

The Local complex of O and B stars. II. Kinematics

Jay A. Frogel

Cerro Tololo Inter-American Observatory^{a1}, Casilla 63-D, La Serena, Chile

Richard Stothers

NASA Institute for Space Studies, Goddard Space Flight Center, 2880 Broadway, New York, New York 10025

(Received 31 May 1977; revised 16 August 1977)

The space velocities of O–B5 stars in the solar neighborhood are analyzed in the present paper. After a short presentation of the historical background, the stars are divided, following Paper I, into members of the Gould and galactic belts on the basis of their positions in space. This permits a homogeneous kinematical comparison to be made between the two belts. The local galactic velocity field is found to be perturbed by the presence of the Gould belt, as reflected in the derived values of the Oort constant B and of the K term for the youngest stars. If the Gould belt is expanding as a unit (which is uncertain), the expansion age must be about 7×10^7 yr. Systematic translational motion of the Gould belt with respect to the galactic belt is negligibly small in directions parallel to the galactic plane. But a coherent “seesaw” motion of the Gould belt in the direction perpendicular to the galactic plane can explain the observed vertical velocities of stars in this belt. It is suggested that the Gould belt had a violent origin close to the galactic plane and that each passage of this belt through the galactic plane initiated a major period of star formation. Inferred kinematical ages for most of the stars concerned are 2×10^7 or 6×10^7 yr, which agree well with the observed frequency distribution of their nuclear ages.

INTRODUCTION

THE majority of O and B stars in the solar neighborhood are distributed in two flat systems or “belts.” A study of the structural properties of the “Gould belt” (sometimes called the *local system*) and of the more familiar “galactic belt” has revealed that these belts can be separated by means of a least-squares solution for the two best-fitting planes that pass through the O and B star distributions (Stothers and Frogel 1974, Paper I). Although the assignment of stars to each belt is not always unique, the method of assignment is certainly objective and permits a reliable test to be made for possible kinematical differences between the two belts by using stars of very nearly the same distance and spectral type in each belt.

In the present paper, we give first a historical review of the earlier kinematical investigations in order to supplement our previous account of the positional investigations. Then we study the motions themselves: the local solar motion, differential galactic rotation, mean peculiar stellar velocities, and systematic stellar motions parallel and perpendicular to the galactic plane.

I. HISTORICAL SURVEY

A. Stars

Originally, the local system of apparently bright stars was regarded kinematically as an outlying fragment of a violently rotating Galaxy (Alexander 1852), although

later it came to be regarded as a scarcely differentiable substrate of an ellipsoidal Galaxy approaching statistical equilibrium (Ohlsson 1927). When Kapteyn (1905) discovered that the observed motions of the nearby stars could be represented in terms of “two star streams,” further kinematical work (Turner 1912; Strömberg 1918; Kapteyn 1922; Jeans 1922) implicitly began to isolate the local system from the rest of the Galaxy.

In the “two-stream” approximation, some of the stars of spectral type B appeared to be associated with Stream I, lying along the galactic belt (Halm 1911); but other B stars seemed to compose a “third” stream, lying along the southern part of the Gould belt and moving in the direction of the solar antapex (Eddington 1910a; Halm 1911). L. Boss (1911) and Campbell (1911) confirmed that the motions of the B stars were directed roughly parallel to the galactic plane and Kapteyn (1916) even found a slight acceleration of the motion of Stream I. Smaller moving groups of B stars were also discovered in relatively confined areas of the sky, viz. in Taurus (Mädler 1846), Sco–Cen (Eddington 1910a), Perseus (Boss 1910; Eddington 1910b; Kapteyn 1910a), Vela (Plummer 1913, Kapteyn 1914), Orion (Kapteyn 1918), Cas–Tau (Rasmuson 1921), and possibly other regions (Markowitz 1948). B. Boss (1910) found that some of the groups seemed to be radially expanding, as if from a point. Thus, the notion of an expanding stellar association arose (Ambartsumian 1949).

Two crucial discoveries led to the modern picture of the relationship between the local system and the Galaxy as a whole. First, Shapley in 1919 conclusively showed that the local system is only a small, outlying wisp in the

^{a1} CTIO is supported by the National Science Foundation under contract No. NSF-C866.

greater galactic system (although he incorrectly regarded the local star streaming as due to a radial translation of the local system away from the galactic center). Second, Lindblad and Oort, in the years 1925–1927, demonstrated the differential rotation of the Galaxy, including the local system, around Shapley's distant center in Sagittarius. Thus, the idea that the local system might be an object of high peculiar velocity (Strömberg 1924; Oort 1926; Shapley 1930) soon became untenable.

There persisted, nonetheless, a small positive residual in the peculiar radial velocities of the brighter B stars (Oort 1927; Plaskett 1928). This residual, amounting to about $+5 \text{ km sec}^{-1}$, had been discovered some years earlier (Frost and Adams 1904; Kapteyn and Frost 1910) and had been called the K term by Campbell (1911). Various interpretations of it, connected with the motion of the Gould belt, were subsequently suggested (Kapteyn 1910b; Boss 1911; Campbell 1911; von der Pahlen and Freundlich 1928; Plaskett 1930; Mineur 1930b; Ogorodnikoff 1932; Lindblad 1936; Pismis 1938; Bourgeois and Coutrez 1943; Toronzhadze 1950; Shain and Gaze 1953).

The vertex of the observed velocity ellipsoid for the nearby B stars was unexpectedly found to point somewhat away from the galactic center, although the amount of the deviation is difficult to determine because the velocities of these stars are relatively small and because the velocity ellipse in the galactic plane is nearly circular. On the "two-stream" hypothesis, however, the deviation of the vertex was found to be about 10° – 20° (Tannahill 1954; Clube 1973). There are several interpretations of this deviation based on the assumed motions of the Gould belt (Oort 1928; Lindblad 1935, 1936; Ogorodnikoff 1958; Bonino and Missana 1960; Kato 1968).

The earliest modern work on the internal kinematics of the local system purported to demonstrate a variety of motions, viz. axial rotation (Mineur 1930a), expansion (Mineur 1934), and spiral motion (Kaburaki 1933). The velocities of the stars were found to lie essentially in the plane of the local system (Dziewulski 1931). Further work has frequently indicated some rotational motion (Dziewulski 1938, 1947, 1959; Cuypers 1940; Armellini 1944; Schmidt 1949; Shatsova 1950, 1952, 1955; Ogorodnikoff 1950; Karpowicz and Zonn 1955; Karpowicz 1956a, 1956b, 1961; Filin 1957a, 1957b), but the derived rotational periods cover a wide range, $(5\text{--}200) \times 10^7 \text{ yr}$. In the case of expansion (Schmidt 1949; Toronzhadze 1950, 1953, 1956; Eggen 1961; Bonneau 1964; Clube 1967; Lesh 1968, 1972; Froeschlé 1969), the derived time scales span the range $(3\text{--}22) \times 10^7 \text{ yr}$. Moreover, the dominant direction of expansion may possibly be along an axis pointing toward or away from the galactic center (Eggen 1961; Bonneau 1964; Clube 1967). Individual associations within the local system show expansion ages varying from 3×10^5 to $2 \times 10^7 \text{ yr}$ (Blaauw 1964). This scatter in the ages of the associations, cou-

pled with the fact that "field" stars may possibly have a smaller K term than "association" stars in parts of the local system (Markowitz 1948; Lesh 1968), helps to explain the uncertainty of the overall expansion age for the system. In fact, other studies have preferred no systematic motion at all (Bok 1930; Mohr 1938; Moerdijk 1962; Fricke 1967; Jones 1971; Fricke and Tsoumis 1975).

Apart from the aforementioned expansion age, whose most probable value is $(4\text{--}7) \times 10^7 \text{ yr}$, another kinematical age can be derived. This age is based on the amount of deviation of the vertex of the velocity ellipsoid and is $\sim 5 \times 10^7 \text{ yr}$, according to three different theories (Ogorodnikoff 1958; Bonino and Missana 1960; House and Innanen 1975). The nuclear age of the local system is not unique because star formation is still going on, but the oldest clusters that are possible members of the system have nuclear ages of $8 \times 10^7 \text{ yr}$ (Eggen 1975). On the other hand, the "characteristic" members of the system have an average nuclear age of $3 \times 10^7 \text{ yr}$ (Paper I).

B. Interstellar Matter

Like the B stars, the local clouds of interstellar calcium (Slipher 1909) and of ionized hydrogen (Campbell 1911) have been shown to have very small peculiar motions. The K term derived from nearby neutral hydrogen is similar to that derived from B stars (Erickson, Helfer, and Tatel 1959) and may indicate an expansion of the local gas system (Davies 1960; Helfer 1961). That the outflow of gas is confined to low galactic latitudes and may conceivably be replenished by an inflow of gas from high galactic latitudes was first shown by McGee and Murray (1961). This picture was later elaborated by Weaver (1974) in his "eddy" model of the local system.

