amount of heat transfer from the fire fountain to the entrained air are not accurately known.

2. Turbulent buoyant plumes above line sources

Consider a turbulent plume maintained above a constant line source of buoyancy with a source length L that is much larger than the plume height H. The plume ascends into an infinite, neutral or stably stratified, inviscid, incompressible ambient medium. (The compressible case will be considered below.) Ascent is rapid enough that radiative and conductive exchanges of heat between the plume and the ambient medium can be ignored. For simplicity, vertical turbulent mixing of the plume fluid is also neglected. The irregular, fluctuating part of the flow and the forced development region near the source will not be considered, as the intent here is to look for a simple quasi-similarity solution for the mean flow aloft. Lateral expansion of the plume fluid away from the vertical plane containing the line source occurs by means of turbulent engulfment of ambient fluid. The mean vertical flow velocity at a height z above the source level and at a horizontal distance r from the source plane will be denoted by $w(z, r)$, and the fluid densities in the plume and in the ambient medium by $\rho(z, r)$ and $\rho_0(z)$, respectively. The standard reference density will be taken to be $\rho_1 = \rho_0(0)$, since the simplifying Boussinesq approximation is adopted here, in which density differences are ignored as being small except when involving the specific buoyancy force, $\Delta = g(\rho_0 - \rho)/\rho_1$, where g is the gravitational acceleration.

Horizontal profiles of the vertical velocity and buoyancy force are assumed to be similar at all heights and to have a Gaussian shape, so that

$$w(z, r) = w(z)\exp(-r^2/b^2), \qquad (1)$$

$$\Delta(z, r) = \Delta(z)\exp(-r^2/\lambda^2 b^2). \qquad (2)$$

Here $w(z)$ and $\Delta(z)$ represent values on the source plane. Laboratory experiments with line source plumes in uniform environments suggest the applicability of Gaussian profiles with $\lambda = 0.9$ (Rouse et al. 1952; Lee and Emmons 1961) or $\lambda = 1.35$ (Kotsovinos and List 1977). Since horizontal profiles for turbulent plumes are quite difficult to measure, however, it is simplest to adopt $\lambda = 1$, which in any case agrees well with the best measurements that have been made far from the experimental source (see Fig. 7 of Kotsovinos and List 1977).

The properties of the mean vertical flow can be determined from the three equations that express the conservation of fluxes of mass, vertical momentum, and buoyancy (Batchelor 1954). Mathematical closure of these equations is effected by Taylor's entrainment assumption, according to which the mean horizontal inflow velocity is equal to a constant α times the mean vertical flow velocity on the source plane. In the present two-dimensional geometry, the conservation equations must be integrated over an infinitely long cross-sectional strip area of element $L dr$. The reduced equations become

$$\frac{d}{dz}(bw) = 2\pi^{-1/2}\alpha w, \qquad (3)$$

$$\frac{d}{dz}(bw^2) = 2^{1/2} b\Delta, \qquad (4)$$

$$\frac{d}{dz}(bw\Delta) = -2^{1/2} bwN^2, \qquad (5)$$

where $N^2 = -(g/\rho_1)d\rho_0/dz$, the square of the Brunt–Väisälä frequency (or buoyancy frequency). Boundary conditions at $z = 0$ are $bw = 0$, $bw^2 = 0$, and $f = f_0$, where $f = F/L = (\pi/2)^{1/2} bw\Delta$, the specific buoyancy flux per unit length of source.

In the case of a medium with a uniform density, N^2 vanishes, and so the buoyancy flux remains constant with height. A similarity solution can be sought in this case. Inserting $b \propto z^n$ and $w \propto z^m$ into (3) and (4) shows that $n = 1$ and $m = 0$. This leads to

$$b = 2\pi^{-1/2}\alpha z, \quad w = \alpha^{-1/3} f_0^{1/3},$$
$$\Delta = 2^{-1/2}\alpha^{-2/3} f_0^{2/3} z^{-1} \qquad (6)$$

as was already known in regard to both the various functional dependences (Schmidt 1941; Rouse et al. 1952; Batchelor 1954) and the full forms (Lee and Emmons 1961).

