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A NEW INTERPRETATION OF LUMINOUS BLUE STARS

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ABSTRACT

A major revision of current theoretical ideas about the brightest blue stars must be made if Carson's new radiative opacities are adopted in stellar models. Unlike earlier opacities, the new opacities exhibit a large "bump" due to CNO ionization, which leads to very strong central condensation, convective instability, and pulsational instability in hot, diffuse stellar envelopes (typically those in which $L/M > 10^3$ solar units). Despite a number of theoretical uncertainties, the new picture of the structure of very luminous stars is reasonably successful in accounting for a variety of previously unexplained observations. Thus, the new stellar models for the phase of core hydrogen burning predict large radii and rather cool effective temperatures (which are yet to be observationally confirmed) for O stars, and a spreading out of the main-sequence band in the H-R diagram toward luminous cool supergiants for masses higher than $\sim 20\ M_\odot$, beginning at $M_V = -4.5$ and Sp = B1. They also predict slower surface rotations for O stars compared with B stars; and, in binary systems, slower apsidal motions, closer rotational-revolutional synchronism, and smaller orbital eccentricities. In massive X-ray binary systems, circular orbits and supergiant-like visual companions are expected to be quite common. Radial pulsations of the models have been calculated by employing linearized nonadiabatic pulsation theory. Long-period variability is predicted to exist for massive blue supergiants of luminosity class Ia. The new models for helium stars predict large radii and rather cool effective temperatures for Wolf-Rayet stars, as well as multimodal pulsational instability and, possibly, surface turbulence for these stars. Ultrashort-period variability, observed in many classes of hot luminous stars, may be due, in part, to high radial overtone pulsations (or, possibly, to nonradial pulsational or convective modes).

Subject headings: stars: binary — stars: early-type — stars: interiors — stars: pulsation — stars: supergiants — stars: Wolf-Rayet

I. INTRODUCTION

The main sequence of stars in the H-R diagram is now generally regarded as a classical topic in astronomy, reasonably well understood in its essentials. It is conventionally interpreted as a narrow band of compact hydrogen-burning stars that are constituted, along the upper main sequence, of a convective core and a radiative envelope. Cooler giants are thought to be in the phase of core helium burning.

Very recently, however, suspicion has fallen on this traditional picture, because stellar models constructed with Carson's (1976) new radiative opacities suggest that the blue objects with the highest masses may be far from compact and may have sizable convective envelopes, while still burning hydrogen in their cores (Stothers 1974*a*, *b*, 1976). They may also be pulsationally destabilized by the opacity mechanism operating in the CNO ionization zone. This unexpected situation has prompted us to make the present survey of the equilibrium and pulsational properties of both homogeneous and inhomogeneous stellar models in the bright, hot parts of the H-R diagram. It will be shown (1) that the new opacities do, in fact, lead directly to a new theoretical picture of the most luminous stars (§§ II–IV) and (2) that observational data do tend to support the new theoretical models (and hence the new opacities), but must be reinterpreted in some cases (§§ V–VII).

II. INPUT PHYSICS

The input physics used in constructing the homogeneous and inhomogeneous stellar models of the present paper is the same as that used for the stellar models in our companion study (Stothers 1976). For orientation purposes, we summarize here the essential ingredients.

Carson's (1976) new radiative opacities, based on the hot "Thomas-Fermi" model of the atom for all elements heavier than hydrogen and helium, have been employed in the present work. They supersede the earlier preliminary opacities of Carson, Mayers, and Stibbs (1968), which are considerably larger in magnitude. The new opacities turn out to be quite similar to standard "hydrogenic" opacities, like those of Cox and Stewart (1965, 1970) except at very low densities, where the new opacities show a large "bump" due to the ultimate ionization of the CNO group of elements at moderate temperatures. Carson's opacities are given in the form of tables, within which linear interpolation (and extrapolation where necessary) has been employed between grid points.

Convection in the stellar envelope has been treated by Böhm-Vitense's (1958) version of the mixing-length theory, modified to allow for radiation pressure. The ratio of mixing length to pressure scale height, α, has been left as a disposable parameter, for which we have variously adopted the values $\alpha = 0$ (no convection), 0.5, 2, and ∞ (adiabatic convection). Convection in the core has been assumed to be adiabatic.

The helium abundance has been taken to be either $Y = 0.25$ or $Y = 1 - Z$. Two choices of the metals abundance, $Z = 0.02$ and $Z = 0.04$, have been adopted.

Axial rotation of the models has been neglected here. However, calculations of a few models including the effect of uniform rotation have been presented elsewhere (Stothers 1976).

III. EQUILIBRIUM MODELS

Equilibrium models have been constructed for a number of chemically homogeneous stars on both the hydrogen-burning and the helium-burning main sequences. We note that models for the cases $\alpha = 2$ and $\alpha = \infty$ have already been published (Stothers 1974*a*, *b*, 1976). Evolution off the main sequence, in the special cases $\alpha = 0.5$ and $\alpha = 2$, has also been computed, starting from the homogeneous "zero-age" stage (ZAMS) and running up to a "terminal" stage (TAMS) defined by a central hydrogen abundance of $X = 0.05$ (for the hydrogen-burning models) or a central helium abundance of $Y = 0.10$ (for the helium-burning models). A series of envelope models for various types of stars in more advanced stages of evolution has also been computed, and will be discussed in § IV.

In order to compare our present results based on Carson's opacities with earlier results based on the Cox-Stewart opacities, we may refer to the already published models computed with the Cox-Stewart opacities for hydrogen-burning main-sequence stars (Stothers 1972, 1974*b*). For stars on the helium-burning main sequence, new models have been computed here with the Cox-Stewart opacities, and they turn out to be in good agreement with previous ones computed by other authors (Divine 1965; Giannone 1967; Biermann and Kippenhahn 1971; Paczynski 1971; Arnett 1972; Dinger 1972; Savonije and Takens 1976).

A previous comparison of hydrogen-burning models based on the two sets of opacities has shown that the core structures of models having the same total mass and chemical composition are virtually identical (Stothers 1974*b*). The same result is found here for the helium-burning models. But the envelope structures depend sensitively on the adopted set of opacities, if the stellar mass is high. For high masses, the envelope densities are low, and therefore Carson's opacities are large in the CNO ionization zone. This leads to two effects not found when the Cox-Stewart opacities are adopted: (1) the envelope structure becomes distended and giant-like, and (2) convective instability breaks out in the layers with a high opacity. The smallest stellar mass for which such a convection zone appears is $\sim 10\ M_\odot$ for the homogeneous hydrogen-burning models, $\sim 2\ M_\odot$ for the homogeneous helium-burning models, and $\sim 6\ M_\odot$ for highly evolved supergiant models. In the case of the Cox-Stewart opacities, the run of opacity through the star is found to be nearly constant, and therefore the stellar radius is determined primarily by the nuclear reaction rate.

IV. PULSATIONALLY UNSTABLE MODELS

Pulsational characteristics of the inhomogeneous models have been computed in the same way as for the homogeneous models (Stothers 1976), by employing linear nonadiabatic pulsation theory. Radial pulsations alone are considered. The convective flux is assumed either (1) not to interact at all with the pulsations or (2) to adjust instantaneously to them. Neither assumption is probably very adequate, because the convective and pulsational time scales are of the same order of magnitude within the CNO ionization zone, where most of the driving takes place. Only the outer 0.5 percent of the stellar mass is relevant for the pulsations in the present (inhomogeneous) models, because the pulsation amplitudes drop off very rapidly just below the surface when the central condensation is high. Therefore we need compute only the outer envelope, which is specified by M, L, T_e, X, Z, and α.

The pulsational characteristics of the unevolved models have already been discussed elsewhere (Stothers 1976). To summarize: (1) the nuclear energy (ϵ) mechanism of pulsational instability is found to be unimportant; (2) the opacity (κ) mechanism, which operates here in the CNO ionization zone, destabilizes stars above a certain critical mass; (3) the assumption that convection adapts itself to the pulsations leads to greater instability than does the assumption of noninteracting convection; and (4) pulsational instability is aided by a larger value of L/M, T_e, and Z, or by a smaller value of α. Since the L/M ratio can be raised by various evolutionary effects (viz., by conversion of hydrogen into helium in the stellar core or by mass loss at the stellar surface), the best candidates for pulsational instability are evolved stars with hot effective temperatures.

a) *Supergiants*

The brightest luminosities attained by hot stars of a fixed mass can be estimated from published evolutionary tracks. Luminosities for stars of 15, 30, and 60 $M_\odot$ are entered in Table 1, and correspond (at least formally) to a bright phase of core helium burning if no mass loss has taken place, or to a late stage of core hydrogen burning if the star has lost, say, 20 percent of its mass.

