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EVOLUTIONARY PROBLEMS OF CEPHEIDS AND OTHER GIANTS INVESTIGATED WITH NEW RADIATIVE OPACITIES

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ABSTRACT

New evolutionary tracks, pulsation constants, and pulsational stability coefficients have been calculated for stellar models applicable to the problems of classical Cepheids, by adopting (*a*) the standard Cox-Stewart opacities and (*b*) the new Carson opacities. The theoretical mass-luminosity relation is slightly fainter for the new opacities, but neither this decrease of luminosity nor the somewhat greater duration of the "blue loop" during core helium burning can be discriminated on the H-R diagram by the use of present observational data for giant stars in moderately young clusters of the Galaxy and the Magellanic Clouds. The masses of Cepheids inferred from the pulsation constants and from the pulsational period ratios are significantly smaller than the masses inferred from evolutionary theory, for both sets of opacities, but the masses that can be inferred from the secondary bump on the radial-velocity curve are now predicted to agree with the evolutionary masses if the new opacities are used. In contrast to the older opacities, the new opacities lead to revised pulsational instability results that are in good agreement with observations: thus, convection in the hotter envelopes is much less important than before, so that the phase lag between luminosity and radial velocity at the stellar surface is closer to 90°; and the blue edge of the theoretical instability strip on the H-R diagram is significantly bluer than before, although it may now have too steep a slope and the problem of the theoretically unlimited red edge remains. Nevertheless, the present improvements are achieved with a normal helium abundance, no mass loss, and a conventional treatment of convection. The predicted transition between second-overtone instability and fundamental-mode instability agrees excellently with the well-observed transition in the Small Magellanic Cloud. Axial rotation is found to increase the effective temperature of the theoretical blue edges by a very slight amount.

Subject headings: clusters: open — stars: Cepheids — stars: evolution — stars: interiors — stars: pulsation — stars: rotation

I. INTRODUCTION

Among the various problems presented by the classical Cepheids, two important ones stand some chance of solution by using improved opacities in the computation of both equilibrium properties and linearized pulsational properties of relevant stellar models. First, the masses of Cepheids have frequently been estimated from their observed periods, luminosities, and effective temperatures by employing a theoretically determined pulsation constant, Q, defined in conventional notation by

$$Q = P(M/M_\odot)^{1/2}(R/R_\odot)^{-3/2}$$
$$= P(M/M_\odot)^{1/2}(L/L_\odot)^{-3/4}(T_e/T_{e\odot})^3 .$$

If the Cepheids are pulsating in the fundamental mode, the "pulsational" masses turn out to be roughly 30 percent smaller than the "evolutionary" masses estimated from the observed luminosities of the Cepheids and a theoretically determined mass-luminosity relation (Cogan 1970; Rodgers 1970; Fricke *et al.* 1971, 1972; Takeuti and Shibata 1971; Cox *et al.* 1972; Iben and Tuggle 1972*a*, *b*; Schmidt 1972*b*, 1973; King *et al.* 1975). Some Cepheids are double-mode pulsators, and the ratio of their two observed periods (presumed to be the fundamental mode and the first overtone) may also be used to infer their masses; although the results so far are not conclusive (Rodgers 1970; Schmidt 1972*b*; Petersen 1973), the "pulsational" masses seem to be, as they are in the other method, too low. Still another method of deriving "pulsational" masses is to compare the observed phase of the secondary bump on the radial-velocity curve of certain Cepheids with the phase predicted from nonlinear models (Christy 1968; Stobie 1969; Rodgers 1970; Fricke *et al.* 1971, 1972); the masses, however, again turn out to be too low.

The second major problem of Cepheids concerns the observed instability strip on the H-R diagram, which seems to be somewhat too blue compared with the theoretically computed instability strips (Cox 1963; Baker and Kippenhahn 1965; King *et al.* 1966; Hofmeister 1967*b*; Stobie 1969; Iben and Tuggle 1972*a*, *b*; Cox *et al.* 1973; King *et al.* 1973; King *et al.* 1975). Involved in these two problems are the uncertainties associated with turbulent convection, axial rotation, and surface boundary conditions.