For an inhomogeneous medium with a constant density gradient, it is convenient to introduce dimensionless variables $x \propto z$, $y \propto bw$, $v \propto bw^2$, and $u \propto bw\Delta$. Then (3), (4), and (5) yield a set of dimensionless equations

$$\frac{dy}{dx} = \frac{v}{2y}, \quad \frac{dv}{dx} = \frac{uy}{2v}, \quad \frac{du}{dx} = -y, \qquad (7)$$

with

$$z = (2^{-1}N^{-1}\alpha^{-1/3} f_0^{1/3})x, \qquad (8)$$

$$b = (2\pi^{-1/2}N^{-1}\alpha^{2/3}f_0^{1/3})y^2/v, \quad (9)$$

$$w = (\alpha^{-1/3}f_0^{1/3})v/y, \quad (10)$$

$$\Delta = (2^{-1/2}N\alpha^{-1/3}f_0^{1/3})u/y, \quad (11)$$

and, at $x = 0$, the boundary conditions $y = 0$, $v = 0$, and $u = 1$.

These equations can be solved once and for all by performing a step-by-step numerical integration forward from $x = 0$. To start the integrations, a series solution around $x = 0$ is adopted from (6), because Morton (1959) has shown that the lowermost layers of a convective plume behave very nearly as if they were embedded in a uniform medium. For small x, we find $y \approx x/2$, $v \approx x/2$, and $u \approx 1$. Numerical integration from this initial point on demonstrates that the buoyancy flux vanishes at $x = 2.04$ and that the vertical velocity does so at $x = 2.96$ (Fig. 1). The distance of overshooting beyond the level of vanishing buoyancy until the fluid finally comes to rest is, therefore, almost a third of the total plume height. However, as the entrainment assumption is very poor for small vertical velocities and as some ambient fluid must be entrained at the top of the plume, the definition of the plume top cannot be made very precise. It is customary to adopt the formal level where the vertical velocity goes to zero, because this choice agrees reasonably well with the results of applicable laboratory measurements for density stratified environments and is often used to deduce the value of the entrainment constant (List 1982).

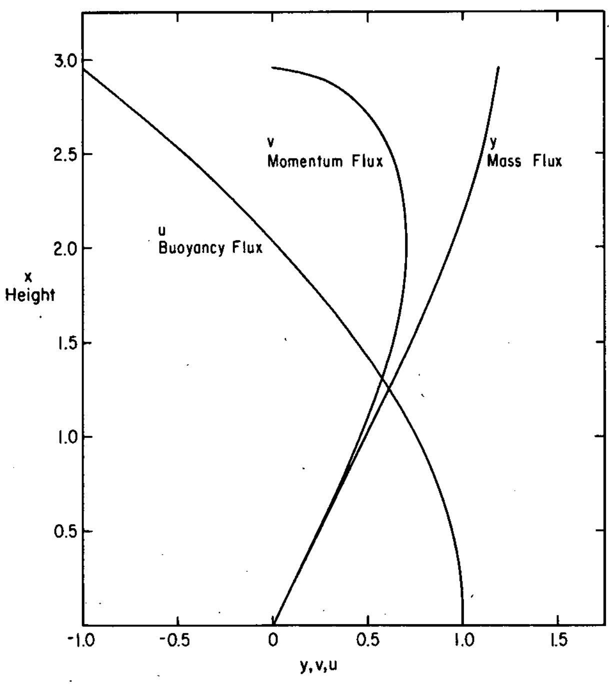

FIG. 1. Profiles of dimensionless fluxes of mass, vertical momentum, and buoyancy, as a function of the dimensionless height for a line source plume.

