

ISOTOPIIC ABUNDANCE OF CO IN INTERSTELLAR CLOUDS

WILLIAM D. LANGER

Goddard Institute for Space Studies, NASA, and Department of Physics, New York University

Received 1976 August 13; revised 1976 November 9

ABSTRACT

The fractional abundances of the isotopic species of carbon monoxide in interstellar clouds are calculated on a basis of gas phase ion-molecule reactions. The ^{13}CO to ^{12}CO ratio varies significantly with extinction of the ultraviolet radiation field, and in the outer regions of dark dense clouds ^{13}CO may be enhanced by a factor of 10. The observational interpretation of the CO to H_2 or A_V relation and the isotopic abundances of carbon are complicated by these effects.

Subject headings: abundances — interstellar: molecules

I. INTRODUCTION

Observations of isotopic abundances of carbon monoxide in dense interstellar clouds suggest that $^{13}\text{C}^{16}\text{O}/^{12}\text{C}^{16}\text{O}$ is enhanced with respect to the solar value of carbon-13 to carbon-12, $[^{13}\text{C}]/[^{12}\text{C}]$. For example, Wannier *et al.* (1976) have surveyed CO in H II associated molecular clouds at a total of 37 positions in 14 regions distributed throughout the galactic plane. The relative abundances of $^{13}\text{C}^{16}\text{O}$ and $^{12}\text{C}^{18}\text{O}$ are derived from the relative intensities of the wings of the line profiles where it is empirically assumed that they are transparent. They conclude that $[^{12}\text{C}]/[^{13}\text{C}]$ is ~ 40 in these clouds, a factor of ~ 2.3 larger than the terrestrial value. Mahoney, McCutcheon, and Shuter (1976) have mapped the dark dust cloud Lynds 134 in the $J = 1 \rightarrow 0$ transition of the CO isotopes. They observe a variation in $^{13}\text{C}^{16}\text{O}/^{12}\text{C}^{18}\text{O}$ over the spatial extent of the cloud, with the outermost values showing greater enhancement. The column density measurements suggest that $^{12}\text{C}/^{13}\text{C}$ ranges from 33 to 88; however, this range may represent the uncertainties in measurement and interpretation. These ratios depend on the determination of CO column densities, $N(\text{CO})$, from the observed line shapes and antenna temperatures; this determination involves a radiative transfer problem which has inherent uncertainties. The interpretation of $[^{12}\text{C}]/[^{13}\text{C}]$ from the CO observations above is complicated by other uncertainties. First, it is not a direct conclusion for $[^{12}\text{C}]/[^{13}\text{C}]$ since it is based on the isotopes $^{12}\text{C}^{18}\text{O}$ and $^{13}\text{C}^{16}\text{O}$, which only yields a double ratio $[^{12}\text{C}][^{18}\text{O}]/[^{13}\text{C}][^{16}\text{O}]$; the $J = 1 \rightarrow 0$ line of $^{12}\text{C}^{16}\text{O}$ is, unfortunately, saturated. The independent determination of $[^{18}\text{O}]/[^{16}\text{O}]$, from another set of measurements, is necessary for determining $[^{12}\text{C}]/[^{13}\text{C}]$. Wannier *et al.* (1976) refer to other double ratio measurements and use theoretical arguments to suggest $[^{18}\text{O}]/[^{16}\text{O}] \sim$ the solar value. Second, it assumes that carbon monoxide reflects the relative abundances of the various carbon and oxygen isotopes. Observational and experimental evidence suggests that this assumption may be incorrect.