In a different model, McGee and Milton (1964) postulated the existence of a very large gas cloud encompassing the whole solar neighborhood and consisting of several lesser clouds, which are located, for example, in Sco-Oph, Pup-Vel, and Ori-Tau-Per. Their model of the gas, not tied too specifically to the Gould belt and certainly not ignoring the observed irregularities, is rather similar to Hansen's (1968) later model.

A third model of the local gas distribution (Lindblad 1967, 1974a; Lindblad, Grape, Sandqvist, and Schober 1973; Grape 1975) invokes simply an expanding ring of gas, centered near the Pleiades (or in Puppis according to Harten 1971). This ring is distorted into an elliptical shape by differential galactic rotation. The expansion age of the ring turns out to be $6 \times 10^7 \text{ yr}$. Dust clouds, seen in interstellar OH (Hughes and Routledge 1972), as well as interstellar formaldehyde (Lindblad *et al.* 1973; Dieter 1973; Sandqvist and Lindroos 1976) and also H I (Knapp 1974), share in the general expansion of the hydrogen gas. Grape (1975) has modified this picture into one of an expanding cloud whose ejected

TABLE I. Least-squares solutions for the local kinematical constants based on early-type stars and luminous supergiants.

Belt	Object	Sp.T.	Lum.	r (pc)	l_{\odot}	b_{\odot}	Radial velocities			Proper motions			N	
							V_{\odot} (km sec ⁻¹)	K (km sec ⁻¹)	l_{\odot}	b_{\odot}	V_{\odot} (km sec ⁻¹)	B (km sec ⁻¹ kpc ⁻¹)		
Gould	Stars	O-B5	All	0-200	69° ± 5	22° ± 8	21.8 ± 1.8	4.6 ± 0.7	53° ± 4	18° ± 3	20.7 ± 1.2	-20.6 ± 7.8	96	
		O-B2.5	All	0-200	65 ± 13	32 ± 23	20.2 ± 5.7	5.7 ± 2.1	64 ± 8	19 ± 6	19.7 ± 2.4	-34.0 ± 15.8	31	
	B3-B5	All	0-200	70 ± 5	19 ± 9	22.4 ± 1.9	4.8 ± 0.8	50 ± 5	18 ± 3	20.5 ± 1.5	-15.8 ± 9.8	65		
		O-B5	All	0-400	69 ± 4	25 ± 9	19.8 ± 1.6	2.4 ± 0.6	53 ± 3	22 ± 3	21.5 ± 0.9	-17.1 ± 3.6	305	
	O-B2.5	All	0-400	64 ± 7	13 ± 18	18.1 ± 2.2	5.5 ± 0.9	53 ± 4	22 ± 5	21.6 ± 1.6	-27.3 ± 6.1	84		
		B3-B5	All	0-400	68 ± 5	27 ± 10	20.3 ± 2.0	1.2 ± 0.7	53 ± 3	22 ± 3	21.1 ± 1.2	-12.9 ± 4.5	221	
	O-B5	All	0-800	66 ± 5	38 ± 8	21.3 ± 2.3	2.2 ± 0.6	51 ± 3	23 ± 3	21.3 ± 1.0	-20.7 ± 3.0	480		
		O-B5	I-V	0-800	67 ± 6	35 ± 10	19.5 ± 2.4	4.1 ± 0.7	53 ± 3	20 ± 3	21.2 ± 1.1	-22.7 ± 4.1	311	
	Galactic	Stars	O-B5	All	0-200	59 ± 5	19 ± 9	25.6 ± 2.3	2.6 ± 1.6	64 ± 3	14 ± 3	23.0 ± 1.2	-12.5 ± 9.6	71
			O-B2.5	All	0-200	58 ± 13	24 ± 27	32.3 ± 9.1	5.8 ± 6.1	38 ± 9	12 ± 3	22.8 ± 3.7	-25.7 ± 26.9	8
B3-B5		All	0-200	59 ± 6	18 ± 10	25.6 ± 2.6	2.4 ± 1.8	65 ± 3	13 ± 3	23.3 ± 1.2	-15.2 ± 10.6	63		
		O-B5	All	0-400	61 ± 4	16 ± 10	19.6 ± 1.6	1.7 ± 1.0	62 ± 3	17 ± 3	21.9 ± 0.9	-14.1 ± 4.6	208	
O-B2.5		All	0-400	50 ± 9	45 ± 19	29.8 ± 9.9	2.1 ± 2.8	56 ± 8	17 ± 8	21.7 ± 2.7	-0.3 ± 9.9	25		
		B3-B5	All	0-400	64 ± 4	12 ± 11	19.3 ± 1.5	1.6 ± 1.0	62 ± 3	16 ± 3	22.0 ± 1.0	-16.3 ± 5.0	183	
O-B5		All	0-800	61 ± 3	18 ± 9	18.5 ± 1.3	1.6 ± 0.7	61 ± 3	18 ± 3	21.8 ± 1.0	-10.1 ± 3.6	366		
		O-B5	I-V	0-800	58 ± 4	14 ± 11	21.1 ± 1.6	2.1 ± 1.0	66 ± 3	16 ± 3	23.3 ± 1.0	-14.8 ± 4.4	213	
Combined		Stars	O-B5	All	0-200	64 ± 3	11 ± 5	22.6 ± 1.1	4.2 ± 0.7	61 ± 2	19 ± 2	21.6 ± 0.8	-14.9 ± 6.2	167
			O-B2.5	All	0-200	59 ± 7	11 ± 15	20.7 ± 2.6	5.1 ± 1.6	60 ± 5	22 ± 5	20.2 ± 1.9	-31.1 ± 11.7	39
	B3-B5	All	0-200	64 ± 3	11 ± 5	23.0 ± 1.3	4.1 ± 0.8	61 ± 3	18 ± 2	21.7 ± 0.9	-13.4 ± 7.6	128		
		O-B5	All	0-400	64 ± 2	15 ± 5	19.5 ± 0.8	2.1 ± 0.5	59 ± 2	22 ± 2	21.4 ± 0.7	-15.9 ± 2.9	513	
	O-B2.5	All	0-400	57 ± 5	5 ± 13	17.9 ± 1.5	5.6 ± 0.8	53 ± 4	26 ± 5	21.6 ± 1.5	-21.3 ± 5.2	109		
		B3-B5	All	0-400	65 ± 3	17 ± 6	19.9 ± 1.0	1.3 ± 0.6	59 ± 2	21 ± 2	21.3 ± 0.8	-14.5 ± 3.4	404	
	O-B5	All	0-800	62 ± 2	25 ± 5	19.9 ± 0.9	1.8 ± 0.4	57 ± 2	23 ± 2	21.4 ± 0.7	-16.4 ± 2.3	846		
		O-B5	I-V	0-800	60 ± 2	10 ± 6	19.6 ± 0.9	3.2 ± 0.6	61 ± 2	21 ± 2	21.8 ± 0.7	-20.7 ± 3.0	524	
	Supergiants	O-M	Bright	0-800	37 ± 16	10 ± 63	17.7 ± 6.1	5.1 ± 3.0	42 ± 12	-6 ± 20	18.2 ± 3.8	-26.5 ± 6.5	24	

debris is now falling back to the galactic plane from high galactic latitudes.

In a fourth model, the apparent local expansion of the

hydrogen gas is found to be a minor perturbation in the overall gas flow predicted on the basis of the density-wave theory, in which two spiral arms are present near

the Sun (Burton and Bania 1974). Hence, the Gould belt system would be, in this model, more illusory than real. Lindblad (1974b), however, believes that the density wave has formed a true local system, and he invokes, for this purpose, a galactic shock in the Carina spiral arm 5×10^7 yr ago.

Substructure in the local gas distribution was once thought of as being lumpy or cloudy, but it may actually be wispy or filamentary (Weaver 1974; Heiles and Jenkins 1976; Sandage 1976). A small "tilted disk" of gas in the solar neighborhood has recently been identified (Fejes and Wesselius 1973), but it may not be real (Grape 1975; Heiles and Jenkins 1976). Nevertheless, strong concentrations of gas are known to be flowing out of young stellar clusters and associations (e.g., Kerr 1968). Other gas motions in the local system, like rotation, may also be present (Helfer 1961; Takakubo 1967), but in all cases the random gas velocities are very small, being equal to about half the random velocities of the stellar associations.

Pronik (1966) and Bingham and Shakeshaft (1967) have suggested, on the basis of interstellar polarization measurements, that the Gould belt has little or no magnetic field. Either the expansion (Bingham and Shakeshaft 1967) or the translation through space (Clube 1968) of the local system is postulated by these authors to have pushed aside and distorted the surrounding spiral arm (or interarm) field lines. Davies (1968), however, found possible evidence for a magnetic field parallel to the Gould belt, while Mathewson and his collaborators (Mathewson 1968; Mathewson and Nicholls 1968; Mathewson and Ford 1970) suggested that, superimposed on the longitudinal magnetic field of the general galactic disk, there is a helical magnetic field associated with the Gould belt. The large value of 59° found for the deviation of the vertex of the velocity ellipsoid for the nearby neutral hydrogen has also suggested the influence of a local magnetic field (Venugopal and Shuter 1969). Finally, magnetic "loops," "spurs," and "filaments" seem to abound in the local gas, particularly near the borders of the Gould belt (e.g., Heiles and Jenkins 1976).