Accordingly, the physical height of the plume top is

$$H = 1.48N^{-1}\alpha^{-1/3}f_0^{1/3}. \quad (12)$$

Here, H is probably relatively well determined because of its weak dependences on the details of the entrainment physics (through $\alpha^{-1/3}$) and on the density stratification (through N^{-1}). The characteristic rise time of the plume fluid follows from (6) and (12) as

$$\tau = H/w_0 = 1.48N^{-1}. \quad (13)$$

In the present two-dimensional geometry, the vertical velocity has a finite value at the source, diminishes slowly to a fraction 0.75 of its original value at the level of zero buoyancy, and then decreases rapidly at higher levels (Fig. 2). The aspect ratio b/z is approximately equal to 1.13α near the source and increases to 1.39α where the buoyancy force vanishes. It rapidly grows above this level and becomes formally infinite where the fluid comes to rest. If the real line source has a finite width $2b_0$, the "virtual" line source lies at a distance $z_0 = 2^{-1}\pi^{1/2}\alpha^{-1}b_0$ below the finite source in a lowest-order calculation; the solutions just obtained are valid as long as $H \gg z_0$.

Laboratory measurements of α for line sources in uniform environments have yielded values of 0.16 (Rouse et al. 1952; Batchelor 1954; Lee and Emmons 1961) and 0.11 (Kotsovinos and List 1977). Ellison and Turner's (1959) measurements were made with the express assumption of top-hat horizontal profiles of vertical velocity and buoyancy force, and so are not directly applicable. A mean value of $\alpha = 0.13$ will be adopted here.

3. Atmospheric and heat-source assumptions

Before applying the present plume theory to observations of volcanic line source plumes in real atmospheric environments, all of the physical assumptions have to be checked as to their approximate validity, or else they must be modified.

1) The emitting heat source is essentially constant in time. Since the rise time of a volcanic plume is only a few minutes whereas sustained eruption intensities typically persist for hours to days, this assumption is usually justified. Discrete explosions occurring very close together in time may act like a continuous source.

2) The volcanic vent is very narrow, long, and straight. What matters most are the ratios H/b_0 and H/L. If these are large, the present theory is applicable. A very closely spaced series of circular vents may behave like a line source.

3) The gas thrust region at the bottom of the plume can be ignored. Provided that the plume height is sufficiently large, memory of all but the amount of buoyancy flux produced at the source is lost in the overlying plume structure (Morton 1959).

4) The convective plume decouples from the fire

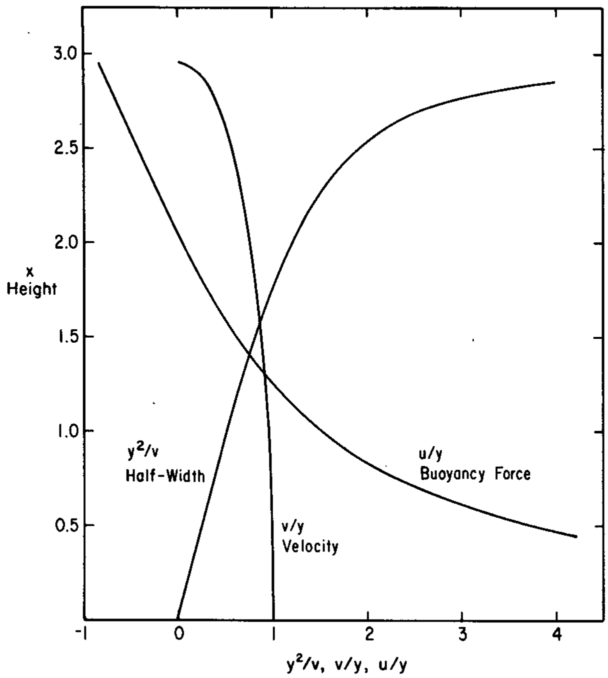

FIG. 2. Profiles of dimensionless half-width, vertical velocity, and buoyancy force, as a function of the dimensionless height for a line source plume.

fountain at the top of the fountain. If it does not decouple, entrained finely divided solids can be expected to complicate the heat balance in the plume by continuing to surrender thermal energy to the entrained air above the fountain. This will extend the plume further. However, the contribution from entrained solids is expected to be relatively small.

5) The plume flow is turbulent. Volcanic eruptions release so much heat into the atmosphere that enormous, fast-moving eddies form. The Reynolds number, $\mathrm{Re} = wb\rho/\eta$ where η is viscosity, is usually much larger than 10^2.

