A Comparison of Estimated and Directly Measured Turbulent Heat Fluxes in the Lower Stratosphere

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ABSTRACT

The contribution of small-scale motions to the vertical heat flux in the lower stratosphere is determined from wind and temperature fluctuation statistics obtained during the High Altitude Clear Air Turbulence investigation. Analysis of the cospectra suggests a horizontal wavelength of 3 km as the appropriate long-wave cutoff of small-scale motion. With this scale restriction, the measured vertical heat fluxes are in good agreement with the values of heat flux estimated by Lilly *et al.* (1974) from the kinetic energy dissipation rate. Hence, the hypothesis of Lilly *et al.* for estimating heat flux from the dissipation rate is considered to be reliable. It follows that, in the lower stratosphere, subsynoptic vertical transport is small compared to large-scale transport.

1. Purpose

Lilly *et al.* (1974) estimated subsynoptic vertical mixing coefficients for heat from HICAT[1] data in the lower stratosphere. The theory for obtaining these estimates required two assumptions: 1) the flux Richardson number in the turbulent region is 0.25, and 2) the turbulent energy budget is locally balanced. Both these assumptions are reasonable but not necessarily valid. For example, since turbulence can exist with a Richardson number less than 0.25, the first assumption may not be satisfied. The second assumption would be violated when some turbulent energy is exported from the region of its generation. The goal of this paper, therefore, is to determine the eddy mixing coefficient for heat as independently as possible without making these assumptions in order to check the hypothesis of Lilly *et al.* (1974).

2. Observations and data handling

A brief description of the HICAT program can be found in Lilly *et al.* The variables from HICAT flights used in this study are the fluctuations of temperature T and the true gust velocity components u, v and w. Here u is the longitudinal velocity component, v the lateral velocity component and w the vertical velocity component. These parameters were recorded at 0.08 s intervals at aircraft speeds of $\sim$200 m s^{-1}. Data were excluded when the aircraft was climbing, descending, turning, or when only derived gust velocities were available. Of the remaining usable data, the 3200 km of turbulent flight segments analyzed in this study are approximately 25% of the total turbulent flight records (Waco, private communication) all of which were analyzed by Lilly *et al.* (1974).

As a preliminary step, vertical velocity and temperature information were plotted as a function of time. Visual inspection of these plots for any particular run showed large variation in the magnitude of both the vertical velocity and temperature fluctuations. It was obvious that turbulent intensity based on vertical velocity fluctuations varied greatly within any one run even though there existed only one turbulence intensity classification for that run. Most of the HICAT runs were classified as having "light" turbulence. To increase the sample size of moderate to severe turbulence, it was necessary to restrict analysis to segments within runs with relatively constant turbulence intensity, but yet containing the highest turbulence intensity for each run. In this way the more severe turbulence would not be averaged out in the data analysis. Only segments shorter than 655 s (131 km) and longer than 82 s (16 km) were analyzed.

Visual inspection of the time series showed trends with no significant curvature; this motivated the removal of the linear trend prior to analysis. After removing linear trends from u, v, w and T records, spectra of u, v and w and cross spectra of wT, uw and vw were computed by fast Fourier transform methods. The spectral estimates were obtained by linearly averaging spectral densities. The spectral decomposition of the vertical velocity provided one more criterion for data selection: the segment was used only if the contribution to the standard deviation by wavelengths less than 3 km exceeded 0.15 m s^{-1}.

[1] High Altitude Clear Air Turbulence.

TABLE 1. Averages of ϵ_u not weighted by length of run for various terrain types, units 10^{-4} m² s⁻³.