The critical mass dividing pulsationally stable and unstable models (with the luminosities of Table 1) is found to be $\sim 25\ M_\odot$ if $Z = 0.04$ and if convection adapts itself to the pulsations. This value drops to $\sim 15\ M_\odot$ if we adopt $\alpha = 0.5$ instead of $\alpha = 2$, as was used for Table 1. At the opposite extreme, the critical mass becomes $\sim 60\ M_\odot$ if $Z = 0.02$ and if

TABLE 1

BLUE AND RED EDGES OF THE PULSATIONAL INSTABILITY STRIP FOR VERY LUMINOUS SUPERGIANTS*

$M/M_{\odot}$	$\log (L/L_{\odot})$	X	Z	Convection $\alpha = 2$	Mode	$\log T_e$ blue	$\log T_e$ red	P (hr) red	Q (day) red
15........	4.85	0.73	0.02	Nonadaptive	0,1,2	Stable	Stable	...	...
15........	4.85	0.71	0.04	Nonadaptive	0,1,2	Stable	Stable	...	...
15........	4.85	0.71	0.04	Adaptive	0,1,2	Stable	Stable	...	...
30........	5.53	0.73	0.02	Nonadaptive	0,1,2	Stable	Stable	...	...
30........	5.53	0.71	0.04	Nonadaptive	0,1,2	Stable	Stable	...	...
30........	5.53	0.71	0.04	Adaptive	0	(>4.6)	4.46	28	0.055
					1	Stable	Stable	...	...
					2	(>4.6)	4.43	18	0.029
60........	6.07	0.73	0.02	Nonadaptive	0	4.47	4.47	48	0.058
					1	4.53	4.53	22	0.039
					2	4.45	4.45	29	0.029
60........	6.07	0.71	0.04	Nonadaptive	0	(>4.6)	4.47	57	0.068
					1	4.45	4.30	126	0.047
					2	4.57	4.45	30	0.031
60........	6.07	0.71	0.04	Adaptive	0	(>4.6)	4.49	46	0.064
					1	4.49	(<4.1)	(?)	(?)
					2	(>4.6)	4.47	25	0.030

* Results for $\alpha = 0.5$ are not tabulated, but are shown in Fig. 6.

convection does not adapt itself to the pulsations. Despite its uncertainty, the critical mass is unquestionably lower than the critical mass for homogeneous main-sequence stars (which is $\sim 45\ M_{\odot}$ for $Z = 0.04$ with adaptive convection, or $> 400\ M_{\odot}$ for $Z = 0.02$ with nonadaptive convection). Notice also, in Table 1, that the first and second overtones (in the linearized theory) tend to be slightly more stable than the fundamental mode. A full-amplitude nonlinear study would be necessary to determine which mode(s) the star actually pulsates in.

Although the blue edge of the pulsational instability strip is too hot to be observed, the red edge nearly always falls near $\log T_e \approx 4.5$ for $\alpha = 2$. If α is reduced (as above) to 0.5, the red edge is shifted to $\log T_e = 4.45$, 4.28, and 4.18 for stellar masses of 15, 30, and 60 $M_{\odot}$, respectively. If $\alpha = 0.5$ and if the luminosity is also reduced, say, to a value appropriate to the phase of core hydrogen burning with no mass loss, the red edge is shifted back nearly to the ZAMS line, all models less massive than $\sim 45\ M_{\odot}$ being pulsationally stable. In Table 1, pulsational Q-values, defined by $Q = \text{Period}\ (M/M_{\odot})^{1/2}(R/R_{\odot})^{-3/2}$, are listed for the red edges based on $\alpha = 2$; they do not vary very much with effective temperature across the strip.

Not to be confused with the CNO-ionization strip is the helium-ionization (Cepheid) strip, whose fundamental *blue* edge, for Carson's opacities, is found to lie near $\log T_e = 3.9$, 4.0, and 4.1 for stellar masses of 15, 30, and 60 $M_{\odot}$, respectively. This derived edge is relatively insensitive to the choices of L, Z, and α. A detailed discussion of Cepheid models can be found in Carson and Stothers (1976).

b) *Helium Remnants of Supergiants*

It is now generally recognized that massive stars during their evolution can lose virtually all of their outer envelopes, under certain circumstances. Thus, a massive supergiant may be stripped all the way down to its helium core. In the event that a small hydrogen mantle remains, or that evolution toward or away from the helium main sequence is occurring, the effective temperature can be considerably lower than on the helium main sequence itself; therefore it is meaningful to compute a theoretical pulsational instability strip for helium stars possessing cool, extended envelopes of small mass.

Since it is already known that models of helium stars with masses less than 2–4 $M_{\odot}$ are pulsationally stable for the new opacities (Stothers 1976), only higher masses need be considered. Stellar masses of 8 and 15 $M_{\odot}$ will be adopted here, with the corresponding luminosities assumed, as a good approximation, to be equal to the equilibrium luminosities for homogeneous helium stars with the same total masses.[1] Characteristics of the derived pulsational instability strip, given in Table 2, turn out to be qualitatively similar to those determined for the supergiants. A detail worth remarking is that the red edge for the helium stars is somewhat hotter ($\log T_e \approx 4.6$ for $\alpha = 2$) because the stellar masses are smaller.

Three cautionary notes about the foregoing results for supergiants and helium stars should be sounded. First, convective modes in the CNO ionization zone may interfere with (and produce irregularity of) the otherwise periodic radial pulsations. Second, a linear nonadiabatic study of the lowest *nonradial* modes of pulsation (Vemury and Stothers 1976) shows that many of these modes, too, are unstable in the helium stars (but not in the supergiants, unless convection is somehow able to drive them) for degrees of the spherical harmonics up to at least $l = 5$. Third, for the higher L/M ratios, significant differences exist between the nonadiabatic pulsation periods and the

[1] The original masses of helium stars having present masses of 3, 8, and 15 $M_{\odot}$ would have been approximately 15, 30, and 45 $M_{\odot}$ on the hydrogen-burning main sequence.

TABLE 2

BLUE AND RED EDGES OF THE PULSATIONAL INSTABILITY STRIP FOR HELIUM REMNANTS OF SUPERGIANTS

$M/M_\odot$	$\log(L/L_\odot)$	X	Z	Convection $\alpha = 2$	Mode	$\log T_e$ blue	$\log T_e$ red	P (hr) red	Q (day) red
8	4.91	0.73	0.02	Nonadaptive	0	(>4.9)	4.62	6.8	0.061
					1	(>4.9)	4.77	1.3	0.032
					2	(>4.9)	4.83	0.6	0.022
8	4.91	0.00	0.02	Nonadaptive	0	(>4.9)	4.61	5.9	0.049
8	4.91	0.00	0.02	Adaptive	0	(>4.9)	4.58	7.1	0.049
					1	(>4.9)	4.66	2.3	0.027
					2	(>4.9)	4.72	1.0	0.019
8	4.91	0.00	0.04	Nonadaptive	0	(>4.9)	4.61	7.2	0.061
15	5.45	0.73	0.02	Nonadaptive	0	(>4.8)	4.51	32.1	0.073
					1	(>4.8)	4.73	3.6	0.039
					2	(>4.8)	4.76	2.2	0.029
15	5.45	0.00	0.02	Nonadaptive	0	(>4.8)	4.61	15.2	0.070
15	5.45	0.00	0.02	Adaptive	0	(>4.8)	4.32	102.4	0.062
					1	(>4.8)	4.62	6.5	0.033
					2	(>4.8)	4.71	2.5	0.023
15	5.45	0.00	0.04	Nonadaptive	0	(>4.8)	4.56	42.4	0.138

adiabatic pulsation periods, indicating that a full-amplitude study of the models is desirable.

c) *Comparison with Results Based on Cox-Stewart Opacities*

Models for luminous hot stars constructed with the comparatively small opacities of Cox and Stewart are pulsationally destabilized by the ϵ-mechanism (in the fundamental radial mode only) if the stellar mass is high and the L/M ratio is low. Results for homogeneous stellar models have been described elsewhere (Stothers 1976), but, to summarize them here, the critical mass for pulsational instability is $\sim 80\ M_\odot$ for hydrogen-burning stars and $\sim 15\ M_\odot$ for helium-burning stars. However, even a slight evolution away from the homogeneous state leads to great pulsational stability (Schwarzschild and Härm 1959; Stothers and Simon 1968, 1970). Addition of a thin hydrogen envelope on top of the helium-burning models also stabilizes them (Simon and Stothers 1969; Van der Borght 1969). Largely because the stellar envelopes are in radiative equilibrium, the degree of pulsational stability or instability is reliably determined for the Cox-Stewart (and other earlier) opacities.

As already mentioned, a CNO-ionization instability strip does not exist for the Cox-Stewart opacities, but the blue edge of the helium-ionization (Cepheid) instability strip occurs at $\log T_e \approx 3.75$ in the mass range of interest in this paper.

V. COMPARISON WITH O AND B MAIN-SEQUENCE STARS

Beginning with this section, a critical comparison of the new theoretical models will be made with relevant observational data. The various classes of stars discussed are those for which the theoretical models built with the Carson and Cox-Stewart opacities differ in some important respects.

a) *Mass-Luminosity Relation*

It has already been determined (Stothers 1974*b*) that, for a fixed mass and chemical composition, models of main-sequence stars constructed with (i) the Cox-Stewart opacities and (ii) the Carson opacities have almost identical luminosities. Therefore, the Carson opacities preserve the classical agreement obtained between theory and observation in the (mass, luminosity)-plane, including the correct inference of the gross chemical composition determined spectroscopically (see, e.g., Stothers 1972, 1973*b*).