6) Laboratory measurements of α and λ (as well as of the basic horizontal profile shapes for velocity and buoyancy force) are applicable to turbulent atmospheric plumes. The plume height predictions and basic profile shapes have been confirmed at least for axisymmetric plumes rising above virtual point sources, in the case of both man-made fires (Briggs 1969) and volcanic eruptions (Wilson et al. 1978; Sparks and Wilson 1982).

7) The fluid is incompressible. Compressibility of the gases can be easily incorporated into the theory by using potential density instead of actual density (Morton et al. 1956).

8) The Boussinesq approximation is adequate. In reality, atmospheric density declines with increasing altitude to satisfy hydrostatic equilibrium between the pressure force and gravity. The effect of this decrease is somewhat, though not wholly, taken into account when the actual density is replaced by the potential density in the expression for N^2, which contains the relative density gradient. Observations suggest that the relative gradient of potential density is roughly constant both in the troposphere and in the nearly isothermal stratosphere, although it has a significantly larger value is the stratosphere. Very tall plumes, therefore, will have their predicted heights somewhat overestimated if tropospheric conditions are assumed. On the other hand, relevant theoretical calculations for point source plumes confirm that these additional density effects exert only a small influence on H (Sparks 1986).

9) Shock effects are negligible. The plume flow becomes supersonic only for eruption rates that exceed roughly 10^2 times that of a very large flood basalt eruption like the Roza flow eruption (see below).

10) The gravitational acceleration is constant. Even the tallest volcanic plumes have heights that are only a small fraction of the Earth's radius; in any case the dependence of plume height on gravity is very weak ($H \propto g^{-1/6}$).

11) A quiet atmosphere prevails. The mean vertical velocity at the base of a tall volcanic plume is so great that all but the strongest ambient cross-winds can usually be ignored in comparison. Only the upper part of the plume might be significantly bent and reduced in height.

12) Local environmental conditions such as wind patterns, air temperatures, and air densities are not significantly affected by the plume well outside the plume. The largest known flood basalt eruptions, however, have possibly violated this assumption (Stothers et al. 1986; Tripoli and Thompson 1988).

To evaluate the total plume height in a compressible, dry, stable atmosphere, it is necessary to reexpress N^2 and F_0 as

$$N^2 = \frac{gS}{T_1}, \quad F_0 = \frac{gQ}{s_a T_1 \rho_1}, \tag{14}$$

where $S = \Gamma + dT_0/dz$ = static stability, $\Gamma = g/s_a$ = absolute value of the dry adiabatic lapse rate, T_1 is the atmospheric temperature at the source level, s_a the specific heat at constant pressure for air, and $Q = (\pi/2)^{1/2} b_0 L w_0 (\rho_0 - \rho)(s_a T_1)$ = enthalpy flux at the source; N^2 and F_0 can also be reexpressed in terms of the local pressure scale height, $h_1 = RT_1/g$ where R is the gas constant for air.

Thermal energy release and consequent heating of entrained air by hot pyroclasts, fine ash particles, and magmatic gases within the volcanic fire fountain provide the enthalpy flux at the "source" (the fountain top). Subsequent heating by the fallen magma is probably a relatively minor contribution (Stothers et al. 1986). The physics of what goes on in the fire fountain is complex, but most of the energy released probably is used up in driving the plume (Sparks and Wilson 1976). From the present point of view, the fire fountain can simply be regarded as a "black box," which supplies a certain heat output. Accordingly, the enthalpy flux may be here represented by

$$Q = \sigma(1 - X - Y)Vs_m\Delta T_f + \sigma XVs_m\Delta T_a + \sigma YVs_w\Delta T_a, \quad (15)$$

where σ is the magma density, V the total volumetric rate of eruption of magma (mass rate divided by σ), s_m the specific heat at constant pressure for basaltic magma, s_w the specific heat at constant pressure for steam, ΔT_a the temperature drop of fine ash particles and magmatic gases in the plume, ΔT_f the temperature drop of clasts in the fountain, X the weight fraction of fine ash particles, and Y the weight fraction of volatiles (assumed to be pure H_2O). It will be of sufficiently general validity to adopt $\sigma = 2900$ kg m^{-3}, $s_m = 1100$ J kg^{-1} K^{-1}, $s_w = 2000$ J kg^{-1} K^{-1}, and $\Delta T_a = 1150$ K (Stothers et al. 1986). Then, since Y is always small,

$$Q = 6.7 \times 10^9[Y + 0.55X + 0.00048(1 - X)\Delta T_f]V, \quad (16)$$

where Q is in watts and V is in cubic meters per second.