In order to determine how sensitive the theoretical predictions are to the adopted stellar opacities, we

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shall make, in this paper, a detailed comparison of evolutionary tracks, pulsation constants, and linearized pulsational stability coefficients for giants whose structure is calculated with (*a*) the standard Cox-Stewart (1965) opacities and (*b*) a new set of opacities recently computed by one of us (Carson 1976). Since our main purpose is to assess the limitations of opacity, we shall adopt conventional physical assumptions in computing the stellar models, in particular the assumption that no significant mass loss has occurred from the star. We shall, however, incidentally reassess the importance of convection, and shall also briefly study the effect of axial rotation, in connection with the pulsational properties of Cepheids.

II. OPACITIES

The new opacities are based on the hot "Thomas-Fermi" statistical model of the atom for all elements heavier than hydrogen and helium, and constitute a considerable improvement over the earlier test calculations published by Carson *et al.* (1968). The two light elements, hydrogen and helium, are treated on a more accurate basis with the help of detailed experimental and theoretical data. Differences from the standard "hydrogenic" opacities, like those of Cox and Stewart (1965), occur in the following temperature regimes, for the density conditions appropriate to stars with intermediate to high masses. Between log $T = 3.8$ and 4.5, the new opacities are somewhat smaller than the Cox-Stewart values; the maximum deviation occurs in the domain of hydrogen ionization. From log $T = 4.5$ to 5.4, the new opacities are slightly larger than the Cox-Stewart values, particularly in the region of second helium ionization. Just above log $T = 5.4$, a very large opacity arises, in the new computations, from the last stage of ionization of the elements in the CNO group. But, by log $T = 6.5$, the opacity is close to the electron-scattering limit, where it remains, at high temperatures, in both sets of opacity computations. Since the new opacities are not yet completed for temperatures below log $T = 3.8$, we have arbitrarily adopted, in this paper, the Cox-Stewart opacities for temperatures less than log $T = 3.85$.

Interpolation of the opacity tables has been performed as follows: (1) linear interpolation in the new opacity tables, for all the stellar models based on them throughout this paper, with the exception of certain Cepheid models in § V; (2) linear interpolation in the