II. SOLAR MOTION, DIFFERENTIAL GALACTIC ROTATION, AND K TERM

The nearby early-type stars have already been assigned, in Paper I, to the Gould and galactic belts on the basis of their positions in space. Presumably, if there exists a significant difference between the average kinematical properties of each belt, this difference should show up in solutions for the local solar motion, differential galactic rotation, and K term, based on the stars in each group. Since the errors of measurement of the radial velocities and proper motions are known to be systematically dependent on apparent magnitude (i.e., on distance), our present approach is expected to lead to more trustworthy results than the results of earlier

studies that generally did not employ groups of object and comparison stars with similar distances and spectral types. The present analysis is based on the following data for O–B5 stars: positions in space (Paper I), radial velocities having average mean errors of less than ± 3.7 km sec $^{-1}$ (see Appendix), and proper motions referred to the FK4 system (Smithsonian Astrophysical Observatory 1966). A mean error of ± 3.7 km sec $^{-1}$ in the inferred tangential velocities is attained at a distance of about $r = 400$ pc. Only stars having both radial velocities and proper motions known will be used here. N will denote the number of stars.

Earlier results have consistently indicated that Oort's constant A and the longitude of the axis of galactic rotation l_0 cannot be accurately determined by using stars as close to the Sun as are the members of Gould's belt. Therefore, we simply adopt here the standard values of $A = 15$ km sec $^{-1}$ kpc $^{-1}$ and $l_0 = 0^\circ$, although any other reasonable choice is found to make no significant difference in our final results. Furthermore, we also adopt, rather than solve for, the corrections to Newcomb's precessional constants, by using Fricke's (1972) recommended values. Since Fricke and Tsioumis (1975) found no significant difference between their separate solutions for the solar motion based on FK4 proper motions in right ascension and in declination, we have directly transformed the equatorial proper motion components into galactic proper motion components. The equations of condition for the radial velocities and for the proper motion components in galactic longitude and latitude are now set up in the usual way [e.g., Smart 1968, Eqs. 8.41(2), 8.42(2), and 8.42(3)]. In our notation and arrangement,

$$\begin{aligned} v_r - Ar \sin 2l \cos^2 b \\ = K - C_1 \cos l \cos b - C_2 \sin l \cos b \\ - C_3 \sin b, \end{aligned} \quad (1)$$

$$\begin{aligned} \mu_l \cos b - A\kappa^{-1} \cos 2l \cos b \\ = B\kappa^{-1} \cos b + C_1 (\kappa r)^{-1} \sin l \\ - C_2 (\kappa r)^{-1} \cos l, \end{aligned} \quad (2)$$

$$\begin{aligned} \mu_b + A(2\kappa)^{-1} \sin 2l \sin 2b \\ = C_1 (\kappa r)^{-1} \cos l \sin b + C_2 (\kappa r)^{-1} \sin l \sin b \\ - C_3 (\kappa r)^{-1} \cos b, \end{aligned} \quad (3)$$

where

$$C_1 = V_\odot \cos b_\odot \cos l_\odot, \quad (4)$$

$$C_2 = V_\odot \cos b_\odot \sin l_\odot, \quad (5)$$

$$C_3 = V_\odot \sin b_\odot, \quad (6)$$

and $\kappa = 4.74$, if r is measured in parsecs, V in kilometers per second, and μ in seconds of arc per year. Solutions are obtained by the method of least squares, with all stars assigned equal weight. The solution based on radial velocities v_r yields the K term, the Sun's velocity V_\odot , and the galactic coordinates of the apex of the Sun's motion, l_\odot and b_\odot . From the proper motions in galactic long-

itude, μ_l , are derived Oort's constant B , l_\odot , and the composite quantity $V_\odot \cos b_\odot$, while the proper motions in galactic latitude, μ_b , yield l_\odot , b_\odot , and V_\odot . Within the errors, the separate solutions based on the separate proper motion components do not differ very much and so the results from these two solutions have been combined. Final values for the derived kinematical constants are given in Table I, where mean errors (as elsewhere in this paper) are quoted.

It is comforting to notice that the derived kinematical constants are relatively insensitive to the inclusion or omission of stars without known luminosity classes (class V was assumed for these stars). There also appears to be no significant difference in the derived values of V_\odot , between the Gould belt and the galactic belt, and the small differences in l_\odot and b_\odot are probably caused by the different spatial concentrations of the stars in the two belts (at the longitude of the solar apex, the Gould belt lies close to its maximum latitude of 20° above the galactic belt).

Despite the rather large mean errors, the derived values of Oort's B for the Gould belt seem to be considerably more negative than for the galactic belt. This difference is due almost entirely to the contribution from the O-B2.5 stars (and presumably from the youngest of the B3-B5 stars), which are relatively more numerous in the Gould belt than in the galactic belt. Thus, it is the youngest component of the Gould belt that seems to represent a local perturbation of the general galactic velocity field, as other authors (e.g., Fricke and Tsioumis 1975) have also found. This does not necessarily imply a systematic rotation of the Gould belt, however.

There is also a definite indication that the K term for the Gould belt is larger than that for the galactic belt. Again, the difference arises primarily from the O-B2.5 stars, whose deviant behavior is also reflected in other very young objects, for example, the luminous supergiants and the B3-B5 stars of the Sco-Cen group. This latter group of stars dominates the Gould belt solutions for $r < 200$ pc (see also Froeschlé 1969). Thus, in the case of the youngest members of the Gould belt, the average K term is about 5 km sec^{-1} , which is to be compared with only $1-2 \text{ km sec}^{-1}$ for the typical nearby galactic belt B stars, which tend to be older objects and not members of associations.

In general, the most likely sources for the K term are group expansion and the gravitational redshift (e.g., Jones 1971). The latter source alone is expected to contribute, on the average, about 1.4 km sec^{-1} for the O-B2.5 stars and about 1.1 km sec^{-1} for the B3-B5 stars, if we employ the latest available data on stellar masses and radii in calculating the gravitational redshift for an average star in each of our two spectroscopic groups of stars. [This prediction for the gravitational redshift seems to be consistent with the small observed K term for the stars of the galactic belt, which is not generally considered to be expanding. More distant B stars in the galactic belt also show a K term of about 1 km sec^{-1}

TABLE II. Least-squares solutions for the local kinematical constants based on the radial velocities of dark dust clouds as seen in molecular emission.

Molecule	l_\odot	b_\odot	V_\odot (km sec^{-1})	K (km sec^{-1})	N
$^{12}\text{C}^{16}\text{O}$	$61^\circ \pm 2$	$31^\circ \pm 4$	25.8 ± 1.2	5.9 ± 0.5	90
$^{13}\text{C}^{16}\text{O}$	60 ± 2	25 ± 6	25.2 ± 1.5	5.6 ± 0.7	68
OH	58 ± 2	31 ± 6	22.6 ± 1.6	4.1 ± 0.5	72
H_2CO	61 ± 2	26 ± 4	23.2 ± 1.0	4.3 ± 0.4	185

(Feast and Shuttleworth 1965), which is further supported by the measured redshifts of B stars in clusters (Thackeray 1967).] For the Gould belt, then, the part of the K term that cannot be explained by the gravitational redshift is approximately 4 km sec^{-1} , if only the youngest members of the belt are used. Interpreted as being due to linear expansion of the Gould belt as a whole, such a velocity implies an expansion time of about 7×10^7 yr. But, since only the youngest stars seem to be expanding, Lesh (1972) and Fricke and Tsioumis (1975) have suggested that, possibly, the expansion arises from the individual expansions of the young associations within the Gould belt. (A simple geometrical argument shows that if all the associations have about the same number of stars, are expanding isotropically, and are moving, insofar as their centroids are concerned, randomly with respect to the local standard of rest, then a net expansion will be observed if one averages the measured peculiar radial velocities of all the stars.) On the other hand, the numerous dark dust clouds in the solar neighborhood (most of which lie along the Gould belt) also show a K term of about 4 km sec^{-1} , even when the solar motion is solved for simultaneously (see Appendix and Table II), rather than assumed, as in the work of earlier authors (Sec. I). If the K term arises from the individual expansions of associations, then the expansions must get under way before star formation begins.