4. Terrestrial fissure eruptions

Adopting standard constants for the Earth's atmosphere, together with $S = 3.3$ K km^{-1}, we find

$$H = 8.1q^{1/3}, \quad (17)$$

with H in meters and $q = Q/L$ in watts per meter length of source. The numerical coefficient differs slightly from that quoted by Stothers et al. (1986) owing to their use of $\alpha = 0.093$.

Measurements of plume heights and volume eruption rates in the case of observed basaltic fissure eruptions are rather sparse (for collected data and a discussion, see Stothers et al. 1986). Scattered observations for Mauna Loa 1984 and Askja 1961 suggest that H/L was $\sim$4 and $\sim$10, respectively, so that a line source model is inappropriate in these two cases. Only Laki 1783 might be useful here (but see Thordarson et al. 1987). This large Icelandic eruption sent up 12 km^3 of lava from a closely spaced series of vents on a fissure 13 km long and a few hundred meters wide. Typical peak eruption rates were $V \sim 10^4$ m^3 s^{-1} (Thorarinsson 1969). According to (16) with likely values of $\Delta T_f = 20$°K, $X = 1\%$, and $Y = 0.5\%$ (Stothers et al. 1986), the volcanic heating rate must have been $q \sim 10^8$ watt m^{-1}, which Eq. (17) predicts would have raised a plume to $H \approx 4$ km if the whole fissure was simultaneously active. [Possibly, however, fissure lengths of only 2 to 5 km were active at any one time during the peak phase (Thordarson et al. 1987).] Observationally, the Laki plume is estimated to have attained at least lower Alpine heights ($H \geqslant 3$ km) but not the summertime Icelandic stratosphere ($H < 11$ km) (Stothers et al. 1986). General compatibility between theory and observation is probably the most that one can expect here in view of the extreme crudeness of the observational data. Vertical flow velocities near the base of the plume would have averaged $w_0 \approx 30$ m s^{-1} according to (13).

The enormous eruption that produced the Roza basalt flow (700 km^3) of the Columbia River Basalt Group of lava flows in the Miocene Epoch probably generated lava at an average rate of 10^4 to 10^5 m^3 s^{-1} from 1 to 10 km lengths of fissure, along a total interval of 100 km (Swanson et al. 1975). If X was 1% or greater, the implied heating rate of $q \sim 10^9$ W m^{-1} would theoretically have led to plumes reaching into the lower stratosphere (Stothers et al. 1986). However, this conclusion is not very firm because the data are very rough and small changes of the estimated heating rate or of the static stability can prevent having the plume penetrate the stratosphere.

An even greater basalt eruption produced the Greenstone flow (1650 km^3) of Michigan's Portage Lake Lava Series during the Precambrian Era (Longo 1983). If this flow was in fact a single eruptive unit, the total fissure length must have been at least 80 km. The average rate of lava production would have been of the same order of magnitude as that of the Roza flow.

5. Martian fissure eruptions

Evidence for past volcanism on Mars, as inferred from observations by the Mariner and Viking spacecraft, is very extensive (e.g., Carr 1981). In addition to large structures characteristic of central-vent eruptions, landforms include vast volcanic plains, which are analogous to the flood basalt plateaus on Earth and have apparently been formed episodically throughout Martian history (Greeley and Spudis 1981). Long sinuous rilles, extending for over 50 km near the boundaries of the lava-flooded plains, also exist and were possibly caused by huge eruptions involving basaltic mass fluxes of 10^8 to 10^9 kg s^{-1}, which continued for several months (Wilson et al. 1982, 1987). Since, however, considerable thermal energy must have been used up in melting and eroding the ground surface (Carr 1974), the amount of heat released to the atmosphere cannot be accurately estimated without further data on initial ground heating and on the ratio Q/V for Martian fire fountains. With no initial ground heating and under terrestrial conditions, q would have been of the order of 10^8 to 10^9 watt m^{-1}.