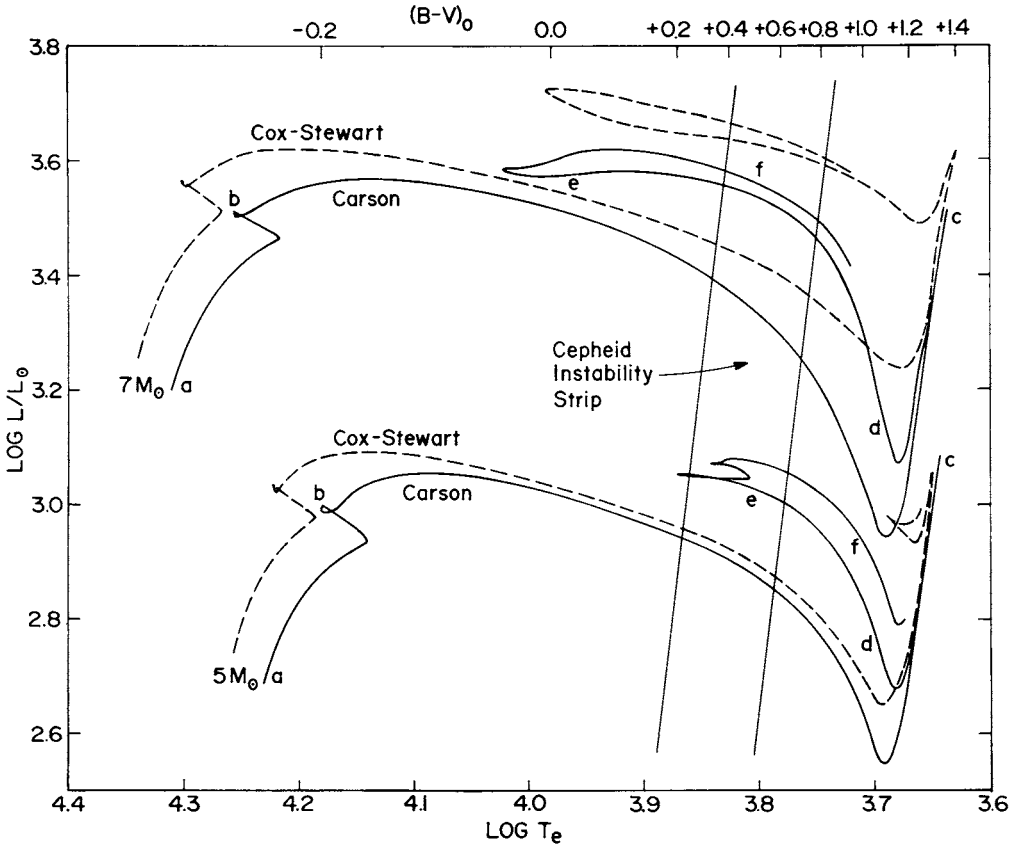

FIG. 1.—Evolutionary tracks on the theoretical H-R diagram for stars of 5 and 7 $M_\odot$ with $(X_e, Z_e) = (0.73, 0.02)$ and (*a*) the new Carson opacities (*solid lines*) and (*b*) the Cox-Stewart opacities (*dashed lines*). The slow phases of evolution occur along the following segments of the tracks: *a–b*, core hydrogen burning; *c–d*, "red" phase of core helium burning; *e–f*, "blue" phase of core helium burning. The empirical Cepheid instability strip is also schematically indicated. The scale of $B - V$ colors is only approximate.

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Cox-Stewart opacity tables, for the relevant Cepheid models in §§ IV and V; and (3) a detailed interpolation formula (Christy 1966), fitted to the Cox-Stewart opacity tables for low-temperature opacities but modified here slightly for a better fit to the high-temperature opacities, for the relevant evolutionary tracks in § III. The grid intervals in the opacity tables are roughly the same for both sets of opacities.

III. EVOLUTION OF GIANTS OF MODERATE MASS

a) Theoretical Tracks

Evolutionary sequences of stellar models for 5 and 7 $M_\odot$, with an initial (hydrogen, metals) content of $(X_e, Z_e) = (0.73, 0.02)$, have been computed from the zero-age main sequence to the end of core helium burning, by using an automatic stellar-evolution program described elsewhere (Stothers and Chin 1973). For the helium-burning nuclear reactions, we have adopted the "new" 3α rate (Austin *et al.* 1971) as well as $\theta_\alpha^2 = 0.1$ for the reduced α-particle width of the 7.12-MeV level in ^{16}O. The convective mixing length has been taken to be 0.4 times the density scale height. Opacities are the newly calculated ones described in § II. The evolutionary tracks on the theoretical H-R diagram are shown in Figure 1. To afford a useful comparison, we have also computed analogous sequences for 5 and 7 $M_\odot$ based on the Cox-Stewart opacities; these tracks are also shown in Figure 1. At the top of the figure an approximate scale of $B - V$ colors has been added (Johnson 1966, Table 3).

Notice that the luminosity at each stage of evolution varies rather little between the two sequences at a given mass. The reason is that the interior opacities ($\log T > 6.5$) are nearly the same in the two cases; they would have to be smaller by a factor of ~ 4 in order to raise the luminosity sufficiently to explain the Cepheid mass discrepancy (Fricke *et al.* 1971). This would require them to be substantially lower than the electron-scattering limit! Since the "hydrogenic" and "Thomas-Fermi" atomic opacities probably bracket the "true" atomic opacities in the deep interior, it is unlikely that further revisions of the atomic model will significantly change the evolutionary mass-luminosity relation. The new relation is approximately $L \propto M^{3.3}$, while it was $L \propto M^4$ in the case of the Cox-Stewart opacities; the slight change in slope is due to the fact that the more massive stars now have larger CNO opacities, but the zero-point of the mass-luminosity relation, taken at 5 $M_\odot$, is about the same for the two sets of opacities. In this connection, we note that Robertson's (1972*a*) adaptation of the early opacity results of Carson *et al.* (1968) gave very faint luminosities, because these preliminary opacities were rather high even at log $T = 7$.