The $^{13}\text{C}^{16}\text{O}$ molecule has also been suggested as a tracer of the molecular hydrogen column densities in dark clouds. In part the work of Encrenaz, Falgarone,

and Lucas (1975) and Dickman (1975) has been directed toward establishing a relation between $N(^{13}\text{C}^{16}\text{O})$ and $N(\text{H}_2)$. Dickman (1975) observes the $J = 1 \rightarrow 0$ line of $^{13}\text{C}^{16}\text{O}$, calculates LTE values of $N(^{13}\text{C}^{16}\text{O})$, and compares these to the extinction in the visual, A_V , as determined from star counts. In addition to the previously discussed uncertainties in determining $N(^{13}\text{C}^{16}\text{O})$, there is the further uncertainty in A_V as derived from star counting; the upper limit for this method is $A_V \lesssim 10$. Encrenaz, Falgarone, and Lucas (1975) derive column densities from fits of observed $^{12}\text{C}^{16}\text{O}$ and $^{13}\text{C}^{16}\text{O}$ line intensities with a microturbulent cloud non-LTE model. The model (necessarily) contains an assumed isotopic ratio $[^{12}\text{C}^{16}\text{O}]/[^{13}\text{C}^{16}\text{O}]$ which they took equal to 40, the value suggested by the work of Wannier (1974) and Wannier *et al.* (1976). If one wants to compare Dickman's (1975) results with those of Encrenaz, Falgarone, and Lucas (1975) one should multiply by 40. When this is done, the results lead to a number of discrepancies at small extinctions with theoretical predictions of $N(^{12}\text{C}^{16}\text{O})$ based on models of molecule production by gas phase ion-molecule reactions (Langer 1976a). For $10 \gtrsim A_V \gtrsim 3$ these model predictions of $N(^{12}\text{C}^{16}\text{O})$ versus A_V are in reasonable agreement with the observations. With decreasing A_V the comparison with the $N(^{12}\text{C}^{16}\text{O})$ values determined from the $^{13}\text{C}^{16}\text{O}$ observations becomes much worse (by as much as a factor of 10) down to the observational limit $A_V \approx 1$. In contrast, however, some estimates of $N(^{12}\text{C}^{16}\text{O})$ by Encrenaz, Falgarone, and Lucas (1975) based on a radiative transfer model for the optically thick $^{12}\text{C}^{16}\text{O}$ at $A_V \lesssim 1.6$ are in rough agreement with theoretical models (Langer 1976a). Clearly, a number of puzzles arise in the interpretation of isotopic abundances of CO. Those aspects of this problem which arise from the chemistry will be dealt with here; we shall show that the several observational results just described can be explained in part by the isotopic ion-molecule chemistry.

II. ISOTOPIIC ABUNDANCES

There are two important gas phase ion-molecule chemistries which produce CO in dense interstellar clouds, and these divide the cloud into two separate

regions. In the interior regions of dark dense clouds where the ultraviolet radiation field is attenuated, the H_3^+ chemistry suggested by Herbst and Klemperer (1973) initiates molecule production. In the outer regions of dark clouds where there is sufficient ultraviolet radiation to keep carbon partially ionized, the C^+ radiative association with H_2 suggested by Black and Dalgarno (1973) initiates molecule production. The transition region between these two chemistries is fairly narrow and will be neglected here (see Langer 1976a for a discussion of this region). These two chemistries have different consequences for molecule production in general and isotopic abundances in particular. Without fractionation reactions there would be no differences between the chemistries of the different isotopic species; however, in the case of $^{13}C^{16}O$ and $^{12}C^{16}O$ the fractionation reaction,



studied by Watson, Anicich, and Huntress (1976) can enhance ^{13}CO with respect to ^{12}CO .

Abundance calculations for CO, excluding fractionation effects, have been carried out for a number of situations. In the C^+ chemistry Oppenheimer and Dalgarno (1975) and Langer (1976a) have calculated time-dependent and steady abundances of CO, respectively. The steady-state and time-dependent features of the H_3^+ chemistry have been considered by Langer (1976a, b) and Langer and Glassgold (1976), with the following two important results: (1) complete conversion of C^+ to CO cannot occur in the C^+ radiative association chemistry for the conditions present in interstellar clouds; and, (2) in the H_3^+ chemistry essentially complete conversion can occur, but the time scale necessary to achieve this condition (a few times 10^6 yr) may be comparable to, or greater than, dynamical time scales for cloud evolution. These results have important implications for isotopic abundances because $N(^{13}C^{16}O)/N(^{12}C^{16}O)$ reflects the true isotopic abundance of carbon in dense clouds only if there is complete conversion of carbon to CO (Langer 1976b); otherwise fractionation effects must be included (Watson, Anicich, and Huntress 1976).