On a very similar geologic scale and extending up to distances of 200 km are a number of long strings of dark patches, which appear to consist of mafic pyroclastic erupted material, possibly produced in a Martian analog of terrestrial fire fountain activity (Lucchitta 1987). These dark patches occur within the tectonic troughs of the Valles Marineris rift system and are almost certainly young, probably less than a million years old. Linear constructional features on Alba Patera are older, but of comparable length, and probably have a similar origin (Cattermole 1986).

Since it is likely that most Martian lavas are mafic in composition (Carr 1981) with their volatiles dominated by a juvenile water content of <1% by weight (Greeley 1987) (although a significant CO_2 volatile content cannot yet be ruled out) (Mouginis-Mark et al. 1982), the ratio Q/V for hypothetical Martian fire fountain activity probably differs from the analogous ratio on Earth mainly as a result of the different planetary near-surface atmospheric conditions. The smaller surface atmospheric pressures on Mars will cause a faster rise, greater fragmentation, and wider dispersal of the erupting magma than on the Earth (Wilson and Head 1983). Mars's lower surface gravity, together with the greater eruption velocities, should raise the heights of fire fountains above the levels attained on Earth, and should also reduce the rate of fall of fountain clasts. Increased exposure time in the air, a larger total surface area presented by the more-highly dispersed and fragmented fountain clasts, and the colder air than prevails on Earth are all expected to promote a larger transfer of heat from the Martian fire fountains to the entrained air, despite the lower Martian air density. This will increase Q for a given V, although probably not by an order of magnitude.

Atmospheric parameters for average Martian conditions are listed in Table 1. They have been taken from Allison and Travis's (1986) updated version of Zurek's (1976) empirical model of the dust-free CO_2 Martian mean atmosphere. The basic state temperature decreases exponentially from its surface maximum to an asymptotic minimum about 30% lower, which is very nearly attained by an altitude of 40 km. There is thus no distinct tropopause. The near-surface atmospheric layers are close to being adiabatic, but the static stability averaged over the lowest two scale heights (22 km) is $S \approx 2$ K km^{-1}. Therefore, for a turbulent line source plume on Mars,

$$H = 53q^{1/3}, \quad (18)$$

TABLE 1. Empirical atmospheric parameters and theoretical plume heights. Q is in watts, q is in watts per meter, and H is in meters. For a line source plume $\alpha = 0.13$ was used, and for a point source plume $\alpha = 0.083$ was adopted.

Quantity	Earth	Mars	Venus
g (m s^{-2})	9.80	3.71	8.87
P_1 (bar)	1.0	0.007	92
ρ_1 (kg m^{-3})	1.3	0.017	66
T_1 (K)	288	214	731
h_1 (km)	8.4	11	16
Γ (K km^{-1})	9.8	4.4	10.5
S (K km^{-1})	3.3	2	1
s_a (J kg^{-1} K^{-1})	1000	845	859
$2\pi N^{-1}$ (min)	9.9	18	30
Line, $Hq^{-1/3}$	8.1	53	5.0
Point, $HQ^{-1/4}$	8.7	35	6.0