The most important feature of the evolutionary tracks is the blue loop that develops on the H-R diagram during core helium burning, when the star moves off the red-giant branch. This loop forms earlier when the new opacities are adopted, because the increased opacity near the base of the outer convection zone ($\log T \approx 6$) drives the convection zone deeper, and therefore brings the fully mixed hydrogen-rich region closer to the hydrogen-burning shell; this promotes the development of a blue loop (e.g., Lauterborn *et al.* 1971*b*). According to arguments presented elsewhere (Stothers and Chin 1968), the increased chemical homogeneity of the envelope leads to a shrinkage of the star's radius, but the large values of the opacities in the envelope tend to keep the star expanded; the net result is that the maximum effective temperature attained on the blue loop for 7 $M_\odot$ is not much higher than that attained when the Cox-Stewart opacities are adopted. However, the failure of the sequence at 5 $M_\odot$ based on the Cox-Stewart opacities to develop a blue loop at all is due to the shallowness of the outer convective envelope at the tip of the red-giant branch, since the envelope opacities are small. The loop properties are summarized in Table 1, where τ_b/τ_r is the ratio of time spent in the blue and red segments of the loop (during core helium burning). We should remark here that all crossings of the Cepheid instability strip for our sequences at 7 $M_\odot$ occur on the fast thermal time scale of the envelope, while only the first and third crossings at 5 $M_\odot$ do so.

TABLE 1

PROPERTIES OF THE BLUE LOOP ON THE H-R DIAGRAM DURING CORE HELIUM BURNING

$M/M_\odot$	Opacity	log T_e (tip)	τ_b/τ_r	Source
5.....	Carson	3.87	1.2	Present paper
	Cox-Stewart	(cool)	0.0	Present paper
	Cox-Stewart	3.70–3.88	0.3–2.0	Other papers*
7.....	Carson	4.02	1.3	Present paper
	Cox-Stewart	3.98	0.5	Present paper
	Cox-Stewart	3.85–3.96	1.7–2.1†	Other papers*

* The sequences taken from other papers were based on a variety of input physics, including initial chemical composition (see text for references). In some of these sequences, no blue loop occurred.

† Based on only one sequence at 7 $M_\odot$ (Hofmeister *et al.* 1964) and on an average of two sequences at 6 and 8 $M_\odot$ (Robertson 1972*b*).

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The rapid phases of evolution take place between points *b* and *c*, between points *d* and *e* (for 7 $M_\odot$ only), and after point *f* in Figure 1.

Unfortunately, the differences at a given mass between the evolutionary tracks constructed with the two different sets of opacities may not be significant, in an absolute sense, when one recognizes the larger differences that are caused by uncertainties in the initial chemical composition, nuclear reaction rates, convective mixing length, amount of mass loss, rotation rate, atmospheric structure, and convective modifications of the helium-core boundary. These effects have been studied by numerous authors in the case of the Cox-Stewart (or other hydrogenic) opacities, and we list here the relevant papers covering the mass range 4–8 $M_\odot$:

4 $M_\odot$: Hayashi *et al.* (1962*a*); Hayashi *et al.* (1962*b*); Meyer-Hofmeister (1969); Fricke *et al.* (1971); Robertson (1973); Eggleton (1973).

5 $M_\odot$: Kippenhahn *et al.* (1965); Iben (1966, 1972); Hofmeister (1967*a*); Forbes (1968); Schlesinger (1969); Meyer-Hofmeister (1969, 1972); Hallgren and Cox (1970); Paczynski (1970); Fricke *et al.* (1971); Lauterborn *et al.* (1971*b*); Robertson (1971, 1972*a*); Fricke and Strittmatter (1972); Johnson and Whitaker (1975).

6 $M_\odot$: Meyer-Hofmeister (1972); Robertson (1972*b*).

7 $M_\odot$: Hofmeister *et al.* (1964); Paczynski (1970); Lauterborn *et al.* (1971*a*); Fricke and Strittmatter (1972); Iben (1972); Gabriel *et al.* (1974).

8 $M_\odot$: Noels and Gabriel (1972); Robertson (1972*b*).