A small part of the K term may arise from the following two additional effects, one of them geometrical and the other kinematical. Among the nearby stars, as compared with distant stars, the component of motion perpendicular to the galactic plane contributes rather heavily to the measured radial velocity, simply because the average nearby star lies at a higher galactic latitude than does the average distant star. Since stars as young as O-B2.5 stars have lifetimes that scarcely exceed the time necessary for these objects to reach the apices of their vertical motions, it might be expected that most of these stars will be observed moving away from, rather than toward, the galactic plane, if they are, in fact, born very near the plane. This preferred motion ought to show up as a positive K term which decreases with increasing distance from the Sun. It might also be expected that since the more distant O-B2.5 stars lie, on the average, farther from the galactic plane as a result of the tilt of the Gould belt, their vertical velocities, and hence their apparent K term, would be *a fortiori* smaller. However, almost all the O-B2.5 stars are actually found at or near

TABLE III. Mean peculiar velocities of B0–B5 stars with MK classifications (units of velocity are kilometers per second).

Belt	Sp.T.	r (pc)	$\langle \dot{x} \rangle$	$\langle \dot{y} \rangle$	$\langle \dot{z} \rangle$	$\langle V \rangle$	$\langle \dot{x}^2 \rangle^{1/2}$	$\langle \dot{y}^2 \rangle^{1/2}$	$\langle \dot{z}^2 \rangle^{1/2}$	$\langle V^2 \rangle^{1/2}$	$\epsilon(V)$	N
Gould	B0–B5	0–200	1.1	–4.7	0.3	12.3	8.6	9.3	5.5	13.8	2.8	81
	B0–B2.5	0–200	2.0	–5.8	–0.1	11.5	7.1	10.1	3.8	12.9	2.3	30
	B3–B5	0–200	0.5	–4.1	0.5	12.7	9.4	8.7	6.3	14.3	3.1	51
	B0–B5	200–400	0.6	–1.6	–2.0	19.7	13.3	15.4	11.8	23.5	6.0	111
Galactic	B0–B5	0–200	–1.2	–4.7	2.1	13.8	10.1	10.5	8.2	16.7	2.8	57
	B0–B2.5	0–200	–0.1	–1.6	0.7	9.1	4.6	7.8	2.9	9.5	2.6	8
	B3–B5	0–200	–1.3	–5.3	2.3	14.6	10.8	10.8	8.8	17.6	2.8	49
	B0–B5	200–400	0.4	–1.4	2.0	20.0	14.1	12.9	11.6	22.3	6.9	67
Combined	B0–B5	0–200	0.1	–4.7	1.0	12.9	9.3	9.8	6.8	15.1	2.8	138
	B0–B2.5	0–200	1.6	–4.9	0.1	11.0	6.6	9.7	3.7	12.3	2.3	38
	B3–B5	0–200	–0.4	–4.6	1.4	13.6	10.1	9.8	7.6	16.0	2.9	100
	B0–B5	200–400	0.5	–1.5	–0.5	19.8	13.6	14.5	11.7	23.1	6.3	178

the apices of their vertical orbits (Sec. III) and, therefore, this particular kinematical contribution to the total K term must be small. Moreover, the galactic concentration of these stars is so strong that the geometrical effect of increasing distance from the Sun must also be rather small.

In order to obtain an average picture of the general velocity field for all the O–B5 stars in the greater solar neighborhood, we have derived another set of solutions by using the stars of both belts (Table I). Our final results, based on a mean of the first, fourth, and seventh solution for the “combined” belts, may be expressed as

$$l_{\odot} = 60^{\circ} \pm 2^{\circ}, \quad (7)$$

$$b_{\odot} = +21^{\circ} \pm 2^{\circ}, \quad (8)$$

$$V_{\odot} = 21 \pm 1 \text{ km sec}^{-1}, \quad (9)$$

$$B = -15 \pm 5 \text{ km sec}^{-1} \text{ kpc}^{-1}, \quad (10)$$

$$K = 2 \pm 1 \text{ km sec}^{-1}, \quad (11)$$

derived under the assumption that $A = 15 \text{ km sec}^{-1} \text{ kpc}^{-1}$. For comparison, standard values of these quantities for stars of all spectral types combined are $l_{\odot} = 56^{\circ}$, $b_{\odot} = +23^{\circ}$, $V_{\odot} = 20 \text{ km sec}^{-1}$, and $B = -10 \text{ km sec}^{-1} \text{ kpc}^{-1}$ (Delhaye 1965; Schmidt 1965). While agreement is fairly good, it is worth emphasizing that the deviant behavior of B and K , shown by the very youngest objects of all, seems to be a kinematical peculiarity of the Gould belt.

III. PECULIAR VELOCITIES

To discuss the peculiar motions of the early-type stars, only the most reliable data that we have at hand will be used, viz. the data for the B0–B5 stars with MK spectral classifications and with distances less than $r = 400 \text{ pc}$. Values of the standard solar motion and of the differential galactic rotation, as quoted at the end of Sec. II for stars of all spectral types combined, will be subtracted from the observed velocities. A K term of 2 km sec^{-1} will also be subtracted.

The coordinate system adopted for the components of the peculiar velocities is analogous to the one adopted

in Paper I for the galactic positions. In rectangular coordinates, the \dot{x} component and \dot{y} component of the total velocity V increase parallel to the direction of the galactic center from the Sun ($l = 0^{\circ}$, $b = 0^{\circ}$) and parallel to the direction of the local galactic rotation ($l = 90^{\circ}$, $b = 0^{\circ}$), respectively, while the \dot{z} component increases parallel to the direction of the North Galactic Pole ($b = 90^{\circ}$).

The derived mean velocities and rms velocities for B stars in the Gould and galactic belts are listed in Table III. Mean errors ϵ of the total velocities are also given (the individual velocity components have nearly the same errors as these). In deriving the mean errors, a possible error of $\pm 0.3 \text{ mag}$ in the distance modulus of each star has been assumed. If the velocities and their errors were distributed normally, the true dispersion σ could be obtained from the formula $\sigma^2 = \langle V^2 \rangle^{1/2} - \epsilon^2$; however, additional small corrections should be introduced for systematic motions as well (see below). The observed distribution function for V is normal only in a crude sense. Since it is affected by clustering and streaming motions of the stars, its true form is probably not precisely representable (cf. Blaauw 1958; Stothers and Tech 1964; Feast and Shuttleworth 1965; Shatsova 1967).

Our numerical results indicate that the rms velocities increase with distance. [This increase continues beyond $r = 400 \text{ pc}$. For example, B0–B5 stars of both belts combined show $\langle V^2 \rangle^{1/2} = 49.4 \text{ km sec}^{-1}$ and $\epsilon(V) = 15.5 \text{ km sec}^{-1}$ for $400 < r < 600 \text{ pc}$.] The formal errors of these velocities are sufficiently small that formal corrections of them would also be very small. We suspect that the true errors at large distances are considerably larger than the formal ones; for example, Shatsova (1967) estimated that there is no increase of the velocities out to at least $r = 500 \text{ pc}$. The present results for $r < 200 \text{ pc}$ are in very good agreement with hers. Of course, the relative motions between associations of stars are considerably greater than the internal motions within individual associations and, therefore, the adoption of progressively larger volumes of space, encompassing a larger number of associations, could possibly lead to the observed general increase of the rms velocities with distance.

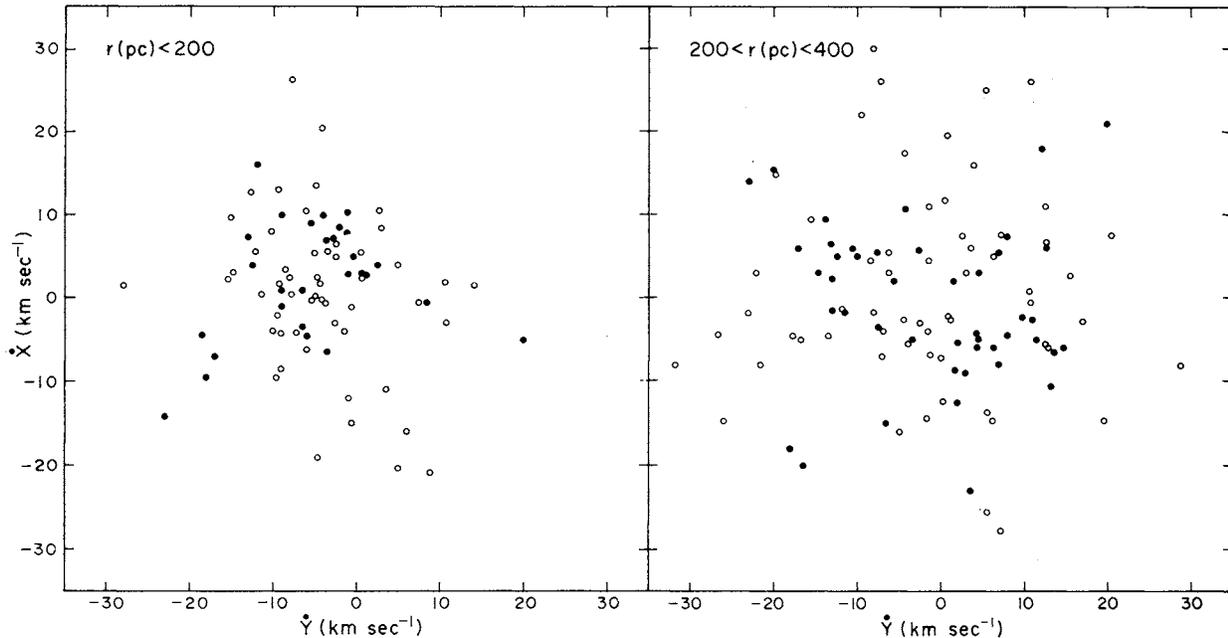


FIG. 1. Rectangular velocity components parallel to the galactic plane are plotted for all early-type stars within $r = 400$ pc assigned to the Gould belt. Filled and open circles refer to spectral types B0–B2.5 and B3–B5, respectively.