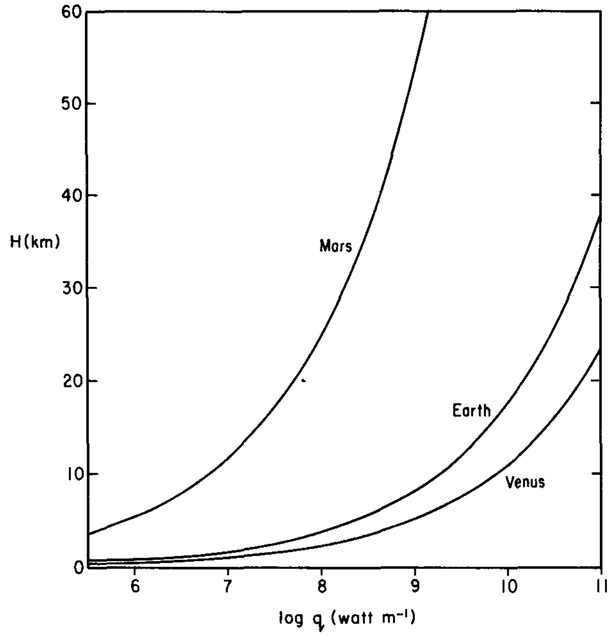

FIG. 3. Height of the top of a line source plume in the atmospheres of Earth, Mars, and Venus, as a function of the logarithm of the heating rate at the source per unit length of source.

which is about six times the analogous terrestrial value (Fig. 3). Most of the difference arises from the much smaller atmospheric densities on Mars.

For a volcanic heating rate of $q \sim 10^8$ W m^{-1} (comparable to Laki's in 1783), a turbulent line source plume on Mars would rise some 25 km. At the larger rate of a terrestrial flood basalt eruption, it would probably attain about 50 km. To put this height in a different perspective, the height that the optically thick dust layers reached during Mars's great global dust storm in 1971 was 40–70 km (Leovy et al. 1973).

6. Venusian fissure eruptions

The cloud-covered surface of Venus, although not yet fully mapped by radar, displays strong evidence of past volcanism on a large scale (e.g., Masursky et al. 1980; McGill et al. 1983; Barsukov et al. 1986). Attention so far has focused on the Beta-Phoebe region, where a linear trough system bears a striking morphological resemblance to the East African Rift system on Earth (Malin and Saunders 1977). Gravity anomaly measurements by the *Pioneer Venus* orbiter also suggest the presence of volcanic terrain in the Beta region (Reasenberg et al. 1982), and measured albedos and radioactivities of exposed rocks sampled in situ by the Venera landers resemble those of certain oceanic basalts on Earth (Golovin et al. 1983; Florenskiy et al. 1983; Surkov 1983). Impulsive low-frequency electrical signals detected by the *Pioneer Venus* orbiter and interpreted (rather controversially) as lightning whistlers appear to emanate from the Beta-Phoebe region and therefore suggest active volcanism there (Scarf and

Russell 1983; but see Taylor et al. 1987). Even more speculatively, volcanic eruptions have been suspected as being responsible for the episodic increases of SO_2 gas and H_2SO_4 aerosol haze that have been observed spectroscopically and polarimetrically above the Venusian cloud layer, for example in 1959 and 1978 (Esposito 1984; Esposito et al. 1988; Ksanfomaliti 1984). Comparison of atmospheric and surface-rock abundances of sulfur also suggests that volcanism has been active during the past one million years (Fegley and Prinn 1989).

Although current volcanic activity, if it exists, seems to be confined to Beta Regio's irregular shields, such a geographically restricted interpretation is certainly not required. Moreover, the apparent riftlike nature of some of Venus' visually prominent troughs (Schaber 1982), the topographic evidence for basaltic lava flows covering many of Venus's smooth, shallow basins (Masursky et al. 1980), and more general analogies with the Earth (Wood and Francis 1988) suggest that large fissure-type eruptions have probably played a role in the past.

If so, the much higher surface atmospheric temperatures and pressures on Venus as compared to the Earth would ensure smaller eruption velocities and also less fragmentation and less dispersal of the erupted magma (Wilson and Head 1983; Head and Wilson 1986). These characteristics of the Venusian fire fountains, along with the already elevated temperature of the entrained air, would lessen the amount of heat transfer to the atmosphere. On the other hand, the entrained air has a much higher density on Venus, which would enhance the rate of heat transfer. It can be tentatively concluded that the ratio Q/V for Venus would probably not differ by as much as an order of magnitude from its terrestrial value. In any case, it is still not known whether H_2O or CO_2 is the dominant magmatic volatile, or even what their combined abundance is.