The approximate equations which have been derived to describe the fractional abundance of CO, $x(CO)$, in our previous work (Langer 1976a, b) can be generalized to include all the isotopic fractional abundances. The two major additions are the isotopic exchange reaction (1) and separate conservation conditions for each of the available gaseous carbon and oxygen isotopes. For example, for the species $^{13}C^{16}O$ the time-dependent abundance is approximately given by

$$\begin{aligned} \frac{1}{n} \dot{x}(^{13}C^{16}O) = & [kx(H_3^+)x(^{13}C) + \frac{1}{2}k_{26}x(^{13}C^+)]\Gamma \\ & + kx(H_3^+)x(^{13}C)\Gamma' + kx(^{13}C^+)x(M^{16}O) \\ & + k_{CO'}x(^{13}C^+)x(^{12}C^{16}O) - [g_{CO} \\ & + kx(He^+) + k_{CO''}x(^{12}C^+)]x(^{13}C^{16}O) \end{aligned} \quad (2)$$

where the fractional abundance for X is $x(X) = n(X)/n$, $n = n(H) + 2n(H_2)$, $k = 1 \times 10^{-9} \text{ cm}^3 \text{ s}^{-1}$, and $a = 4 \times 10^{-11} \text{ cm}^3 \text{ s}^{-1}$. The other terms are $\Gamma = ax(^{16}O)/[g_{CO} + ax(O)]$, $g_X = G_X/n$, $G_X =$ photodestruction rate for species X in s^{-1} , $\Gamma' = \gamma(OH)ax(^{16}O)/[g_{OH} + ax(O) + ax(C)]$, and $\gamma(OH) = \frac{1}{2}$ is the branching ratio for dissociative recombination $H_3O^+ + e \rightarrow OH + H_2$. Unless otherwise noted, the notation and rates are those listed in Langer (1976a, b). The radiative association rate for $C^+ + H_2 \rightarrow CH_2^+ + h\nu$ is $k_{26} = 10^{-16} \text{ cm}^3 \text{ s}^{-1}$ (see Langer 1976a and Herbst 1976), although much larger values have been suggested (Black and Dalgarno 1973; Black, Dalgarno, and Oppenheimer 1975). The rates for the isotopic exchange reactions (1) are, $k_{CO'} = 2 \times 10^{-10} \text{ cm}^3 \text{ s}^{-1}$ and $k_{CO''} = k_{CO'} \exp(-\Delta E/T)$ (Watson, Anicich, and Huntress 1976), where $\Delta E = 35 \text{ K}$.

In order to solve equations (2) and (3) for the CO isotopes the additional equations for $x(^{12}CO)$, $x(^{12}C^+)$, $x(^{13}C^+)$, $x(He^+)$, and $x(M^{16}O)$ [$\approx x(H_2^{16}O) + x(^{16}OH) + x(^{16}O_2)$] are required. These are given by simple extensions of the relationships derived in Langer (1976a, b) and will not be elaborated upon here. The approximate conservation conditions for the available gaseous carbon are: (1) $\xi_{^{12}C} \approx x(^{12}C^+) + x(^{12}CO) + x(^{12}C)$; (2) $\xi_{^{13}C} \approx x(^{13}C^+) + x(^{13}CO) + x(^{13}C)$; and (3) $\xi_C = \xi_{^{12}C} + \xi_{^{13}C}$.