A further point about the models based on Carson's opacities should be noted. In the CNO ionization zone, the radiative luminosity is found to exceed the local Eddington luminosity for stellar masses greater than $\sim 27\ M_\odot$ ($Z = 0.02$) or $\sim 19\ M_\odot$ ($Z = 0.04$). If convection is suppressed, then models for higher masses will not exist. But the known orbital masses of stars in binary systems range definitely up to 37 $M_\odot$ and possibly as high as $\sim 60\ M_\odot$ (Sahade 1962); furthermore, the evolutionary masses inferred from the luminosities of the brightest known supergiants are also as high as $\sim 60\ M_\odot$ (Stothers and Simon 1968) and perhaps even a little higher (see § VI*b*; also Osmer 1973). Therefore, it is safe to assert that $\alpha > 0$ in the CNO convection zone if Carson's opacities are adopted.

b) *Mass-Radius Relation*

Carson's opacities also preserve (Stothers 1974*b*) the earlier agreement found between theory and observation in the (mass, radius)-plane for well-observed B stars (Stothers 1973*a*; Popper 1974). Radius data for O stars are less reliable, but are plotted in Figure 1; the stars used are: UW CMa A and B, AO Cas A and B, Y Cyg A and B, and V444 Cyg B (Wood 1963); V448 Cyg B (Sahade 1962); LY Aur A (McCluskey and Kondo 1974); and V453 Sco B (Woodward and Koch 1975). We find that, for very massive stars, the models constructed with

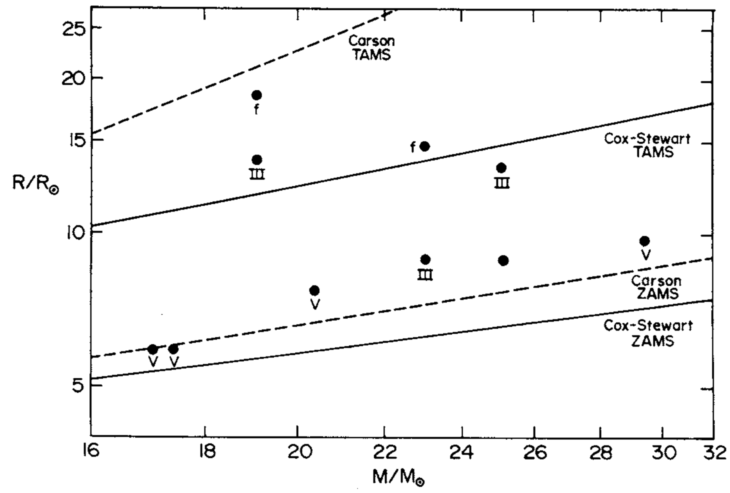

FIG. 1.—(Mass, radius)-relation for O stars. Filled circles represent observed stars, with luminosity classes indicated. Lines represent theoretical stellar models for the beginning and end of the main phase of core hydrogen burning, with $Z = 0.02$ and $\alpha = 2$.

Carson's opacities have significantly larger radii than do the older models, and provide better coverage of the observed stars, especially the more highly evolved ones. The O-type binary system that has the smallest known orbital separation, V382 Cyg, is not shown in Figure 1, but it is probably wide enough to accommodate the large radii predicted by Carson's opacities for two nearly contact stars of $\sim 37\ M_{\odot}$.

c) *H-R Diagram*

Unevolved models for main-sequence stars based on the Carson and Cox-Stewart opacities are compared on the theoretical H-R diagram in Figure 2. The two sequences agree very well up to masses of $\sim 20\ M_{\odot}$. At higher masses, the new models show increasingly larger radii (cooler effective temperatures) than do the older models. The amount by which the effective temperature is reduced in the new models is sensitive to α and Z.

There exists a possibility of inferring the correct values of α and Z by comparing the theoretical sequences with the observed ZAMS, which is also plotted in Figure 2. This line was constructed with the following data. For B-type stars, we accepted the empirical ZAMS of Blaauw (1963); his definition of the ZAMS was based on the *ridge line* of unevolved stars in the (M_V, $B - V$) diagram. For O-type stars, we have adopted the *lower envelope* of the empirical distribution of O5–O9.5 stars in the (M_V, spectral type)-diagram, as given by Conti and Alschuler (1971); the lower envelope seems to be a reasonable approximation to the ZAMS since the majority of the O stars that are available are probably at least partially evolved. The two sections of the ZAMS are found to connect up very smoothly. Bolometric corrections and effective temperatures listed by Morton and Adams (1968) and by Morton (1969) have been tentatively adopted for the B and O stars, respectively.

It appears that the models constructed with Carson's opacities cannot reproduce the observed ZAMS (as we have adopted it) in the range of O stars. On the other hand, the Cox-Stewart opacities seem to produce very good agreement. However, instead of necessarily throwing doubt on the newer opacities, this comparison rather leads us to examine the assumptions under which the adopted relation between spectral type and effective temperature for the O stars has been derived.

The two standard methods of obtaining this relation give, at present, essentially the same results. One of the methods is to compare observed intensities of spectral lines with theoretical predictions based on static model atmospheres, which are specified by the two quantities effective temperature and surface gravity (Peterson and Scholz 1971; Auer and Mihalas 1972; Conti 1973*b*). However, there is growing evidence (e.g., Scholz 1972; Conti 1973*a*; Morrison 1975) that main-sequence O stars show extended-atmosphere features, which are not present to any large degree in main-sequence stars of later spectral type. If the surface gravity really is lower than is usually assumed, then the implied effective temperature will be cooler (Conti 1973*b*). Although such a reduction of effective temperature may come into conflict with the hot effective temperatures implied by the other method, viz., the Zanstra mechanism (Hjellming 1968; Morton 1969), this latter method is clearly much more indirect, being based on Hα fluxes and radio fluxes of nebulae surrounding the O stars.

An independent method of estimating the effective temperature is the very simple approach of basing it on a broad-band color temperature of the continuum. The most thorough discussion of the intrinsic colors of O stars is probably that of Heintze (1973), who has

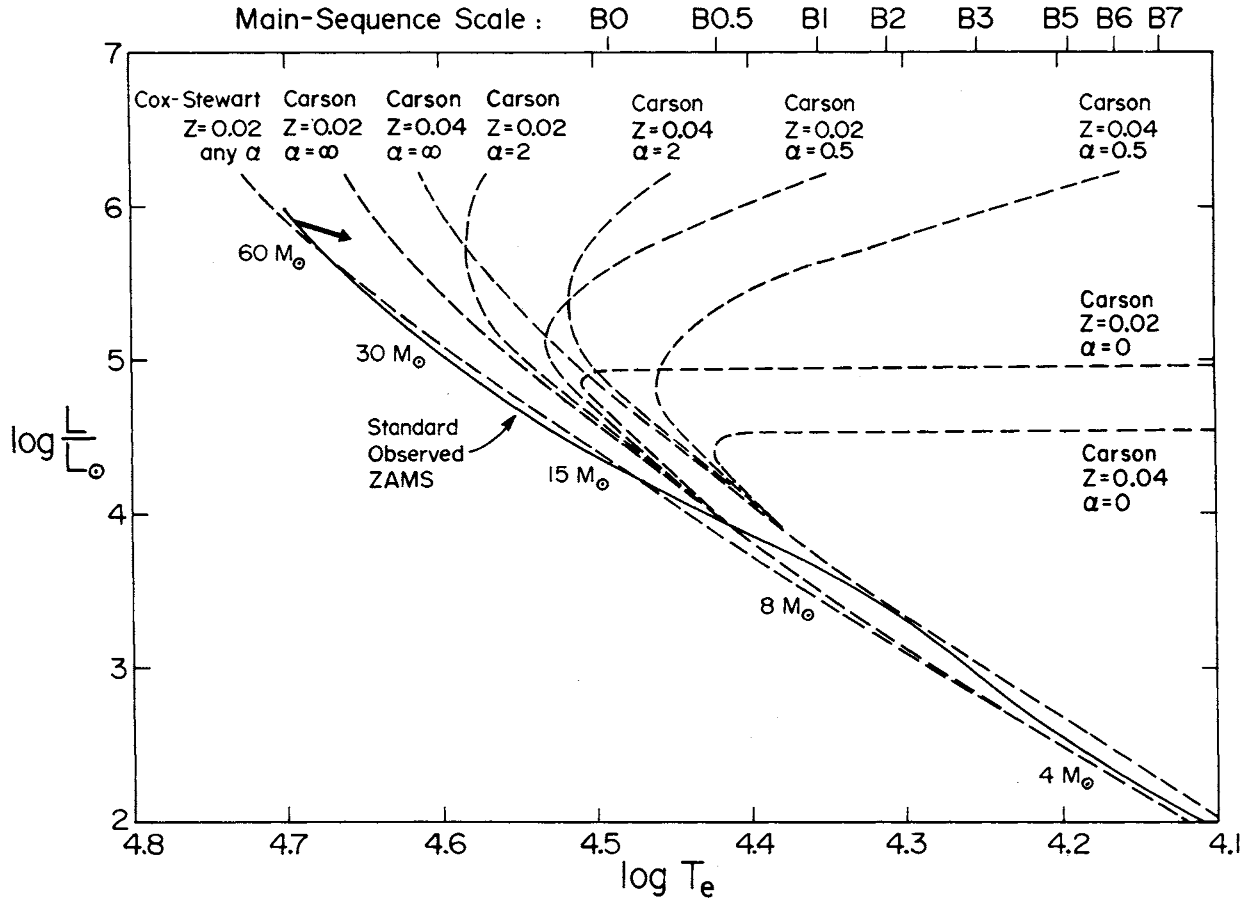

FIG. 2.—Theoretical H-R diagram showing various main sequences of homogeneous hydrogen-burning stellar models. A main sequence for $Z = 0.04$ based on the Cox-Stewart opacities is not shown, but would be cooler than the one shown for $Z = 0.02$ by about 0.03 in log T_e. The standard observed ZAMS is also indicated. The heavy arrow denotes the direction of possible revision of the ZAMS for O stars, as discussed in the text.

systematically unreddened the observed colors of a large number of O stars with the help of the mean color excess of B stars in the same star cluster, and has found that $(B - V)_0$ reaches its most negative value, -0.30, at spectral type O8, contrary to the earlier results of Johnson and Morgan (1953) and others, who found a monotonic decrease of color index with spectral subtype, based on far fewer data. This somewhat surprising result had already been adumbrated by Serkowski (1963) and Burnichon (1973), although, apparently, all observers ultimately rejected it, ascribing the anomalies to residual reddening from material surrounding the O stars, which have always been *assumed* to have hotter effective temperatures and hence bluer $(B - V)_0$ colors at earlier spectral subtypes. It is true that the earlier O stars are possibly ejecting, or are still embedded in, more circumstellar material than are the later O stars, and that their "unreddened" colors lie nearly along the reddening trajectory in the $(B - V, U - B)$-diagram (see Fig. 10 of Heintze 1973); but this trajectory is not very different from the blackbody line. Therefore, we suggest that perhaps massive O stars are intrinsically no bluer than $(B - V)_0 = -0.30$, which would correspond to an effective temperature of about 31,000 K (if standard model atmospheres apply). Similarly cool effective temperatures are indicated by ultraviolet flux measurements of the essentially unreddened O stars 15 Mon (O7) and ζ Pup (O5f) (Stecher 1970; Boksenberg *et al.* 1973), but these ultraviolet temperatures are, unfortunately, rather sensitive to any small amount of reddening that may be present.