Terrain	This paper	Lilly *et al.*
Water	7.40	11.8
Flatland	17.4	9.87
Low mountains	24.9	14.2
High mountains	76.1	29.0

3. Calculation of dissipation rates

The dissipation rate can be computed from measurements of turbulent velocities in the inertial subrange. In this range, theory predicts a $-5/3$ power law for the spectrum of each velocity component. However, experience has shown that the $-5/3$ law for the longitudinal component extends without break into length scales much larger than those of the inertial subrange. For this reason, it is common to base the estimate of the dissipation rate on the u component when the measurements are made outside the inertial subrange. Thus, we assume variance spectra of the u component of the form

$$S(k)=a\epsilon^{2/3}k^{-5/3}, \quad (1)$$

where $S(k)$ is the one-dimensional spectral density of the longitudinal component of velocity and ϵ is the rate of kinetic energy dissipation; a is a dimensionless constant. Most recent estimates of this constant in the inertial range are about 0.50 (Kaimal *et al.*, 1972). This value applies only when k has units of radians per length. The dissipation rates were estimated by solving (1) for ϵ. The dissipation rate based on the longitudinal component of the wind will be denoted by ϵ_u.

The author's estimates of the unweighted dissipation rates, categorized by terrain,[2] are compared with those of Lilly *et al.* (1974) in Table 1. Since the technique of computing ϵ_u was similar to that employed by Lilly *et al.*, the difference in the values of ϵ_u can be explained by the larger data sample used by Lilly *et al.*, and the fact that we restricted the analysis to only the more severe turbulence in each run in which the turbulence varied significantly.

4. Direct computation of the heat flux covariance

The heat flux covariance $\overline{w'T'}$ was evaluated from the cospectra of vertical velocity and temperature. Here the overbar denotes the mean through a region of clear-air turbulence (CAT) and the prime denotes the deviation from the mean. Visual inspection of the cospectra of w and T as a function of wavenumber showed that 1) the contribution to $\overline{w'T'}$ by the low wavenumbers was large and was as likely to be negative as positive, and 2) the cospectral estimates approached zero at wavenumbers much lower than those wavenumbers at which the signals were attenuated due to insufficient instrumental response. The random sign and magnitude of the low wavenumber contribution to $\overline{w'T'}$ was attributed to waves that may be too long compared to the record length to obtain reliable estimates of the heat flux. Therefore, $\overline{w'T'}$ was defined as the sum of the cospectral estimates between w and T for wavelengths <3000 m. This definition occasionally resulted in very small positive values. A typical cospectrum is shown in Fig. 1. (The cospectral estimate corresponding to a k of 1.7×10^{-4} cycles m⁻¹ represents the contribution to $\overline{w'T'}$ by all wavelengths >3 km.)

5. The hypothesis of Lilly et al. (1974)

Following Dutton (1969), we can simplify the turbulent kinetic energy equation by assuming nondivergent motion, small horizontal gradients and small mean vertical velocities within the turbulent patch. We then obtain

$$\frac{\partial\bar{e}}{\partial t}=-\overline{\mathbf{v}'w'}\cdot\frac{\partial\bar{\mathbf{v}}}{\partial z}+\frac{g}{T}\overline{w'T'}-\epsilon-\frac{\partial}{\partial z}\left(\frac{\overline{p'w'}}{\bar{\rho}}+\overline{w'e}\right), \quad (2)$$