A. Motions Parallel to the Galactic Plane

Horizontal velocity components of the B stars are plotted in Figs. 1 and 2 for the Gould and galactic belts, respectively. Two ranges of spectral type, B0–B2.5 and B3–B5, are discriminated symbolically in these diagrams, which contain only stars with $r < 400$ pc in order that the mean observational errors in the velocities not exceed ± 3 km sec $^{-1}$ ($r < 200$ pc) or ± 7 km sec $^{-1}$ (200

$< r < 400$ pc).

It is clear that the scatter in these diagrams prohibits our determination of a meaningful velocity ellipse for the early B-type stars. The bimodal distribution of velocities found earlier by Yuan (1971) would appear to be simply a chance effect due to the small number of stars involved, since he used only 17 stars with $r < 100$ pc.

Very little kinematical difference between the two belts shows up on the \dot{x} , \dot{y} plane, according to our results.

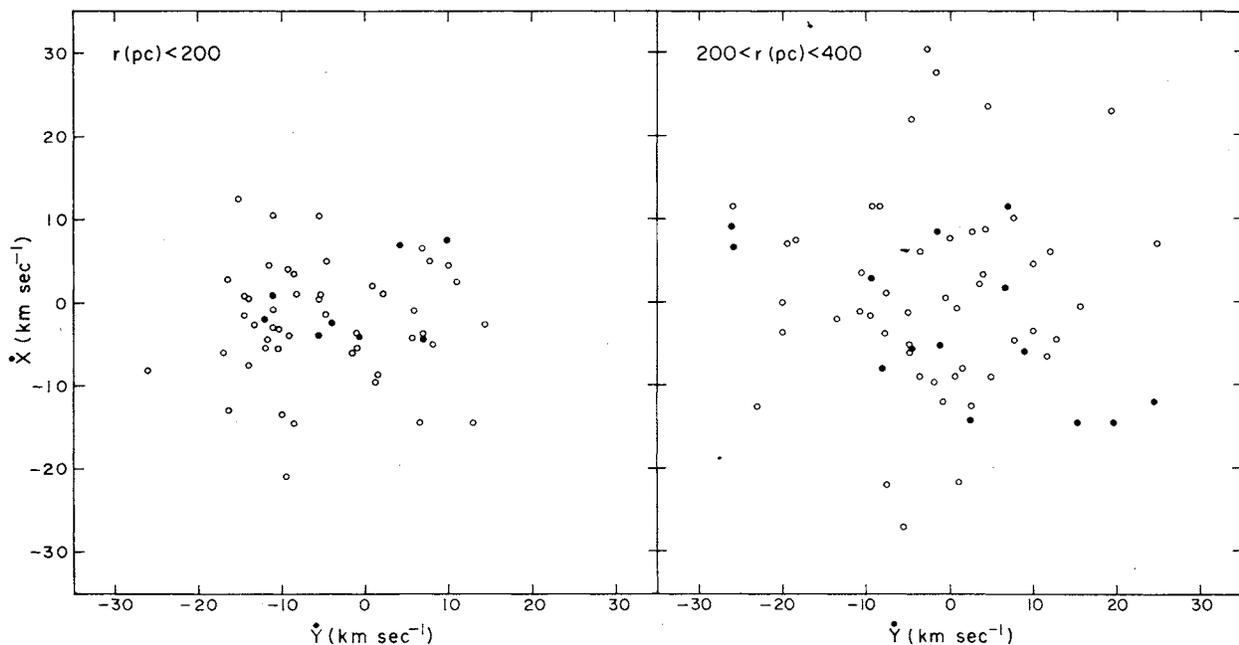


FIG. 2. Same as Fig. 1, but for the galactic belt stars.

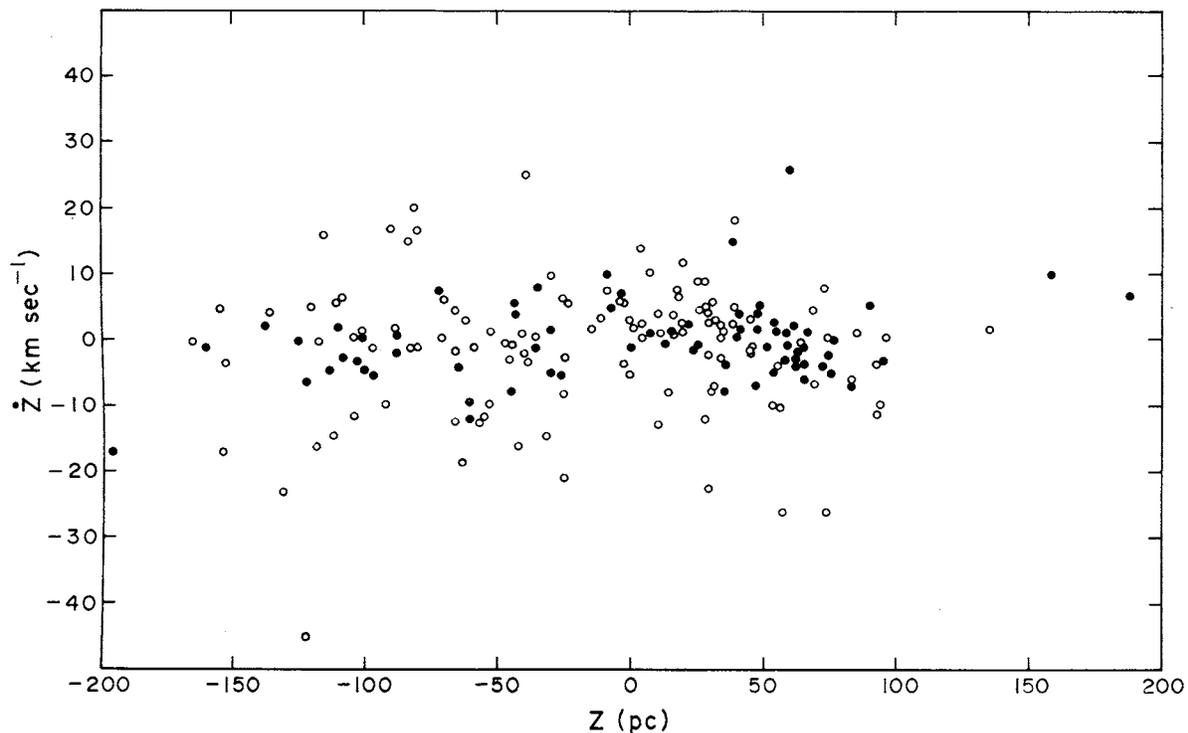


FIG. 3. The component of velocity perpendicular to the galactic plane is plotted against the vertical height above or below the galactic plane for all early-type stars within $r = 400$ pc assigned to the Gould belt. Filled and open circles refer to spectral types B0–B2.5 and B3–B5, respectively.

Eggen (1961, 1975) has attempted to discriminate members of the Gould belt from nonmembers by using kinematical criteria. However, the kinematical identities of some of his assigned stellar groups have been questioned before, viz. the groups in Vela (Blaauw 1946), Her–Lyr (Blaauw 1956), and Cas–Tau (Petrie 1958). Moreover, many of the more distant groups that are definite members of the galactic belt have motions that are not very dissimilar from those of the nearby stars (see Fig. 18 of Eggen 1961). Nevertheless, the nearest stars of all exhibit a small systematic difference in \dot{y} of about -3 km sec^{-1} with respect to the more distant stars—a result found also by Feast and Shuttleworth (1965). This difference is probably caused by the well-known streaming of the stars in Sco–Cen toward the solar antapex. Since our method of assigning stars to the two belts is not very accurate near the Sun (where the belts intersect each other), it is not surprising that some members of the Sco–Cen group seem to have been assigned, incorrectly, to the galactic belt, which appears to exhibit the same sort of streaming among its nearby stars as does the Gould belt.