Another method (Heintze 1973) of estimating the effective temperatures is provided by empirical radii and masses of components of eclipsing binary systems, in conjunction with the mass-luminosity relation. From the most reliable data tabulated by Wood (1963), we find the effective temperatures to be approximately 34,000 K for V444 Cyg B (O6), 24,000 K for UW CMa B (O7f), and 31,000 K for AO Cas B (O9).[2]

[2] It is likely that UW CMa B and AO Cas B are slightly hotter than stated, because their spectroscopic luminosity classifications indicate luminosities brighter than normal.

The method can be checked by applying it to Y Cyg (O9.5–B0), whose effective temperature is found to be 31,000 K—in good agreement with the model-atmosphere result for an O9.5–B0 star. Thus the effective temperatures of early O stars again turn out to be cooler than ordinarily admitted. A final method, based on the measurement of the angular diameter of an O star, has been applied to ζ Pup (O5f), which is found to have an effective temperature of only 31,000 K (Davis *et al.* 1970). It should be noted that the last two methods give the effective temperature directly as it is defined, viz. $T_e = (L/4\pi\sigma R^2)^{1/4}$.

Returning to Figure 2, we may conclude that, if the effective temperatures of O stars are really confined to the range 31,000–35,000 K ($\log T_e = 4.49$–4.55), then only the models constructed with Carson's opacities give agreement with a suitably revised observational ZAMS. In that case, values of $\alpha = 0.5$–2 and $Z = 0.02$–0.04 (it is not possible to be more specific) seem to be appropriate, and agree well with other astrophysical evidence concerning the mixing length in convective envelopes and concerning the abundances of the heavy elements. Furthermore, since the bolometric corrections of O stars will be smaller in absolute value than before, the revised bolometric magnitudes for these stars will be fainter; therefore, the very high luminosities (masses) previously obtained for the brightest known O stars (Conti and Burnichon 1975) will be brought more into line with the luminosities (masses) of the brightest known B and A supergiants.

d) *Axial Rotation*

Main-sequence B stars show average rotational velocities at their surfaces that are virtually independent of their spectral subtypes (Abt and Hunter 1962; van den Heuvel 1965). However, the average rotational velocities of O stars drop off sharply with earlier spectral subtypes (Boyarchuk and Kopylov 1958; Scholz 1972). While this behavior is not obviously explained by models based on the Cox-Stewart opacities, the very large radii and central condensations of the models for O stars constructed with Carson's opacities will lead naturally to a smaller surface rotational velocity if the total angular momentum of an O star is simply proportional to its mass, as seems observationally to be the case for B and A stars (Kraft 1970).

e) *Tidal Effects in Normal Binary Systems*

The tides raised in the components of a close binary system lead to three effects which depend critically on the interior structure of the components and which are also readily observable: (i) the line of apsides precesses at a rate governed by the interior density distribution in the components; (ii) the axial rotation of the stellar envelopes becomes synchronized with the orbital revolution at a rate determined largely by the extent of convection in the envelopes; and (iii) the orbit becomes circularized, again at a rate depending mainly on the extent of convection in the envelopes. The convective cores lie too deep to have much effect on synchronization and circularization times (Zahn 1966).

Apsidal-motion observations of close binary systems that contain main-sequence stars more massive than $\sim 7\ M_\odot$ indicate that these stars are more highly centrally condensed than the models constructed with the Cox-Stewart opacities would indicate. It has recently been shown that the Carson opacities provide about the right amount of central condensation to explain the observations (Stothers 1974*a*).

The masses of main-sequence stars in which deep convective envelopes are expected to exist are indicated in Figure 3. For masses less than $\sim 1.5\ M_\odot$, a deep convective envelope occurs as a result of the ionization of hydrogen and helium extending far below the surface. At higher masses, the envelopes of models constructed with the Cox-Stewart opacities are found to be radiative, except for a thin convection zone due to helium ionization just below the photosphere. But, when Carson's opacities are adopted, a deep convection zone arising from the ultimate ionization of the CNO group of elements develops in stars more massive than $\sim 10\ M_\odot$. (Nevertheless, the *mass* of any convective envelope shown in Fig. 3 is very small, covering less than 10^{-3} of the stellar mass.)

In well-observed binary systems with evolved (but still detached) stellar components, axial rotation of the components appears to be nearly locked into the orbit for periods ranging up to P_{syn} for the divisions of stars shown in Table 3, whose average properties have been determined observationally by Levato (1976). (It is sufficient here to compare orbital periods P instead of the physically more meaningful quantities R/a, because the latter are only weakly dependent on the stellar mass, viz., $R/a \sim M^{0.2}P^{-2/3}$, since $R \sim M^{0.5}$, approximately, for main-sequence stars of 1–15 $M_\odot$.) Theoretical calculations indicate that rotational-revolutional synchronism is attained far more slowly for a radiative envelope than for a convective one (Zahn 1966); for example, the theoretical synchronization time for a radiative envelope in a binary system containing two stars of $\sim 10\ M_\odot$ becomes just equal to the nuclear lifetime of the stars for the very short orbital period of $P = 4$ days (Zahn 1975). Moreover, one would expect P_{syn} to decrease slightly with increasing stellar mass if the envelope is radiative, since less time is available to achieve synchronization in a more massive (i.e., more rapidly

TABLE 3

Observed Limit to the Orbital Period for Synchronous Stellar Rotation in a binary System*

Sp	$M/M_\odot$	P_{syn} (days)
B2	10	11
B7	5	5
A5	2	4
F5	1.5	14

* After Levato (1976).

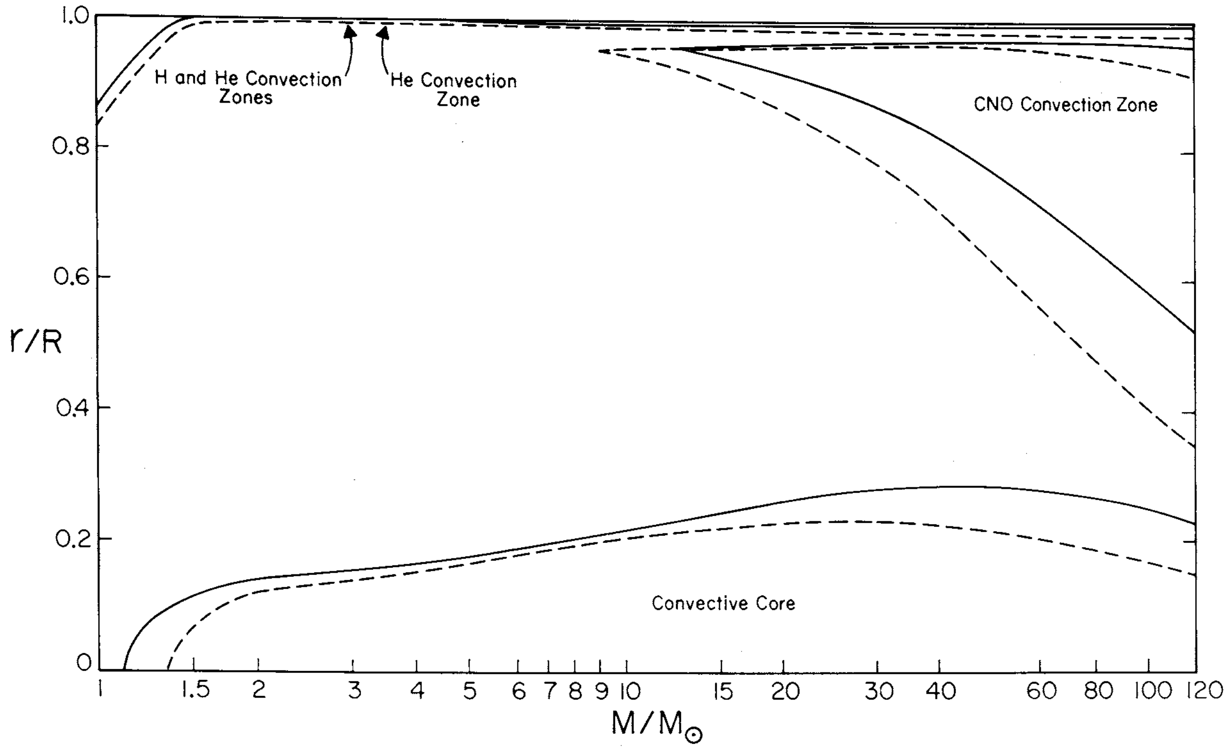

FIG. 3.—Spatial extent of the convective zones in homogeneous hydrogen-burning stellar models that are based on Carson's opacities and $\alpha = 2$. Solid and dashed lines refer to the convective boundaries for metals abundances of $Z = 0.02$ and $Z = 0.04$, respectively. Note that the radius fraction contained in the convective core is not monotonic with increasing stellar mass (however, the mass fraction is).

evolving) star. Therefore, the observational data given above strongly suggest that real stars of high mass, like those of very low mass, have convective envelopes.