III. RESULTS

For the purpose of emphasizing the different aspects of the isotopic abundance problem in the different cloud regions, solutions were obtained for the corresponding chemistries. First the results for the C^+ radiative association chemistry operative in the outer regions will be discussed. The fractional abundances of the 12 and 13 isotopes of CO, C^+ , and C are calculated in the steady state as a function of extinction. In Figure 1 are presented the results of a sample calculation using the following parameters typical of dark dense clouds: $n(H_2) = 10^8 \text{ cm}^{-3}$; $T = 10 \text{ K}$; cosmic ray ionization rate on hydrogen, $\zeta_p = 10^{-17} \text{ s}^{-1}$; $\xi_C = 10^{-4}$; $\xi_O = 2 \times 10^{-4}$; and the photodestruction rates, $G_C = 10^{-10} \exp(-\tau_C) \text{ s}^{-1}$ and $G_{CO} = 10^{-11} \exp(-\tau_C) \text{ s}^{-1}$, where τ_C measures the extinction by grains in the ultraviolet $\sim 1000 \text{ \AA}$ to the cloud edge (using the albedo measurements of Witt and Lillie 1976, $\tau_C \approx 2\tau_V$, the extinction in the visible). These results are presented in terms of the fractional abundance compared to the available gaseous carbon, $x(^A X)/\xi_{A^+}$; throughout it is assumed that $\xi_{^{13}C} \ll \xi_{^{12}C}$. These curves begin at $\tau_C = 1$ where most of the hydrogen is almost certainly in the form of H_2 . For $\tau_C > 6$ the radiative association chemistry becomes less efficient at producing CO and the H_3^+ chemistry must be included. If H_3^+ reactions are neglected, the CO abundance will eventually decrease to zero because of the destructive effects of He^+ (Langer 1976a). Furthermore, time-dependent effects become increasingly important in this region, and steady-state calculations are not valid for the description of isotopic abundances (Langer 1976b). The time scale for conversion of C^+ to CO increases with extinction, reaching a value $\sim 5 \times 10^5 \text{ yr}$ at $\tau_C = 6$. Compared to evolu-

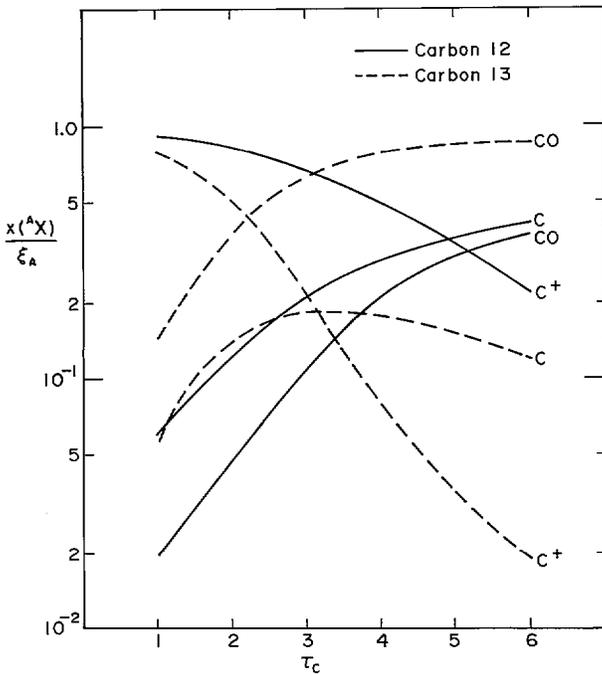


FIG. 1.—The fractional abundances compared to the available gaseous carbon, $x(A)/\xi_A$, are plotted for the isotopes of CO, C⁺, and C as a function of extinction in the ultraviolet ($\sim 1000 \text{ \AA}$), τ_c . These curves are calculated for $n(\text{H}_2) = 10^3 \text{ cm}^{-3}$ and $T = 10 \text{ K}$, typical of dense dark clouds; the remaining parameters are discussed in the text.

tionary times, this time is short enough to make it probable that the steady-state results in Figure 1 are applicable to most observations.

The enhancement of $^{13}\text{C}^{16}\text{O}$, $R = [x(^{13}\text{C}^{16}\text{O})/\xi_{^{13}\text{C}}]/[x(^{12}\text{C}^{16}\text{O})/\xi_{^{12}\text{C}}]$, is plotted in Figure 2 for $n(\text{H}_2) = 10^3 \text{ cm}^{-3}$ and $T = 10$ and 20 K ; the other parameters are those listed for Figure 1. At $T = 10 \text{ K}$, R peaks at $\tau_c = 2$ ($\tau_V = 1$) with a fairly large value of 8 and decreases to 2.3 at $\tau_c = 6$. Only if $R = 1$ will observations of CO measure the true isotopic abundance of carbon.