B. Motions Perpendicular to the Galactic Plane

The vertical velocity components of the early B stars in the Gould belt with $r < 400$ pc are plotted in Fig. 3 as a function of their distances above or below the galactic plane. Notice that, among the B0–B2.5 stars in partic-

ular, several clumpings of stars appear, which reveal individual associations. For example, the clump centered at $z = 50$ pc refers to the Sco–Cen group and the one at $z = -120$ pc refers to the nearer portions of the Ori OB1 and Per OB2 associations. A comparison plot showing the stars of the galactic belt is presented in Fig. 4. The apparent asymmetry in the distribution of the stars in this figure is simply due to the fact that the galactic belt of early-type is inclined by an angle of about 5° to the IAU galactic plane. Notice that the galactic belt stars are located closer to the galactic plane, on the average, and have a much wider dispersion of vertical velocities at all z , than do the members of the Gould belt.

Since both the average values of the absolute velocities and the dispersions around these average values seem to be very small at all z distances in the Gould belt, one might conclude that most of the stars (especially the B0–B2.5 stars) have reached the apices of their vertical orbits, on the assumption that they all began their trajectories through space close to the layer $z = 0$ and at very nearly the same time. The period P of vertical oscillation near the galactic plane is independent of the initial velocity of the stars and equal to about 8×10^7 yr (Stothers and Tech 1964); therefore, the implied kinematical age of the Gould belt would be about $\tau = 2 \times 10^7$ yr, since

$$\tau = (P/2\pi) \tan^{-1}(2\pi z/P\dot{z}). \quad (12)$$

This age is of the same order as the characteristic nuclear

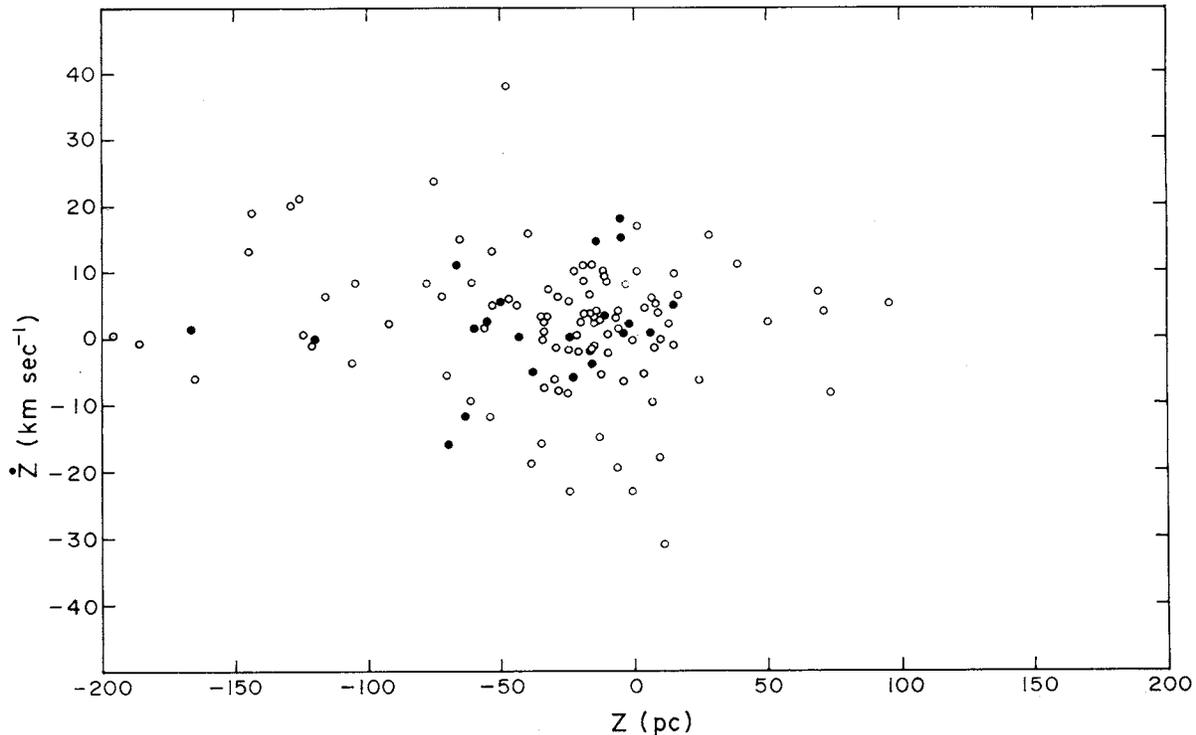


FIG. 4. Same as Fig. 3, but for the galactic belt stars.

age of the belt, 3×10^7 yr (Paper I), and the expansion age of the Sco-Cen group, 2×10^7 yr (Blaauw 1964; Jones 1971).

The slightly decreasing average value of \dot{z} , with increasing positive values of z , suggest that an age spread may exist in depth through the Sco-Cen group. Thus, the more distant members of the group may be the older, with kinematical ages ranging from 1.5×10^6 yr (near members) to 2.5×10^6 yr (far members). One possible explanation for this age spread is that the original disturbance that created the Gould belt moved very slowly across the galactic plane. In that event, the Orion and Perseus stars would be expected to have z motions very much out of phase with respect to the Sco-Cen stars. Since this does not seem to be the case, an alternative explanation of the small phase differences among the Sco-Cen stars is necessary. Perhaps random density fluctuations have distorted the local galactic distribution of mass, which determines the force and, hence, oscillation period for motions perpendicular to the galactic plane (cf. Oort, quoted in Blaauw 1952; Clube 1967). Further information on this point is, unfortunately, buried in the observational errors of Fig. 3, where the mean error is $\pm 5 \text{ km sec}^{-1}$. In any case, we do not confirm the very negative z velocities that Lesh (1968) found among the Orion and Perseus stars; our results for 53 B0-B2.5 stars in this region yield $\langle \dot{z} \rangle = -2.0 \text{ km sec}^{-1}$. For all the foregoing reasons, we believe it likely that the Gould belt was actually formed close to the layer $z = 0$, where heavy concentrations of matter are normally

found, rather than at high z (as in the contrary picture of Strauss and Poeppl 1976; Schmidt-Kaler and House 1976).

A possible interpretation, then, of the Gould belt is that about 2×10^7 yr ago some disturbance knocked a slab of gas and dust out of the galactic plane. Stars condensed as a result of the disturbance. The slab has, ever since, been executing a "seesaw" motion about a line in the galactic plane running from Cassiopeia to Carina. One end of the slab is now protruding out of the plane in the direction of Sco-Cen and the other end is pointing downward toward Orion.

It is necessary to point out that the Gould belt could actually be older than 2×10^7 yr. Equation (12) admits as a solution any age longer by an integer multiple of the half-period of vertical oscillation than the minimum age we selected above; hence, the true age could be 6×10^7 yr (the observed horizontal dissipation of the Gould belt probably excludes an age longer than about 7×10^7 yr). If this is so, it follows that the initial disturbance which created the Gould belt occurred 6×10^7 yr ago and, subsequently, when the belt again passed through the galactic plane, a second wave of star formation began as a result of encounters between the gas and dust clouds of the Gould belt and those of the galactic belt. Hence, the stellar ages in *both* belts could be broadly peaked around values of 2×10^7 and 6×10^7 yr. In fact, there is a noticeable peak in the nuclear age distribution of the somewhat older star clusters in the vicinity of the Sun at about 5×10^7 yr (spectral type B5; see the list of clusters

tabulated by Becker and Fenkart 1971). Within our limit of 400 pc, the sample of clusters is probably very nearly complete for spectral types earlier than B8.

A further prediction based on a greater assumed age for the Gould belt is that the most recent wave of star formation ought to have taken place primarily where the encounters between the gas and dust clouds of the two belts are strongest, i.e., at distances far from the Cassiopeia-Carina line. This may well be the case observationally, since the youngest stars are seen to be concentrated in Ori OB1, Sco-Cen, Per OB2, and regions of Taurus. An alternative interpretation of the absence of very young stars (and dust) close to the Sun is that local material has been used up in star formation and the site of active star formation has moved out into richer regions. In either case, the roughly uniform density of the Gould belt and the high concentration of dark dust clouds and very young stars in the Gould belt, as compared with the galactic belt, are consistent with the hypothesis that the former belt is a chunk expelled from the latter.

The original disturbance that created the Gould belt could have been a strong shock produced by a galactic spiral density wave, if we accept the picture of Roberts (1969) and Lindblad (1974b). The Gould belt would then have to be regarded as more than a statistical fluctuation of the flow field of the Galaxy (Heckmann and Strassl 1934; Burton and Bania 1974; Schmidt-Kaler and House 1976), though certainly less than an independent galactic subsystem (Shapley 1930; Kaburaki 1933; Schmidt 1949). It seems clear that the present inclination of the Gould belt to the galactic plane and the distribution of the stellar z velocities argue for an evolutionary interpretation of the belt's vertical structure, although the evolution of its horizontal structure remains unclear.