The second test for a convective envelope employs the empirical distribution of orbital eccentricities as a function of stellar mass. Data are here taken from the extensive catalogs of Batten (1967), Lucy and Sweeney (1971), Pédoussaut and Ginestet (1971), and Pédoussaut and Carquillat (1973). The sample used is restricted to detached systems having components that are rather similar in spectral type and are more massive than 1 $M_{\odot}$. To increase the statistics for the very massive stars, we have assigned masses from an average (mass, spectral type)-relation for components earlier than B2 which do not have known orbital masses. Finally, we have omitted all systems with Batten's (1967) lowest quality class *e*. Figure 4 shows *both* components of systems with $P < 10$ days; one should note that an orbital eccentricity of less than 0.05 may not be significantly different from zero (Lucy and Sweeney 1971).

Two features of Figure 4 are important in the present context. First is the virtual absence of large eccentricities for masses less than $\sim 1.5\ M_{\odot}$. This is undoubtedly due to the presence of deep convective envelopes in stars with such low masses (Zahn 1966). The second feature is the marked absence of large eccentricities among systems having high masses and short periods. This is surprising, because a number of intermediate-mass systems with equally short periods have large eccentricities (the frequency of such systems is actually higher than Figure 4 would indicate, since the quoted catalogs do not list, for example, many of the apsidal-motion stars). If the stars with masses greater than $\sim 1.5\ M_{\odot}$ all had radiative envelopes, one would have expected to find larger eccentricities, on the average, at the higher stellar masses, since less time is available to circularize the orbit if the mass is high. We conclude that the most massive stars of all may possess deep convective envelopes.

It is possible to proceed further in a quantitative way by employing Zahn's (1966) theoretical analysis for the case of a convective envelope. He has shown that the exponential time scale for the decay of the orbital eccentricity is proportional to $P^{16/3}$ (see also Alexander 1973). The constant of proportionality is very uncertain, but we may adopt Zahn's value for his hypothetical system containing identical stars of 1 $M_{\odot}$, so that

$$\tau_e \sim 900 P^{16/3}\ \mathrm{yr}$$

if P is given in days. By equating the orbit circularization time to the nuclear lifetime, viz.,

$$\tau_{\mathrm{nuc}} = \tau_e\ ,$$

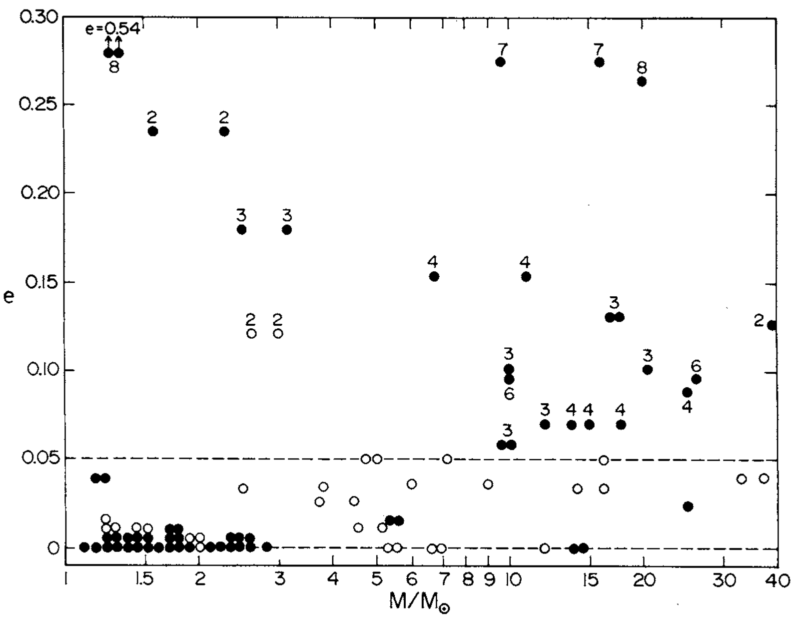

FIG. 4.—Orbital eccentricity as a function of stellar mass, for both components of double-lined spectroscopic binary systems. Symbols refer to the orbital period: $P < 2$ days (*open circles*) and $2 < P < 10$ days (*filled circles*); the orbital periods for systems with eccentricities significantly greater than zero are given explicitly in days.

we can determine a characteristic orbital period below which small eccentricities are expected to be common.

With $\tau_{\rm nuc} = 10^{10}$ yr (appropriate for low-mass systems) the characteristic orbital period turns out to be ~20 days. Among the observed low-mass systems in Figure 4, every one but HD 185912 ($P = 8$ days) has a small eccentricity. Note that the rather unique (and not plotted) eccentric system α CrB has a long period ($P = 17$ days) and that observational selection acts strongly against the detection of systems with periods as long as this. Therefore, we find at least no disagreement with the characteristic orbital period just derived, and we shall proceed to use Zahn's theory for stars of higher mass.

It is probably safe to assume that the constant of proportionality in Zahn's formula for τ_e is only weakly dependent on stellar mass and on details of the stellar convection zone. Therefore, the above formula gives, for a hypothetical system containing identical stars of 15 $M_\odot$ with $\tau_{\rm nuc} = 10^7$ yr, a characteristic orbital period of ~6 days if the stellar envelope is convective. This agrees very well with the distribution of observed periods in Figure 4, and may be taken as confirming the qualitative trend mentioned above.

f) *X-Ray Binaries*

Formation of a compact X-ray source in a close binary system probably involves very rapid mass ejection from the component which is to become later the X-ray source, and therefore probably causes a sudden elongation of the orbit. If the companion is a massive main-sequence or supergiant star, and has a deep convective envelope, the orbit should rapidly become circular again, provided that the period is sufficiently short. The probably small orbital eccentricities of Cen X-3 ($P = 2$ days) (Schreier *et al.* 1972), HD 153919 ($P = 3$ days), SMC X-1 ($P = 4$ days), and Cyg X-1 ($P = 6$ days), and the possibly large orbital eccentricity of Vel X-1 ($P = 9$ days) (Hutchings 1974) tend to confirm our suggestion that the massive early-type companions in these systems have deep convective envelopes.

VI. COMPARISON WITH EARLY-TYPE SUPERGIANTS

Several important tests of the new opacities are provided by observations of O, B, and A supergiants, in conjunction with the new theoretical models for evolving stars of high mass.

a) *H-R Diagram*

The domain of hydrogen-burning main-sequence stars in the H-R diagram is plotted on Figure 5. For both the Cox-Stewart and the Carson opacities, the theoretical main-sequence band widens for higher stellar masses. However, in the case of Carson's opacities, the width of the band suddenly increases enormously above ~20 $M_\odot$; hydrogen-burning stars of very high mass thus appear as early-type supergiants. (The scale of spectral types for supergiants, plotted at the top of Fig. 5, is taken from Stalio 1971 and Humphries, Nandy, and Kontizas 1975.) Regardless of the chosen values of α and Z, the point at which the main-sequence band suddenly flares out occurs around ($\log L/L_\odot$, $\log T_e$) = (4.5, 4.3) or (M_V, Sp) = (−4.5, B1).

In contrast, no such drastic widening occurs in the case of the Cox-Stewart (or any previous) opacities.

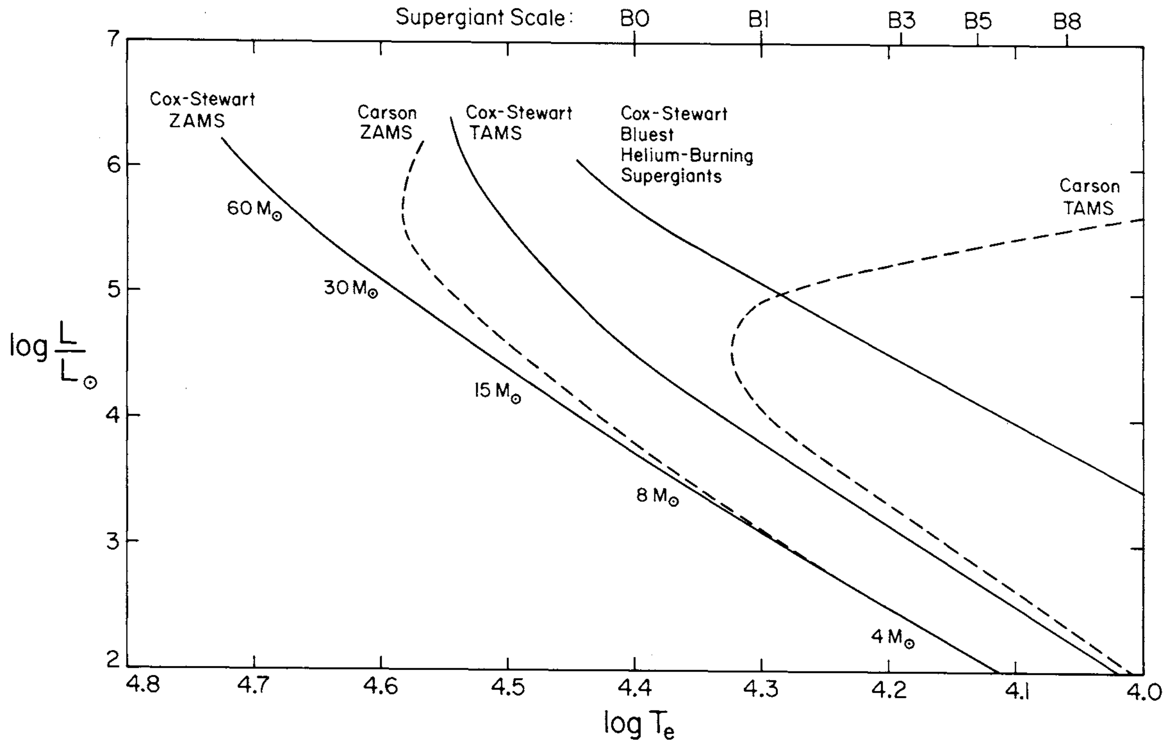

FIG. 5.—Theoretical H-R diagram showing the zone of occupation of stellar models during the main phase of core hydrogen burning, with input parameters $Z = 0.02$ and $\alpha = 2$. The bluest extension of stellar models during the phase of core helium is indicated for the case of the Cox-Stewart opacities.