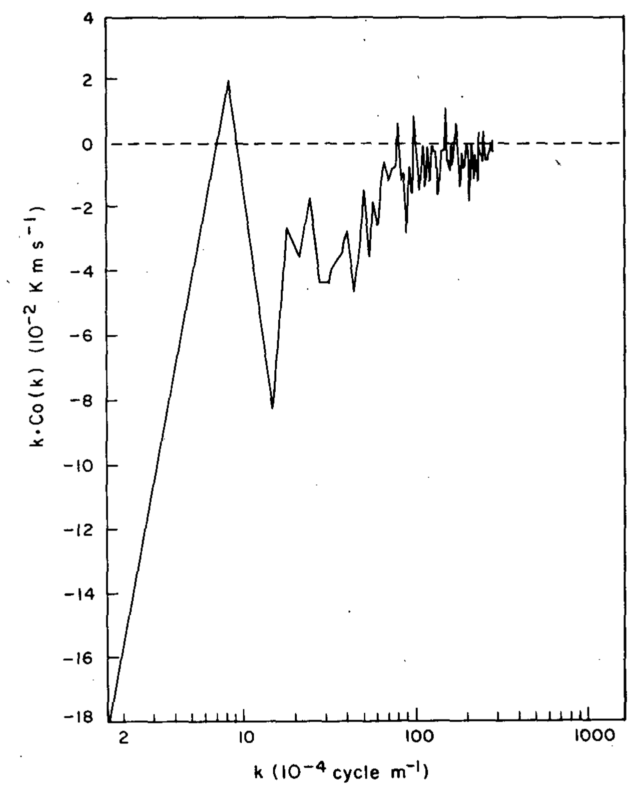

FIG. 1. Cospectrum between w and T as a function of wavenumber for Run 198-12 (moderate–severe turbulence).

[2] Ashburn *et al.* (1970) classify underlying terrain according to local terrain relief which is greater than 2100 m for high mountains, between 900 and 2100 m for low mountains, and less than 900 m for flatland. For the earth as a whole the portion of water, flatland and low and high mountains is, respectively, 71%, 23%, 3% and 3%. For North America and Greenland the approximate corresponding proportions are 10%, 65%, 12% and 13%.

where $\mathbf{v}$ is the horizontal velocity vector, w the vertical velocity, p the pressure and ρ the density. The specific turbulent kinetic energy is $e=(u'^2+v'^2+w'^2)/2$.

Lilly *et al.* (1974) assumed that the first three terms on the right-hand side of (2) locally balance and that all other terms in the equation can be neglected. In addition, they assumed that the flux Richardson number is 0.25 so that (2) reduces to

$$-\overline{w'T'}=\frac{\bar{T}}{g}\frac{\epsilon}{3}. \tag{3}$$

By evaluating ϵ from high wavenumber spectra and by assuming a mean lower stratospheric temperature of 210 K, $\overline{w'T'}$ was computed. Estimates of $\overline{w'T'}$, from Lilly *et al.*, categorized by terrain, are shown in Table 2. Two sets of values for $\overline{w'T'}$ are given in Table 2 for each of the four terrain types. The larger values (column 1) are ensemble averages of $\overline{w'T'}$ in turbulent regions only. The smaller values (column 2) were computed by multiplying ensemble averages of $\overline{w'T'}$ by the ratio of turbulent flight miles to total flight distance for the given terrain type.

Fig. 2 shows the heat flux covariance as a function of the dissipation rate. The following ordinate was adopted for this figure:

$$\text{For } -\overline{w'T'}>0, \quad \log(1-\overline{w'T'}),$$

$$\text{For } -\overline{w'T'}\leqslant 0, \quad -\log(1+\overline{w'T'}).$$

The dashed line was obtained by first choosing appropriate intervals of ϵ_u and weighting the values of $-\overline{w'T'}$ and ϵ_u within each interval according to the length of their respective flight segments. Six intervals were chosen such that the average $-\overline{w'T'}$ in each interval would be greater than zero. Averages were used instead of the individual values because the logarithm of negative numbers is not defined. Finally the method of least squares was used to obtain a regression equation for logarithm $-\overline{w'T'}$ as a function of logarithm ϵ_u from these averages. The resulting relationship was

TABLE 2. Average local heat flux covariances for categories of terrain and estimates of worldwide average, units 10^{-4} K m s^{-1} (from Lilly *et al.*, 1974).

	$-\overline{w'T'}$ (local)	$-\overline{w'T'}$ (worldwide)
Water	48	1.00
Flatland	46	1.10
Low mountain	60	1.90
High mountain	122	6.40

$$-\overline{w'T'}=\frac{\bar{T}}{g}\frac{\epsilon_u^{1.07}}{2.7} \tag{4}$$

in SI units. Since (4) was obtained from all of the data with appropriate weight factors, it is considered a suitable empirical relation for the purpose of estimating $-\overline{w'T'}$ from ϵ_u. The solid line in Fig. 2 represents Eq. (3), the relation postulated by Lilly *et al.* (1974). The agreement between the two curves is good; the observations do not differ significantly from Lilly's relation. The individual values of $-\overline{w'T'}$ as a function of ϵ_u are also shown in Fig. 2.