IV. CONCLUSION

After a historical review of the present subject, we have analyzed the observed space velocities of O-B5 stars lying within about 1 kpc around the Sun. In Paper I these stars were divided into members of the Gould and galactic belts on the basis of their positions in space. The present kinematical determinations of the Oort constant B and the K term confirm results by earlier authors that the Gould belt (or at least its youngest component) comprises a local perturbation in the general galactic velocity field. If the Gould belt is expanding as a unit (which is by no means certain), its expansion age must be about 7×10^7 yr.

Translational motion of the Gould belt relative to the galactic belt and parallel to the plane of the Galaxy seems to be negligibly small. However, our analysis of stellar motions perpendicular to the galactic plane shows some evidence for an approximate "seesaw" motion of the Gould belt about the Cassiopeia-Carina line in the galactic plane. The age of the Gould belt inferred from

the vertical motions is either 2×10^7 or 6×10^7 yr. This would imply the existence of either one or two waves of star formation as the Gould belt passed through the galactic belt either once or twice. The known nuclear ages of stars in the two belts lend some support to our proposed model.

Further progress will obviously depend on the acquisition of more accurate velocity data for O and B stars near the Sun and on the measurement of the velocities of additional stars of faint apparent magnitude.

APPENDIX: RADIAL VELOCITIES

A. Stars

A literature search was made for published radial velocities of the O-B5 stars whose positions were determined in Paper I. The different determinations of the radial velocity for each star were averaged by introducing a (partly subjective) system of weights based on the quoted mean errors, the numbers of plates taken, and an inspection of the radial velocity curves if the star was a variable. Following a comparative study by Petrie (1963), we first corrected the Lick radial velocities of stars fainter than apparent magnitude $m = 6.5$ by adding the following velocity increments: 5 km sec^{-1} if $6.5 < m \leq 7.5$, and 10 km sec^{-1} if $m > 7.5$. However, since few of our stars happen to be fainter than $m = 6.5$, the effect of making these corrections is undoubtedly small. The method that we adopted for the final assignment of quality classes to the radial velocities was that of Wilson (1953). We retained only stars with quality classes a , b , and c (average mean errors of not more than $\pm 3.7 \text{ km sec}^{-1}$). Our compilation was completed in 1966, but an updating of the data at the present time would not be expected to lead to any significant overall improvement. For the luminous supergiants, we simply used the radial velocities quoted by Humphreys (1970).

B. Dark Dust Clouds

A catalog of celestial positions and radial velocities of 242 dark dust clouds has been prepared by searching the literature. Only clouds with low velocities (solar neighborhood objects) have been retained. Most of the published velocities have been determined from the interstellar lines of the molecules $^{12}\text{C}^{16}\text{O}$, $^{13}\text{C}^{16}\text{O}$, OH, and H_2CO . Since these published velocities are given with respect to the local standard of rest as defined by an adopted solar motion of $V_{\odot} = 20 \text{ km sec}^{-1}$, $\alpha_{\odot}(1900) = 18^{\text{h}}$, and $\delta_{\odot}(1900) = +30^{\circ}$ (approximately $l_{\odot} = 56^{\circ}$ and $b_{\odot} = +23^{\circ}$), the original heliocentric velocities have first been recovered. Then, since the distances to only a few clouds are known, an *assumed* contribution from differential galactic rotation has been removed from the data (the characteristic $\sin 2l$ signature appears clearly in the velocities). A semi-amplitude of $\langle Ar \rangle = 4 \text{ km sec}^{-1}$

has been adopted; if $A = 15 \text{ km sec}^{-1} \text{ kpc}^{-1}$, then $\langle r \rangle = 270 \text{ pc}$, which is the average distance, approximately, of 40 dust clouds in Knapp's (1974) Table III. The actual adopted value of $\langle Ar \rangle$ is, however, unimportant for our applications. Most of the dark dust clouds appear to lie along the Gould belt (Lynds 1962), but the radial velocity coverage of the clouds is incomplete in the southern sky, except for some H_2CO observations.

REFERENCES

- Alexander, S. (1852). *Astron. J.* **2**, 95.
 Ambartsumian, V. A. (1949). *Astron. Zh.* **26**, 3.
 Armellini, G. (1944). *Commentat. Pontif. Acad. Sci.* **8**, 461.
 Becker, W., and Fenkart, R. (1971). *Astron. Astrophys. Suppl. Ser.* **4**, 241.
 Bingham, R. G., and Shakeshaft, J. R. (1967). *Mon. Not. R. Astron. Soc.* **136**, 347.
 Blaauw, A. (1946). Groningen Publ. No. 52.
 Blaauw, A. (1952). *Bull. Astron. Inst. Neth.* **11**, 414.
 Blaauw, A. (1956). *Astrophys. J.* **123**, 408.
 Blaauw, A. (1958). *Specola Vaticana Ric. Astron.* **5**, 105.
 Blaauw, A. (1964). *Ann. Rev. Astron. Astrophys.* **2**, 213.
 Bok, B. J. (1930). *Harvard Bull. No.* 876, 8.
 Bonino, C., and Missana, N. (1960). *Mem. Soc. Astron. Ital.* **31**, 459.
 Bonneau, M. (1964). *J. Obs.* **47**, 251.
 Boss, B. (1910). *Astron. J.* **26**, 163.
 Boss, L. (1911). *Astron. J.* **26**, 187.
 Bourgeois, P., and Coutrez, R. (1943). *Bull. Cl. Sci. Acad. R. Belg.* (5) **29**, 230.
 Burton, W. B., and Bania, T. M. (1974). *Astron. Astrophys.* **34**, 75.
 Campbell, W. W. (1911). *Lick Obs. Bull.* **6**, 101.
 Clube, S. V. M. (1967). *Mon. Not. R. Astron. Soc.* **137**, 189.
 Clube, S. V. M. (1968). *Observatory* **88**, 243.
 Clube, S. V. M. (1973). *Mon. Not. R. Astron. Soc.* **161**, 445.
 Cuypers, K. (1940). *Wiss. Natuurkund. Tijdschrift* **10**, 68.
 Davies, R. D. (1960). *Mon. Not. R. Astron. Soc.* **120**, 483.
 Davies, R. D. (1968). *Nature* **218**, 435.
 Delhaye, J. (1965). *Galactic Structure*, edited by A. Blaauw and M. Schmidt (Univ. Chicago P., Chicago), p. 61.
 Dieter, N. H. (1973). *Astrophys. J.* **183**, 449.
 Dziwulski, W. (1931). *Bull. Astron. Obs. Wilno No.* 12, 22.
 Dziwulski, W. (1938). *Bull. Astron. Obs. Wilno No.* 21, 3.
 Dziwulski, W. (1947). *Bull. Astron. Obs. Torun No.* 2, 17.
 Dziwulski, W. (1959). *Studia Soc. Sci. Torunensis (F)* **2**, No. 4, 15.
 Eddington, A. S. (1910a). *Mon. Not. R. Astron. Soc.* **71**, 4.
 Eddington, A. S. (1910b). *Mon. Not. R. Astron. Soc.* **71**, 43.
 Eggen, O. J. (1961). *R. Obs. Bull. No.* 41.
 Eggen, O. J. (1975). *Publ. Astron. Soc. Pac.* **87**, 37.
 Erickson, W. C., Helfer, H. L., and Tatel, H. E. (1959). *IAU Symp.* No. 9, 390.
 Feast, M. W., and Shuttleworth, M. (1965). *Mon. Not. R. Astron. Soc.* **130**, 243.
 Fejes, I., and Wesselius, P. R. (1973). *Astron. Astrophys.* **24**, 1.
 Filin, A. I. (1957a). *Astron. Zh.* **34**, 525.
 Filin, A. I. (1957b). *Astron. Zh.* **34**, 838.
 Fricke, W. (1967). *Astron. J.* **72**, 1368.
 Fricke, W. (1972). *Ann. Rev. Astron. Astrophys.* **10**, 101.
 Fricke, W., and Tsioumis, A. (1975). *Astron. Astrophys.* **42**, 449.
 Froeschlé, C. (1969). *Ann. Obs. Besançon* **8**, 7.
 Frost, E. B., and Adams, W. S. (1904). *Decennial Publ. Univ. Chicago* **8**, 105.
 Grape, K. (1975). Thesis, Univ. Stockholm.
 Halm, J. (1911). *Mon. Not. R. Astron. Soc.* **71**, 610.
 Hansen, H. K. (1968). *Phys. Abstr.* **71**, 3535.
 Harten, R. H. (1971). Thesis, Univ. Maryland.
 Heckmann, O., and Strassl, H. (1934). *Veröff. Univ. Sternw. Göttingen Nos.* 41 and 43.
 Heiles, C., and Jenkins, E. B. (1976). *Astron. Astrophys.* **46**, 333.
 Helfer, H. L. (1961). *Astron. J.* **66**, 160.
 House, F. C., and Innanen, K. A. (1975). *Astrophys. Space Sci.* **32**, 139.
 Hughes, V. A., and Routledge, D. (1972). *Astron. J.* **77**, 210.
 Humphreys, R. M. (1970). *Astron. J.* **75**, 602.
 Jeans, J. H. (1922). *Mon. Not. R. Astron. Soc.* **82**, 132.
 Jones, D. H. P. (1971). *Mon. Not. R. Astron. Soc.* **152**, 231.
 Kaburaki, M. (1933). *Jpn. J. Astron. Geophys.* **10**, 313.
 Kapteyn, J. C. (1905). *Br. Assoc. Adv. Sci. Rep.*, p. 257.
 Kapteyn, J. C. (1910a). *Trans. Int. Solar Union* **3**, 215.
 Kapteyn, J. C. (1910b). Lecture, Mt. Wilson Obs. (quoted by Campbell 1911).
 Kapteyn, J. C. (1914). *Astrophys. J.* **40**, 43.
 Kapteyn, J. C. (1916). *Mt. Wilson Ann. Rep.*, p. 255.
 Kapteyn, J. C. (1918). *Astrophys. J.* **47**, 104, 146, 255.
 Kapteyn, J. C. (1922). *Astrophys. J.* **55**, 302.
 Kapteyn, J. C., and Frost, E. B. (1910). *Astrophys. J.* **32**, 83.
 Karpowicz, M. (1956a). *Bull. Acad. Pol. Sci.* **4**, 587.
 Karpowicz, M. (1956b). *Bull. Acad. Pol. Sci.* **4**, 591.
 Karpowicz, M. (1961). *Postepy Astron.* **9**, 217.
 Karpowicz, M., and Zonn, W. (1955). *Acta Astron.* **5**, 78.
 Kato, S. (1968). *Astrophys. Space Sci.* **2**, 37.
 Kerr, F. J. (1968). *Nebulae and Interstellar Matter*, edited by B. M. Middlehurst and L. H. Aller (Univ. Chicago P., Chicago), p. 61.
 Knapp, G. R. (1974). *Astron. J.* **79**, 527.
 Lesh, J. R. (1968). *Astrophys. J. Suppl. Ser.* **17**, 371.
 Lesh, J. R. (1972). *Stellar Ages*, edited by G. Cayrel de Strobel and A. M. Delplace (Obs. Paris, Meudon), Sec. 23.
 Lindblad, B. (1935). *Mon. Not. R. Astron. Soc.* **95**, 663.
 Lindblad, B. (1936). *Mon. Not. R. Astron. Soc.* **97**, 15.
 Lindblad, P. O. (1967). *Bull. Astron. Inst. Neth.* **19**, 34.
 Lindblad, P. O. (1974a). *Highlights of Astronomy*, edited by G. Contopoulos (IAU, Dordrecht), p. 381.
 Lindblad, P. O. (1974b). *Stars and the Milky Way System*, edited by L. N. Mavridis (Springer, Berlin), p. 65.
 Lindblad, P. O., Grape, K., Sandqvist, A., and Schober, J. (1973). *Astron. Astrophys.* **24**, 309.
 Lynds, B. T. (1962). *Astrophys. J. Suppl. Ser.* **7**, 1.
 Mädler, J. H. von (1846). *Astron. Nachr.* **24**, 213.
 Markowitz, W. (1948). *Astron. J.* **53**, 217.
 Mathewson, D. S. (1968). *Astrophys. J. Lett.* **158**, L47.
 Mathewson, D. S., and Ford, V. L. (1970). *Mem. R. Astron. Soc.* **74**, 139.
 Mathewson, D. S., and Nicholls, D. C. (1968). *Astrophys. J. Lett.* **154**, L11.
 McGee, R. X., and Milton, J. A. (1964). *Aust. J. Phys.* **17**, 128.
 McGee, R. X., and Murray, J. D. (1961). *Aust. J. Phys.* **14**, 260.
 Mineur, H. (1930a). *Mon. Not. R. Astron. Soc.* **90**, 516.
 Mineur, H. (1930b). *Mon. Not. R. Astron. Soc.* **90**, 789.
 Mineur, H. (1934). *Bull. Astron.* (2) **9**, 41.
 Moerdijk, W. (1962). *Verh. K. Acad. Wet. Belg. No.* 68.
 Mohr, J. M. (1938). *Publ. Inst. Astron. Univ. Charles Prague No.* 21.
 Ogorodnikoff, K. (1932). *Z. Astrophys.* **4**, 190.
 Ogorodnikoff, K. (1950). *Publ. Astron. Obs. Leningrad* **15**, 1.