Rather, the older models predict a large gap of ~ 3 mag in luminosity separating the main-sequence band of early-type hydrogen-burning stars from the supergiant band of early-type helium-burning stars (see Fig. 5). Evolution across the gap takes place very rapidly, on the Kelvin time scale of the stellar envelope. The gap may be even wider than we have indicated, because of our deliberate selection of the bluest extension achieved by helium-burning models computed with a variety of free parameters including initial chemical composition (Stothers and Chin 1975, 1976) and because of the relative insensitivity of the location of the main-sequence band to the chosen values for the free parameters (Stothers 1972). Although the drastic assumption of heavy mass loss can cause stellar models to fill in the gap (e.g., Tanaka 1966; Hartwick 1967; Chiosi and Nasi 1974), the observed rates of mass loss (interpreted as being due to radiation pressure, which is the one likely mechanism for hot stars if the Cox-Stewart opacities are adopted) are probably too low to have a significant effect on the evolution (Lucy 1975). It should be cautioned, however, that the gap may appear smaller in an (M_V, spectral type)-diagram because the spectral type at a given effective temperature is earlier for a supergiant than for a main-sequence star; on the other hand, the bolometric correction at a given spectral type is less for a supergiant, and this will moderate the first effect.

It has been noted previously as an unsolved problem (Stothers 1972) that, in well-observed clusters and associations containing supergiants of spectral type B1 and earlier, the predicted gap does not exist. A check of earlier composite H-R diagrams shows, in fact, a remarkably continuous distribution of stars between O5 and B1, and then a sudden bifurcation of the distribution into clearly defined main-sequence and supergiant branches toward later spectral types, starting at $M_V = -4.5$ (Kopylov 1955; Petrie 1965; Andrews 1968). The large observed number of supergiants among O–B1 stars is reflected also in the high incidence of them in early-type binary systems, both optical (Stothers and Lloyd Evans 1970) and X-ray (Pacheco 1975).

If, in fact, these supergiants are in the slow phase of core hydrogen burning (as their numbers would suggest), then the Carson opacities can account remarkably well for their peculiar distribution on the H-R diagram, How far into later spectral types the hydrogen-burning stars extend depends, of course, on the adopted values of α, Z, and stellar mass. Extrapolation of our results for the case $Z = 0.02$ indicates that the last stages of core hydrogen burning would actually occur in the red-supergiant configuration for masses perhaps as low as $\sim 30\ M_{\odot}$ if $\alpha = 2$. For a larger metals abundance or for a smaller value of α, the possible masses would be even lower. Ironically, this strange situation may, for a few of the very brightest objects, partially redeem the numerous (but unsuccessful) efforts during the 1940s to explain luminous red supergiants as stars still burning core hydrogen.

Detection of a dividing line on the H-R diagram between hydrogen-burning supergiants and helium-burning supergiants will not be an easy observational problem. No obvious division (unless it be the Hertzsprung gap itself) appears in the composite H-R diagrams for the Galaxy (Humphreys 1970) and for the Large Magellanic Cloud (Brunet and Prévot 1971). The aid of theoretical models for the helium-burning phase is evidently required. We do know, however, that models constructed with the *older* opacities cannot correctly reproduce the overall distribution of supergiants in the H-R diagram for masses exceeding $\sim 20\, M_\odot$ (Stothers and Chin 1975, 1976). It is also clear that our earlier observational tests for the proper criterion for convection and for the influence of neutrino emission in evolved stars—tests which depended on the ratio of the numbers of blue and red supergiants—will have to be redone, since there will now be a heavy contribution to the total number of blue supergiants coming from stars still burning core hydrogen.

b) *Cluster Dating*

Nuclear dating of young star clusters in the H-R diagram will also be affected by the new opacities, because stars more massive than $\sim 20\, M_\odot$ are now shifted to significantly cooler effective temperatures than was the case with the Cox-Stewart opacities. Main-sequence turnups earlier than B1 will be younger than previously assigned (by an amount dependent on α and Z). Ages based on the observed luminosities of blue supergiants will also be younger, since most of these stars must still be burning core hydrogen and hence must be intrinsically fainter than stars burning core helium. These revisions may help to reduce the current discrepancy between the long "nuclear" ages and the short "kinematic" ages that have been found for young star groups (Blaauw 1964; Stothers 1972). Any remaining discrepancy can probably be ascribed to a random scatter of the masses and of the times of formation of the member stars (Schlesinger 1972; Maeder 1972) and, more hypothetically, to mass loss from the main-sequence members (Tanaka 1966; Hartwick 1967; Chiosi and Nasi 1974).

c) *Variability*

The theoretical instability strip for luminous O stars and early-type supergiants on the H-R diagram is shown in Figure 6. "Instability" in the present context will be taken to refer to radial pulsation in the fundamental mode. In the case of the Cox-Stewart opacities the instability arises from the ϵ-mechanism operating in the hydrogen-burning core, whereas in the case of Carson's opacities it is due to the κ-mechanism operating in the CNO ionization zone of the envelope. Each region of instability, as plotted, is based on choices of the free parameters that are most favorable to pulsational instability. For Carson's opacities, this means bright luminosities, large metals abundances, and low values of α; for the Cox-Stewart opacities, it means small metals abundances and an essentially unevolved state of the interior. Since, for Carson's opacities, unstable radial overtones and various sources of irregularity are also predicted (see § IV), the observable consequences could well be a quasi-regular (possibly multiperiodic) variability of the light and radial velocity. On the other hand, all the modes in comparable stellar models constructed with the Cox-Stewart or earlier opacities are found to be quite stable.

It is well known that irregular or quasi-regular variability is actually observed in Of stars and in other early-type supergiants. The variations of color, light, and radial velocity are most marked among those stars with the hottest effective temperatures and the brightest luminosities (e.g., Appenzeller 1972; Maeder and Rufener 1972, 1974; FitzGerald 1973), just as Figure 6 would predict in the case of the new opacities. But some variability seems to exist all the way across the top of the H-R diagram. This suggests that the CNO-ionization instability strip, at very high luminosities, may merge into the helium-ionization (Cepheid) instability strip, which is not confined exclusively to low effective temperatures at such high luminosities, in the case of Carson's opacities (see § IV). For the most luminous B8–A3 supergiants (those with luminosity class Ia), Abt (1957) has found a single dominant periodicity in the radial-velocity and light variations, which yield a mean pulsation constant of $Q = 0.06$ day. For the B3 Ia supergiant 55 Cyg, which has a cyclic variation of 4–5 days (Granès 1975) and a visual absolute magnitude of -6.1 (from assumed membership in the Cyg OB7 association), we find just about the same Q-value. This suggests that the fundamental mode of radial pulsation may be the dominant mode appearing in most of the variable B3–A3 supergiants. However, it should be pointed out that the theoretical Q-value does depend on what physical quantities are adopted, and can range from 0.04 to 0.12 day (fundamental mode) for the stellar models of interest here, although 0.06 day is the most characteristic value obtained.

VII. COMPARISON WITH WOLF-RAYET STARS

Helium stars of high mass lacking any detectable hydrogen at their surfaces are found among Wolf-Rayet stars of spectral type WN5. This WN subclass and the other WN subclasses showing progressively more hydrogen enrichment at the surface are represented on Figure 7. The data have been taken from Smith (1973*a*), who adopted effective temperatures based on the Zanstra method from Morton (1970), bolometric corrections also from Morton (1969), and mostly her own visual absolute magnitudes. The approximate correctness of Morton's effective temperatures is supported, in part, by the value of 29,100 K derived from the measured angular diameter of the WC8 component of γ_2 Vel (Hanbury Brown