Seiff (1983) has proposed a working empirical model for the CO_2 Venusian mean atmosphere. Below the cloud tops, which lie at an altitude of 65 km, the atmosphere is mostly stable, but possibly contains two or three convective layers (see also Avduevskiy et al. 1983). The average value of the static stability in this complex region of monotonically decreasing temperatures is $S \approx 1$ K km^{-1}. At the cloud tops a distinct tropopause occurs, where the static stability is very high. Parameters for the Venusian atmosphere have been summarized by Allison and Travis (1986), and are reproduced in part in Table 1. A turbulent line source plume in such an atmosphere will rise to a height

$$H = 5.0q^{1/3}. \qquad (19)$$

This is close to the analogous plume height attained on Earth (Fig. 3) since the Venus atmosphere's relatively low static stability is roughly counterbalanced by its very high mass density.

If a line source plume on Venus had a real height that was considerably less than 10 km, Eq. (19) would yield a gross underestimate of it since the planet's lower atmosphere is probably close to being adiabatic ($S = 0$). Nevertheless, for any reasonable assumed heating rate up to 10^{10} W m^{-1}, the height of a line source plume on Venus is unlikely to surpass 15 km, based on the assumption of an average value of $S = 0.5$ K km^{-1} up to 15 km. It would therefore appear that the observed increase of sulfur dioxide above the Venusian cloud deck in 1978 could not have been caused by a large elongate fissure eruption. However, a large central-vent eruption remains a possibility (Esposito 1984), especially if the rise of the plume was aided by convective updrafts in the two or three unstable layers that exist below the cloud tops.

7. Turbulent buoyant plumes above point sources

The theory of a maintained convective plume above a point source of buoyancy has been worked out by Morton et al. (1956). Their result for the height of a turbulent plume in the Earth's atmosphere is listed in Table 1, where the numerical coefficient has been adjusted slightly to accommodate recent revisions of the entrainment constant, which is here taken to be $\alpha = 0.083$, valid for a point source (Fischer et al. 1979; Turner 1986). Analogous results for plumes in the Martian and Venusian atmospheres are also listed, and agree substantially with what Mouginis-Mark et al. (1982) and Esposito (1984) have obtained by similarly using the Morton et al. theory. Notice that H is proportional to the fourth root of Q for a point source of buoyancy. The heights of point source plumes on Earth, Mars, and Venus occur in the ratios 1:4:0.7.

8. Summary and conclusions

Taylor's entrainment assumption has been used to calculate the properties of a turbulent plume maintained above a constant line source of buoyancy. The ambient fluid medium is assumed to be stably stratified with a constant density gradient. The predicted total height of the plume up to the level of zero vertical velocity is $H = 1.48N^{-1}\alpha^{-1/3}f_0^{1/3}$, where N is the buoyancy frequency, α the entrainment constant, and f_0 the specific buoyancy flux at the source per unit length of source. The mean upward flow velocity near the source is $w_0 = 0.675HN$, and the aspect ratio b/z (where z is altitude above the source and b is a characteristic Gaussian half-width) remains close to 1.2α between the source and the level of vanishing buoyancy at $z = 0.688H$.

Erupting volcanoes are heat sources that provide enormous fluxes of buoyancy through their ejection of fountains of hot pyroclasts, fine ash particles, and magmatic gases which warm the turbulently entrained, ambient air. If the volcanic vent is long and narrow, the present theory may be validly applied to the plume that is formed above the fire fountains. Laki, which erupted in 1783, and some prehistoric flood basalt outbursts are the best known examples of such eruptions on the Earth. Mars and Venus appear to offer other examples of large fissure-type eruptions. Since the atmospheric properties of Earth, Mars, and Venus differ, however, the predicted heights of line source plumes on these planets vary in the ratios 1:6:0.6. Thus, on theoretical grounds, it is concluded that the observed abrupt enhancement of sulfur dioxide above the cloud tops of Venus in 1978 was probably not due to an elongate fissure eruption of any reasonable magnitude (although a central-vent eruption is possible, as Esposito has suggested).

The present theory is expected also to be useful in providing estimates of plume height and plume width as input quantities to regional and global climate models that attempt to incorporate the atmospheric effects of large volcanic fissure eruptions. The theory can also be used to model the turbulent plumes rising above smoking chimney rows and above vigorous forest line-fires in a windless atmosphere.

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