- Ogorodnikoff, K. (1958). *Dynamics of Stellar Systems* (Pergamon, Oxford) Chap. 7.
- Ohlsson, J. (1927). *Lund Medd.* (2), No. 48.
- Oort, J. H. (1926). *Publ. Groningen No.* 40, 63.
- Oort, J. H. (1927). *Bull. Astron. Inst. Neth.* **3**, 275.
- Oort, J. H. (1928). *Bull. Astron. Inst. Neth.* **4**, 269.
- Pahlen, E. von der, and Freundlich, E. (1928). *Publ. Potsdam No.* 86.
- Petrie, R. M. (1958). *Mon. Not. R. Astron. Soc.* **118**, 80.
- Petrie, R. M. (1963). *Basic Astronomical Data*, edited by K. Aa. Strand (Univ. Chicago P., Chicago), p. 64.
- Pismis, M. P. (1938). *Publ. Istanbul Univ. Obs. No.* 10.
- Plaskett, J. S. (1928). *Mon. Not. R. Astron. Soc.* **88**, 395.
- Plaskett, J. S. (1930). *Science* **71**, 225.
- Plummer, H. C. (1913). *Mon. Not. R. Astron. Soc.* **73**, 492.
- Pronik, I. I. (1966). *Astron. Zh.* **43**, 291.
- Rasmuson, N. H. (1921). *Lund Medd.* (2), No. 26.
- Roberts, W. W. (1969). *Astrophys. J.* **158**, 123.
- Sandage, A. (1976). *Astron. J.* **81**, 954.
- Sandqvist, Aa., and Lindroos, K. P. (1976). *Astron. Astrophys.* **53**, 179.
- Schmidt, H. (1949). *Veröff. Sternw. Bonn No.* 35.
- Schmidt, M. (1965). *Galactic Structure*, edited by A. Blaauw and M. Schmidt (Univ. Chicago P., Chicago), p. 513.
- Schmidt-Kaler, Th., and House, F. (1976). *Astron. Nachr.* **297**, 77.
- Shain, G. A., and Gaze, V. F. (1953). *Astron. Zh.* **30**, 130.
- Shapley, H. (1930). *Harvard Circ. No.* 350.
- Shatsova, R. B. (1950). *Publ. Astron. Obs. Leningrad* **15**, 113.
- Shatsova, R. B. (1952). *Publ. Astron. Obs. Leningrad* **16**, 67.
- Shatsova, R. B. (1955). *Astron. Zh.* **32**, 61.
- Shatsova, R. B. (1967). *Astron. Zh.* **44**, 396.
- Slipher, V. M. (1909). *Lowell Obs. Bull.* **2**, 1.
- Smart, W. M. (1968). *Stellar Kinematics* (Wiley, New York).
- Smithsonian Astrophysical Observatory (1966). *Star Catalog* (Smithson. Inst., Washington, DC).
- Stothers, R., and Frogel, J. A. (1974). *Astron. J.* **79**, 456.
- Stothers, R., and Tech, J. L. (1964). *Mon. Not. R. Astron. Soc.* **127**, 287.
- Strauss, F. M., and Poeppel, W. (1976). *Astrophys. J.* **204**, 94.
- Strömberg, G. (1918). *Astrophys. J.* **47**, 7.
- Strömberg, G. (1924). *Astrophys. J.* **59**, 228.
- Takakubo, K. (1967). *Bull. Astron. Inst. Neth.* **19**, 125.
- Tannahill, T. R. (1954). *Mon. Not. R. Astron. Soc.* **114**, 460.
- Thackeray, A. D. (1967). *IAU Symp. No.* 30, 163.
- Toronzhadze, A. P. (1950). *Doklady Acad. Sci. USSR* **74**, 441.
- Toronzhadze, A. P. (1953). *Bull. Abastumani Astrophys. Obs. No.* 15, 115.
- Toronzhadze, A. P. (1956). *Bull. Abastumani Astrophys. Obs. No.* 20, 45.
- Turner, H. H. (1912). *Mon. Not. R. Astron. Soc.* **72**, 387, 474.
- Venugopal, V. R., and Shuter, W. L. H. (1969). *Mon. Not. R. Astron. Soc.* **143**, 27.
- Weaver, H. (1974). *Highlights of Astronomy*, edited by G. Contopoulos (IAU, Dordrecht), p. 423.
- Wilson, R. E. (1953). *General Catalogue of Stellar Radial Velocities* (Carnegie Inst. Wash., Washington, DC).
- Yuan, C. (1971). *Astron. J.* **76**, 664.