The equations for $x(\text{CO})$ can be expanded to include the isotopic species of oxygen; isotopic exchange reactions involving oxygen are not likely to be important since it is neutral in interstellar clouds and the neutral exchange reaction, $^{18}\text{O} + ^{12}\text{C}^{16}\text{O} \rightarrow ^{16}\text{O} + ^{12}\text{C}^{18}\text{O}$, is probably too slow to contribute. From an expanded set of equations it can be shown that, $x(^{12}\text{C}^{18}\text{O})/x(^{12}\text{C}^{16}\text{O}) \approx x(^{18}\text{O})/x(^{16}\text{O})$ since $x(^{18}\text{O}) \ll x(^{16}\text{O})$ (similarly for $^{13}\text{C}^{18}\text{O}$ and $^{13}\text{C}^{16}\text{O}$). Thus, the oxygen isotopes of CO should reflect their true isotopic abundances in interstellar clouds.

For $\tau_c > 6$, the evolutionary history of the material will determine the molecular abundances, and steady-state results are not appropriate for comparison with the observations. The material in the dark dense regions of interstellar clouds with $n \gtrsim 10^3\text{--}10^4 \text{ cm}^{-3}$ evolves from a lower density configuration, and a correct analysis of molecular abundances requires a full

hydrodynamic treatment. For the purpose of illustrating the possible behavior of R in dark dense regions, a simple time-dependent model has been adopted (see Langer 1976*b* for details) which neglects variations in n and T for n greater than a few $\times 10^3 \text{ cm}^{-3}$. Assume that initially, at large $\tau_c (> 8)$, $x(\text{C}^+) = \xi_c$, $x(\text{C}) = x(\text{CO}) = 0.0$. Within 10^5 years the C⁺ primarily recombines to C, though the radiative association chemistry produces some CO: $x(^{12}\text{C}^{16}\text{O})/\xi_{^{12}\text{C}} \approx 0.1$ and $x(^{13}\text{C}^{16}\text{O})/\xi_{^{13}\text{C}} \approx 0.35$ ($R = 3.5$). Shortly afterwards, the H_3^+ chemistry dominates molecule production, ^{12}C and ^{13}C are slowly converted to CO, and $R \rightarrow 1$ as $t \rightarrow \infty$ (after $\sim 4 \times 10^6 \text{ yr}$, $R \approx 1.1$). Another possible scenario would involve the evolution of the material at low densities and $\tau_c \approx 6$ to higher densities and optical depths. Thus, at the time when the H_3^+ chemistry dominates molecule production the initial conditions for $x(\text{CO}, t)$ would correspond to the steady state values derived for $\tau_c \approx 6$ (the maximum CO that can be produced by radiative association). Afterwards the value of R will decrease from 2.3 to 1.1 within $\sim 4 \times 10^6 \text{ yr}$. Unfortunately, without the hydrodynamic models the evolutionary times and chemical history cannot be known for this material. The CO in the dark regions of dense clouds probably reflects the true isotopic abundances only if they are $\sim (4\text{--}6) \times 10^6$ years older than the low density clouds from which they evolved.