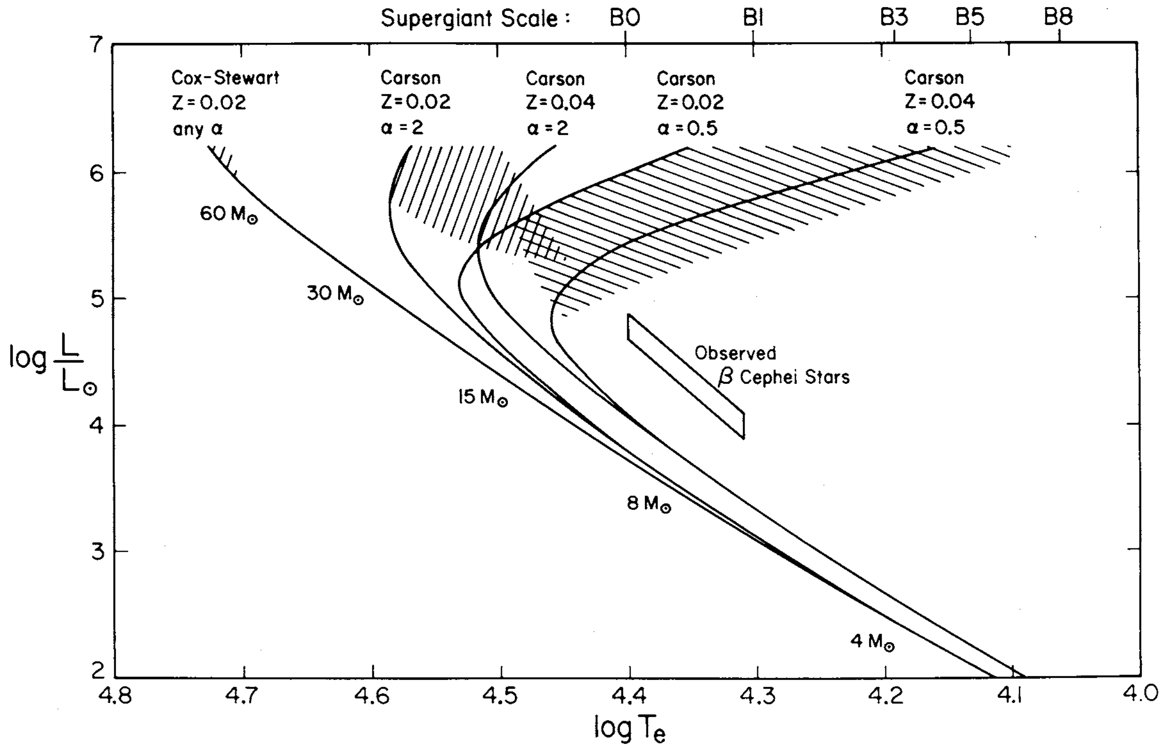

FIG. 6.—Theoretical H-R diagram showing the predicted regions of pulsational instability in the fundamental radial mode (under optimal conditions) for ordinary main-sequence stars and luminous supergiants. The two regions shown for stellar models constructed with Carson's opacities refer to $\alpha = 2$ (left) and $\alpha = 0.5$ (right). Corresponding main sequences of homogeneous hydrogen-burning stellar models are indicated for reference.

et al. 1970) and by the fact that significantly higher effective temperatures would lead to implied luminosities of these stars that are in excess of the Eddington limit (see Rublev 1975). The orbital masses of Wolf-Rayet members of binary systems seem to cluster around 11 $M_\odot$ (Kuhi 1973). If such masses are typical of Wolf-Rayet stars in general, then the high luminosities of these stars are almost certainly due to core helium burning (see Fig. 7, as well as Smith 1973*a*; Paczynski 1973).

This result leads to a number of important inferences. First, the brightest known Wolf-Rayet stars have luminosities that imply masses of only $\sim 20\ M_\odot$ (Smith 1973*a*). This would tend to rule out theories of these stars that rely on extremely high masses and small opacities (like the Cox-Stewart opacities) in order to permit nuclear-energized pulsations to be excited (e.g., Simon and Stothers 1970). In fact, since Wolf-Rayet stars represent almost certainly the exposed helium cores of stars that were at one time on the hydrogen-burning main sequence, the present upper mass limit of 20 $M_\odot$ implies an original mass of $\sim 60\ M_\odot$, which agrees well with the observed upper mass limit for main-sequence stars and supergiants. Probably the most realistic explanation of the origin of Wolf-Rayet stars (at least those in binary systems) is that they are the former primaries in systems that have experienced a mass exchange between the components (Paczynski 1967, 1973; Smith 1973*b*).

A second important inference from Figure 7 is that the effective temperatures of the hydrogen-exhausted WN5 stars are much too cool to be explained by models constructed with the Cox-Stewart opacities, but are comfortably explained by models built with Carson's opacities, particularly if reasonable values of α and Z are adopted. Moreover, if the observed Zanstra temperatures have been slightly overestimated (see § V*c*), then the new opacities are even more to be preferred. The same conclusion is indicated by the one directly observed radius for a Wolf-Rayet star, namely, 2.1 $R_\odot$ (Kron and Gordon 1950) for the WN5 component of V444 Cyg, which has a mass of about 10 $M_\odot$ (Kuhi 1973); the theoretical radius of a helium star with this mass and with $Z = 0.02$ is 2.3 $R_\odot$ (for $\alpha = 2$) based on Carson's opacities, but only 0.9 $R_\odot$ based on the Cox-Stewart opacities. Finally, the smallest known orbital separation in a Wolf-Rayet binary system, that in CQ Cep, is found to be not incompatible with the large radii predicted by the new opacities.

Since the addition of a hydrogen-rich envelope always increases the theoretical radius of a helium star, the observed trend of effective temperature with surface hydrogen abundance is in qualitative accord with theory (Smith 1973*a*). This does not imply that a hydrogen-rich envelope model constructed with the Cox-Stewart opacities (which have been adequately approximated by a purely electron-scattering opacity

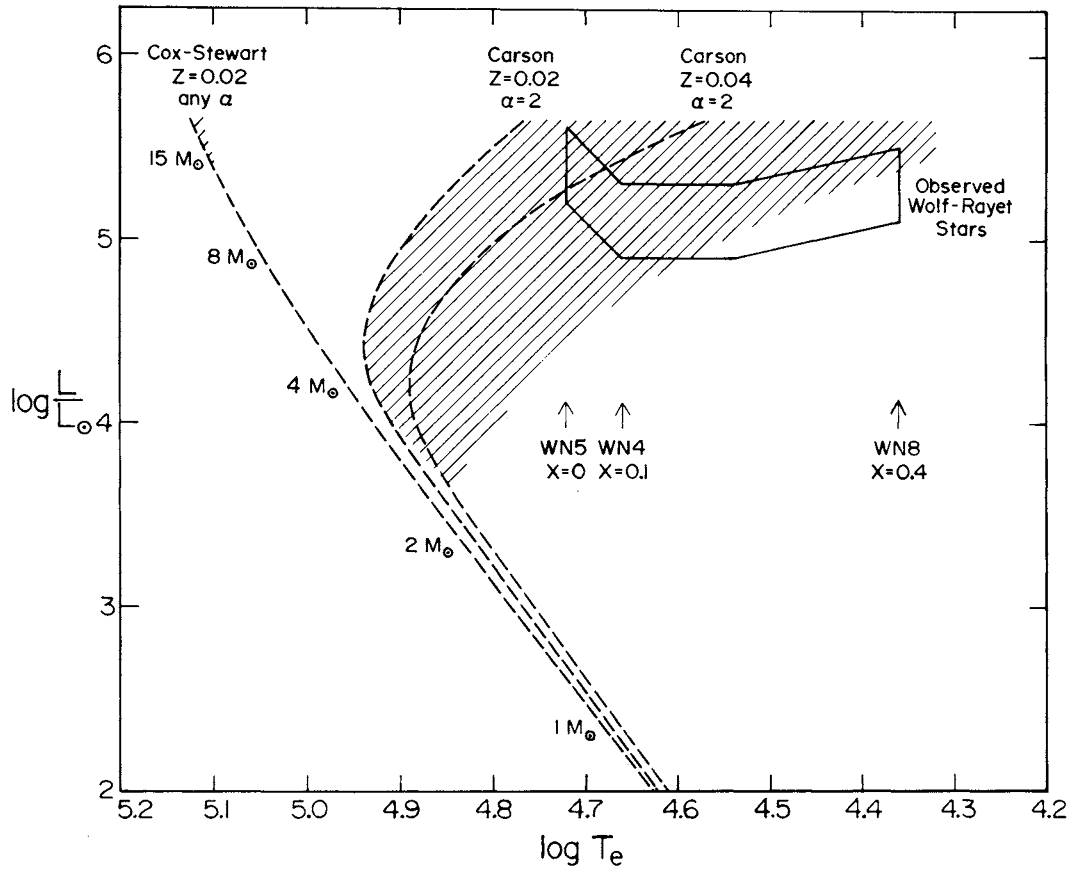

FIG. 7.—Theoretical H-R diagram showing the predicted region of pulsational instability in the fundamental radial mode (under optimal conditions) for helium remnants of supergiants, with $\alpha = 2$. Main sequences of homogeneous helium-burning stellar models are indicated for reference. A main sequence for $Z = 0.04$ based on the Cox-Stewart opacities is not shown, but would be cooler than the one shown for $Z = 0.02$ by about 0.01 in log T_e. The zone of occupation of most observed Wolf-Rayet stars is also shown; it is subdivided by spectral subtype and surface hydrogen abundance (Smith 1973*a*).

in the work of Simon and Stothers 1969) can reproduce the observations, because even a surface hydrogen abundance of $X = 0.1$ would be too low for such models to be satisfactory. Nor can evolutionary effects in the interior of the models constructed with the Cox-Stewart opacities be invoked to explain the low observed effective temperatures, because the theoretical "main-sequence" band for evolving helium stars is very narrow (see Fig. 8). The narrowness of the band is due to the rather small difference in mean molecular weight between the carbon core and the helium envelope. In the case of Carson's opacities, however, we may predict that further evolution of the models beyond the "main sequence" will take them into the region of massive, cool R CrB stars (Schmidt-Kaler 1961; Bessell *et al.* 1970); massive members of the R CrB class are impossible to explain by the very hot evolved models constructed with the Cox-Stewart opacities (Biermann and Kippenhahn 1971; Paczynski 1971; Trimble and Paczynski 1973).

The orbits of binary systems containing a Wolf-Rayet component are nearly all circular. This fact is usually attributed to the circularizing tendency of the mass-exchange process. However, it may also be due, in part, to tidal friction acting efficiently on the envelope of the Wolf-Rayet star (and of the massive companion) if, as is found in the new models, the envelope has a thick convection zone (see § V*e*). We may also expect the axial rotation of the Wolf-Rayet star to be synchronized with the orbital revolution, but nothing is known observationally about this question.