Table 3 shows the variability of the unweighted mean heat flux covariance with different underlying terrain. Specifically, $-\overline{w'T'}$ values in column 1 were determined directly from cospectra of vertical velocity and temperature; those in column 2 were obtained using Eq. (4) with ϵ_u taken from Table 1, column 1. The two sets

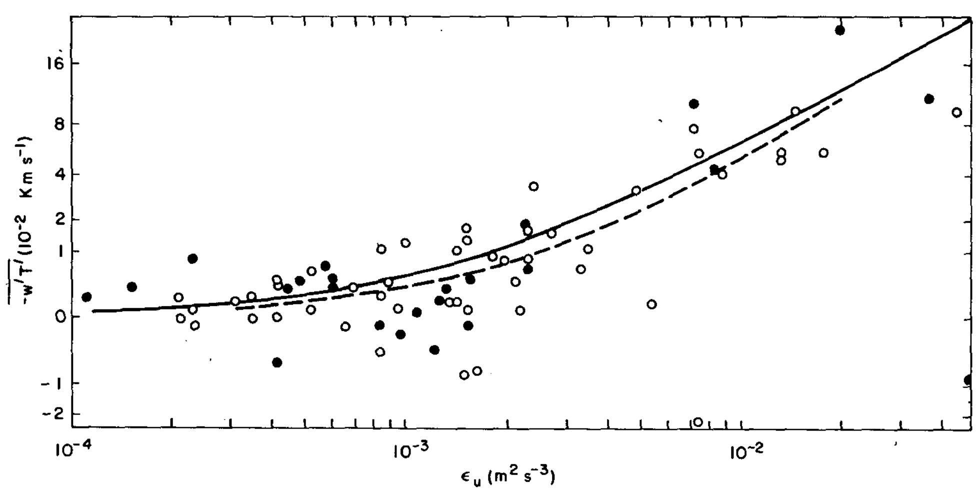

FIG. 2. Vertical heat flux as function of dissipation. Solid line, Eq. (3); dashed line, Eq. (4). Closed circles represent 16 km flight segments and open circles longer segments.

TABLE 3. Averages of $-\overline{w'T'}$ not weighted by length of run, for this paper, for categories of terrain, units 10^{-2} K m s^{-1}.

	Observed $-\overline{w'T'}$	Computed $-\overline{w'T'}$ by (4)
Water	0.11	0.36
Flatland	0.99	0.89
Low mountains	1.19	1.30
High mountains	3.06	4.31

of values of $-\overline{w'T'}$ evaluated in this paper agree well with each other for all categories of terrain except over water. However, over water the individual values are small, and this disagreement may be due to observational error. Since there is no physical reason for the relation between $-\overline{w'T'}$ and ϵ_u to be different over water from that over other terrain types, (4) was used to estimate $-\overline{w'T'}$ from ϵ_u over water. For all practical purposes, the computed averages of $-\overline{w'T'}$ over flatland and low mountains are about the same—close to 1.0×10^{-2} K m s^{-1}, corresponding to a local eddy coefficient K_h of 1.0 m^2 s^{-1} when an isothermal lapse rate is assumed. In contrast, typical turbulent patches over high mountains have values of $-\overline{w'T'}$ about 3–4 times as large.

6. Summary

By choosing a horizontal wavelength of 3 km as the long-wave cutoff of small-scale motion, the measured heat fluxes as determined from the cospectra vertical velocity and temperature for individual patches of CAT have shown good agreement with values of heat flux of Lilly *et al.* (1974) estimated from the kinetic energy dissipation rate in the patch. We conclude that the assumption by Lilly *et al.* of a locally balanced energy budget and the assumption of a Richardson number equal to 0.25 give reliable estimates of heat flux in turbulent regions provided this particular wavelength cutoff has been selected. (Some experimentation with other limiting wavelengths may be desirable, however.) Therefore, we also agree with the conclusion by Lilly *et al.* (1974) that vertical mixing by CAT in the lower stratosphere is negligible compared to large-scale vertical transport.

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