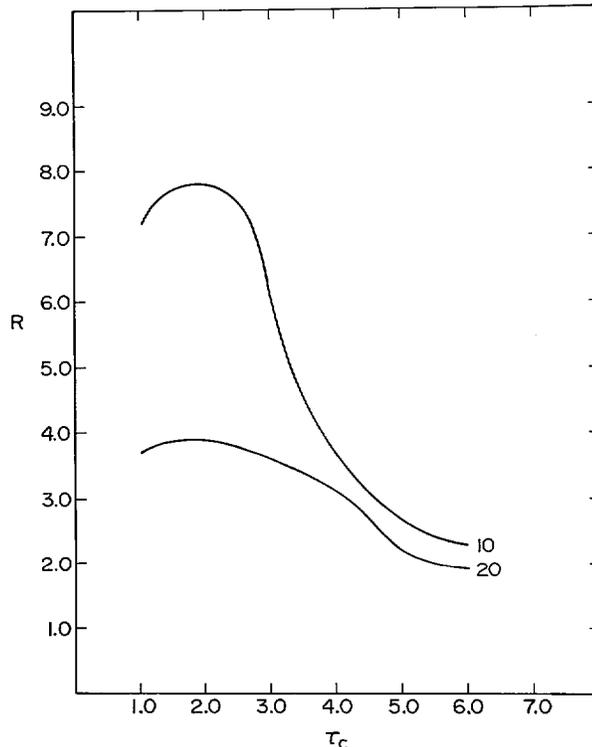


FIG. 2.—The enhancement, $R = [x(^{13}\text{C}^{16}\text{O})/\xi_{^{13}\text{C}}]/[x(^{12}\text{C}^{16}\text{O})/\xi_{^{12}\text{C}}]$, is plotted as a function of extinction, τ_c , for temperatures of 10 K and 20 K at a density $n(\text{H}_2) = 10^3 \text{ cm}^{-3}$; the remaining parameters are discussed in the text.

IV. DISCUSSION

It can be concluded from the results of § III that fractionation causes an enhancement of $x(^{13}\text{C}^{16}\text{O})/x(^{12}\text{C}^{16}\text{O})$ and $x(^{13}\text{C}^{16}\text{O})/x(^{12}\text{C}^{18}\text{O})$ in regions where the ultraviolet radiation field keeps carbon partially ionized, and that this enhancement varies with extinction. In these regions the CO formation time is short enough that the enhancement R probably reflects the present values of n , T , and τ_{C} , whereas in denser darker regions, where the H_3^+ chemistry initiates molecule production, R may reflect the time-dependent evolution of the cloud.

The variation of R with τ_{C} and its relatively large value at small extinctions may explain the $N(\text{CO})/A_V$ measurements discussed in § I. In the case of the observations in ρ Oph by Encrenaz, Falgarone, and Lucas (1975) the measurements of $N(^{12}\text{C}^{16}\text{O})/A_V$ for $1 \leq A_V \leq 1.6$ are in excellent agreement with the model calculations corresponding to $n(\text{H}_2) \sim 500\text{--}10^3 \text{ cm}^{-3}$ (Langer 1976a). For $A_V > 3$, both the $N(^{12}\text{C}^{16}\text{O})$ and $N(^{13}\text{C}^{16}\text{O})$ data are in good agreement with these model calculations (within a factor of 2). For $1.6 \leq A_V < 3$, however, the values of $N(\text{CO})$ obtained from the $^{13}\text{C}^{16}\text{O}$ measurements [recall their assumption $N(\text{CO}) = 40 N(^{13}\text{C}^{16}\text{O})$] are much larger than those predicted by theory. Encrenaz, Falgarone, and Lucas (1975) quote values for $N(\text{CO})/A_V$ of $\sim 3 \times 10^{16}$ and $\sim 10^{17} \text{ cm}^{-2} \text{ mag}^{-1}$ at $A_V = 1.8$ and 2.5, respectively; the corresponding theoretical calculations give 1.5×10^{16} and $2 \times 10^{16} \text{ cm}^{-2} \text{ mag}^{-1}$ when $n(\text{H}_2) = 10^3 \text{ cm}^{-3}$ [increasing $n(\text{H}_2)$ to $5 \times 10^3 \text{ cm}^{-3}$ at $A_V = 2.5$ still gives only $N(\text{CO})/A_V \sim 5 \times 10^{16} \text{ cm}^{-2} \text{ mag}^{-1}$]. According to the results in Figure 2 for $T = 10 \text{ K}$, appropriate to ρ Oph, a better choice for $N(\text{CO})$ would be $\approx 13N(^{13}\text{C}^{16}\text{O})$ in the region $A_V \approx 1\text{--}2.5$, which would bring the observations into agreement with the model calculations (Langer 1976a). (Throughout we use the enhancement R determined from local densities; the corresponding enhancement for column densities is found to be only slightly smaller.) Dickman (1975) has surveyed many locations within 37 interstellar dark clouds in $^{13}\text{C}^{16}\text{O}$. For each of these, LTE values of $N(^{13}\text{C}^{16}\text{O})$, kinetic temperatures (typically 10 K), and star-count determinations of A_V were calculated. The comparison with theory again suggests that $N(^{12}\text{C}^{16}\text{O})/N(^{13}\text{C}^{16}\text{O})$ varies