In some Wolf-Rayet stars the observed emission lines, and, less noticeably, the visual continua, show irregular or quasi-periodic variations of small amplitude on a time scale of minutes to hours (e.g., Underhill 1968; Moffat and Haupt 1974; Sanyal, Weller, and Jeffers 1974; Bahng 1975). However, stronger continuum variations may be occurring in the ultraviolet, if one allows for the effect of bolometric correction (Simon and Stothers 1970). The great breadth of the emission lines implies energetic atmospheric motions, although the precise nature of these motions remains uncertain, except for a definite component of radial expansion.

We suggest that these phenomena, taken in conjunction with the location of the theoretical instability strip on the H-R diagram (Fig. 7), can be explained

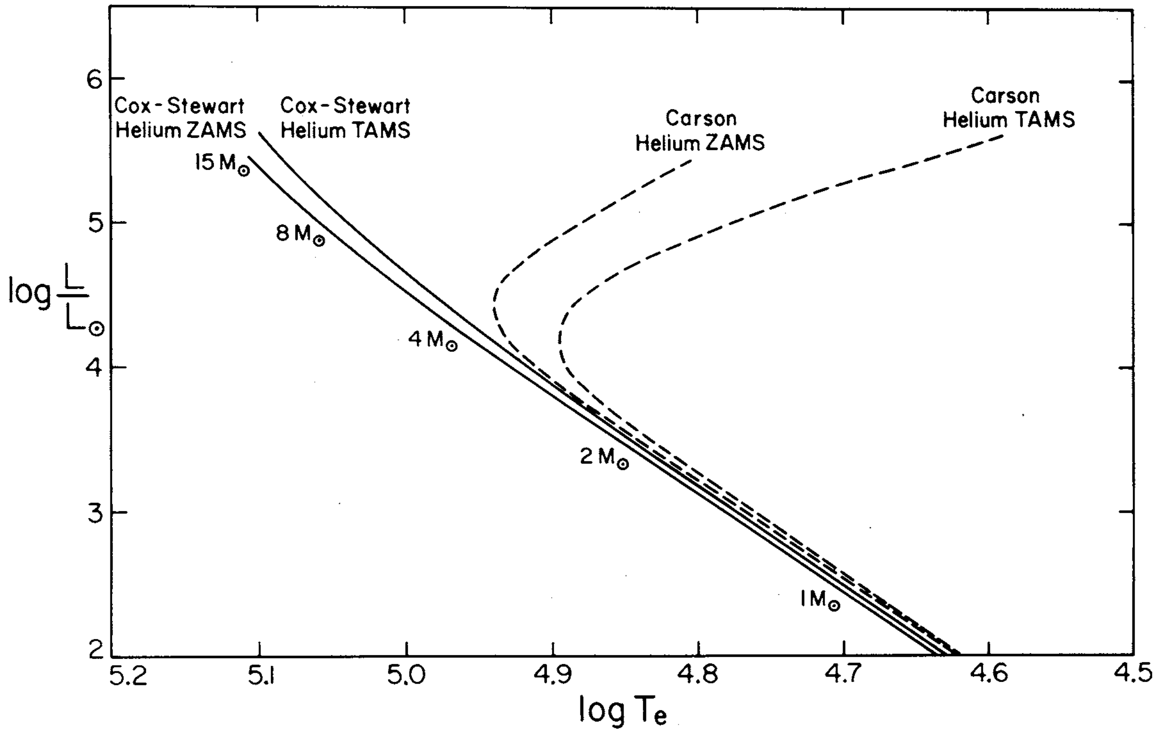

FIG. 8.—Theoretical H-R diagram showing the zone of occupation of helium-star models during the main phase of core helium depletion, with input parameters $Z = 0.02$ and $\alpha = 2$.

by multimodal pulsational instability in a massive helium star, if Carson's opacities are adopted. Pulsation was originally suggested for Wolf-Rayet stars by Paczynski (1967) and, independently, by Simon (1967) and Simon and Stothers (1969), on the basis of the pulsational instability found (for the fundamental radial mode only) in models of pure-helium stars more massive than $\sim 8\ M_\odot$, in which the opacity source was assumed to be purely electron scattering and in which destabilization was provided by the ϵ-mechanism (Boury and Ledoux 1965). But this idea was later discarded when it was discovered that use of the Cox-Stewart opacities raises the critical mass to $\sim 15\ M_\odot$ (Stothers and Simon 1970; see also Fig. 7).

The theoretical instability strip for models constructed with Carson's opacities, as plotted in Figure 7 for the fundamental radial mode, is only schematic, being dependent on the choices of α, Z, and pulsation mode. However, it is not sensitive to the hydrogen abundance in the envelope or to the evolutionary state of the interior. If, in fact, a large number of radial and nonradial modes are excited as linear theory predicts, there may also be resonances, beat periods, and an appearance of great irregularity. Moreover, since strong envelope convection in the models may reach the surface for masses higher than $\sim 5\ M_\odot$, atmospheric turbulence due to convection is also predicted. However, the pulsational instability is expected to disappear for $\log T_e \leq 4.3$. Perhaps significantly, this is just where one finds stars that are evolutionarily similar to, but spectroscopically very different from, Wolf-Rayet stars; these apparently stabler stars are the cool B-type components of massive semidetached binary systems.

VIII. CONCLUSION

The astrophysical implications of Carson's (1976) new radiative opacities have been explored for stellar models that lie in the hot, luminous portion of the H-R diagram. These new opacities, unlike older ones (e.g., Cox and Stewart 1965, 1970), exhibit a remarkable "bump" due to the ultimate ionization of the CNO group of elements at moderate temperatures and low densities. This "bump" turns out to have a maximum effect in diffuse stars with a high luminosity-to-mass ratio, and can lead to the following effects: (1) a large central condensation and a high radiation pressure in the envelope, (2) convective instability inside the CNO ionization zone, and (3) pulsational instability via the κ-mechanism. In most cases, instability is found to develop if $L/M > 10^3$ solar units.

Unfortunately, pulsational instability seems to occur only in models having a convectively unstable CNO ionization zone. Since the problem of convection and of its interaction with pulsation has only been solved in a crude, approximate way, our results are subject to much quantitative (although, it is hoped, not qualitative) uncertainty, owing to the sensitivity of the models to the adopted choice of metals abundance, convective mixing length, degree of interaction between convection and pulsation, possible importance of surface running waves, and grid size used in the opacity tables. Furthermore, a survey of the nonradial modes has not yet been made in sufficient detail. But, after this has been allowed, a number of surprising predictions of the stellar models have emerged, even in areas long considered well understood. A summary of what the new theoretical picture for stars of high luminosity can explain in areas where the old picture (based on earlier opacities) fails now follows.

1. *O and early B main-sequence stars.*—The predicted high central condensations of these stars seem to be confirmed by observations of the (mass, radius)-relation, axial-rotation rates, and apsidal-motion constants. The following predictions can be made: rather cool effective temperatures of O stars (if "reasonable" values of α and Z are adopted) and an increase of the minimum allowable orbital period in massive binary systems. Furthermore, the deep convective envelopes predicted to exist for stars more massive than $\sim 10\, M_{\odot}$ seem to be confirmed by observations of the close rotational-revolutional synchronism and of the small orbital eccentricities in massive binary systems.

2. *Early-type supergiants.*—The predicted spreading out of the main-sequence band of hydrogen-burning stars, beginning at $M_V = -4.5$ and Sp = B1 for masses higher than $\sim 20\, M_{\odot}$, is supported very well by the observational H-R diagram. (The term "main sequence" therefore loses its customary meaning at very high masses.) The observed quasi-periodic variability of supergiants of luminosity class Ia is tentatively accounted for, as is the general tendency for stronger variability among the brighter, hotter stars. Cluster ages are predicted to be younger than with earlier opacities.

3. *Wolf-Rayet stars.*—The predicted high central condensations of these stars are confirmed by the rather cool effective temperatures and large radii that are observed. Reasonable values of α and Z are implied. Convective envelopes are suggested by the atmospheric turbulence that is observed, as well as by the circular orbits of binary systems containing a Wolf-Rayet star. Multimodal pulsational instability is predicted to be a major cause of the observed variability.

4. *X-ray binary systems.*—The small orbital eccentricities and the supergiant character of the massive visual companions in these systems can be simply understood in terms of the new stellar models, as discussed above.

At the very least, the available observational data seem to corroborate the existence of a large source of stellar opacity around a temperature of 10^6 K under conditions of low density, i.e., in the diffuse outer envelopes of highly luminous stars. But uncertainties remain. Theoretical desiderata have already been enumerated; yet it should not be overlooked that further improvements in the opacities can be expected, too. In fact, a further *increase* of the opacities at intermediate temperatures could significantly improve agreement with observations, because (1) the region occupied by the quasi-periodic early-type supergiants in the H-R diagram extends to somewhat fainter luminosities and cooler effective temperatures than have been predicted theoretically, (2) the masses of the β Cephei stars are not high enough for these stars to be pulsationally destabilized by the present opacities, and (3) the observed synchronism between axial rotation and orbital revolution in very massive close binary systems may extend to slightly lower stellar masses than we have predicted. The most important evidence favoring *smaller* opacities (such as those of Cox and Stewart) consists of the hot observed spectroscopic and Zanstra effective temperatures of O stars; but we have pointed out (§ V*c*) that more direct ways of obtaining the effective temperatures yield lower values. On the observational side, more precise information is certainly needed on the effective temperatures of O stars and Wolf-Rayet stars. Further studies of massive binary systems could provide improved data on the shortest known orbital periods, the distribution of orbital eccentricities, the degree of rotational-revolutional synchronism in long-period systems, direct stellar radii, and apsidal-motion constants. A more intensive search for periodicities in luminous O–F supergiants and Wolf-Rayet stars would also be very valuable.

It is a pleasure to thank T. Richard Carson for his kind permission to use his new radiative opacities in advance of their publication.

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