with extinction. When the $N(^{13}\text{C}^{16}\text{O})$ measured by Dickman (1975) are multiplied by $89/R$ for $T = 10 \text{ K}$, in order to compensate for the fractionation and isotopic ratio of available gaseous carbon, the resultant $N(^{12}\text{C}^{16}\text{O})$ versus A_V are brought into very good agreement with the model calculations for densities in the range, $10^3 \lesssim n(\text{H}_2) \lesssim 5 \times 10^3 \text{ cm}^{-3}$ (Langer 1976a). The choice $89/R$ is not unique for these observations because the scatter in the data is such that a smaller ratio (e.g., $40/R$) would work just as well. While the observed variation of the ratio with extinction is readily explained, the absolute carbon isotopic ratio cannot be determined precisely.

The variations in isotopic abundances within Lynds 134 (Mahoney, McCutcheon, and Shuter 1976) are in qualitative agreement with the previous discussion. A better comparison might be made, however, if extinction measurements were also available for comparison with column densities of $^{12}\text{C}^{16}\text{O}$, $^{13}\text{C}^{16}\text{O}$, and $^{12}\text{C}^{18}\text{O}$. Further observations of this kind are required to understand the effects of isotopic fractionation. Taken together, the CO measurements at small extinction ($A_V \leq 10$) suggest that fractionation is present in accord with the results presented here.

The measurements of Wannier *et al.* (1976) are essentially of regions shielded from the ultraviolet radiation field, and the time-dependent discussion of R in § III is relevant here. If these regions are less than $\sim 4 \times 10^6$ years older than the lower-density clouds from which they evolved, then some fractionation will be present. Watson, Anicich, and Huntress (1976) have also suggested that $[^{13}\text{C}]/[^{12}\text{C}]$ is enhanced in a time-dependent evolution since C^+ and C preferentially freeze onto grains compared to CO. In conclusion it is difficult to assess the enhancement effects for the measurements of Wannier *et al.* (1976), but it is possible that some part of their $[^{13}\text{C}]/[^{12}\text{C}]$ value is due to the effects of chemical fractionation, grain depletion, and cloud evolution.

The author is grateful for financial support as an NRC-NAS Senior Research Associate. I would also like to thank Drs. R. Dickman, A. Penzias, and P. Goldsmith for many useful conversations on the subject of this Letter.

REFERENCES

- Black, J., and Dalgarno, A. 1973, *Ap. Letters*, 15, 79.
 Black, J., Dalgarno, A., and Oppenheimer, M. 1975, *Ap. J.*, 199, 633.
 Dickman, R. 1975, Ph.D. thesis, Columbia University.
 Encrenaz, P. J., Falgarone, E., and Lucas, R. 1975, *Astr. Ap.*, 44, 73.
 Herbst, E. 1976, *Ap. J.*, 205, 94.
 Herbst, E., and Klemperer, W. 1973, *Ap. J.*, 185, 505.
 Langer, W. D. 1976a, *Ap. J.*, 206, 699.
 ———. 1976b, *Ap. J.*, 210, 328.
 Langer, W. D., and Glassgold, A. E. 1976, *Astr. Ap.*, 48, 395.
 Mahoney, M. J., McCutcheon, W. H., and Shuter, W. L. H. 1976, *A. J.*, 81, 508.
 Oppenheimer, M., and Dalgarno, A. 1975, *Ap. J.*, 200, 419.
 Wannier, P. G. 1974, Ph.D. thesis, Princeton University.
 Wannier, P. G., Penzias, A. A., Linke, R. A., and Wilson, R. W. 1976, *Ap. J.*, 204, 26.
 Watson, W. D., Anicich, V. G., and Huntress, W. T. 1976, *Ap. J. (Letters)*, 205, L165.
 Witt, A. N., and Lillie, C. F. 1976, *Ap. J.*, 208, 64.

WILLIAM D. LANGER: Department of Astronomy E1, University of Pennsylvania, Philadelphia, PA 19174