The variations of the derived loop properties for the two most intensively studied cases, 5 and 7 $M_\odot$, are listed in Table 1 for those sequences that are based on the Cox-Stewart opacities and that have a blue loop of any significance. Comparison with our new sequences reveals that, although the effective temperature of the tip of the loop may be slightly hotter for the new opacities, this is not a very good discriminant of opacity. However, the ratio τ_b/τ_r may be. Assuming that, other things being equal, the new opacities always yield a significantly larger value of τ_b/τ_r for the loop (if it occurs), we can examine published cluster data to see whether the corresponding ratio of the numbers of evolved giants, n_b/n_r, ever exceeds ~ 2, which seems to be the largest ratio theoretically predicted by models constructed with the Cox-Stewart opacities.

b) *Observed Distribution of Cluster Giants*

In the Galaxy, no single open cluster of moderately young age is known to contain more than 11 evolved giants lying outside the main-sequence band. Therefore, in order to study the distribution of evolved giants in the H-R diagram, it is necessary to construct a composite H-R diagram for many clusters, even at the risk of introducing some scatter due to possibly different chemical compositions in the various clusters.

A compilation of evolved giants belonging to clusters whose main-sequence turnups lie in the spectral range B3–B8 has been assembled from published data. Wherever possible, we have determined cluster membership for any giant on the basis of its spectral type, luminosity class, reddening, apparent magnitude, radial velocity, and proper motion. Although we omitted A-type giants entirely

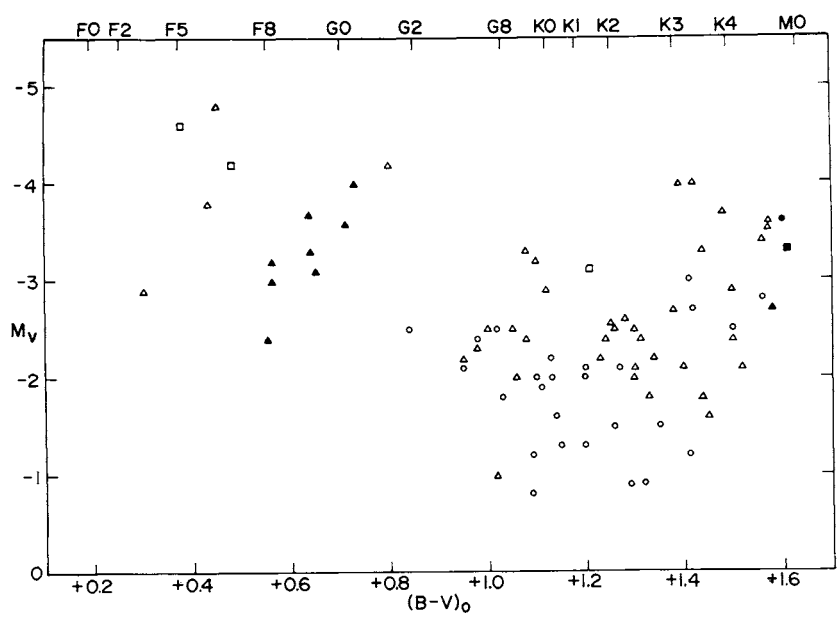

FIG. 2.—A composite color-magnitude diagram for giants in moderately young Galactic open clusters. Symbols refer to the age groups defined in the text: *squares*, young; *triangles*, middle-age; *circles*, old. A filled symbol designates a Cepheid or an M-type giant. The scale of spectral types is only approximate.

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(since their evolutionary status, before or during core helium burning, is unclear), there are only three of them in our sample: two in NGC 2345 and one in NGC 6067. The observed $B - V$ color of an evolved giant was unreddened by adopting the mean color excess of early-type stars either in the immediate neighborhood of the giant or in the cluster as a whole.

In order to date a cluster, the main-sequence turnup in the color-magnitude diagram was examined and assigned a spectral type, with due regard to the possible occurrence of blue stragglers and to the occasional paucity of stars on the turnup, which would tend to make the assigned spectral type too early or too late, respectively. The clusters were then divided into three age groups, for which the spectral type of the main-sequence turnup and the initial mass of the evolved giants are roughly correlated as follows: young clusters (turnup at B3–B4, $\sim 7\ M_\odot$), middle-age clusters (turnup at B5–B6, $\sim 5\ M_\odot$), and old clusters (turnup at B7–B8, $\sim 4\ M_\odot$). Figure 2 shows the composite color-magnitude diagram for the evolved giants in the three age groups just defined; the attached scale of spectral types has been taken from Johnson (1966, Table 3).

The ranges of color occupied by the "blue" and "red" giants in the middle-age group of clusters are virtually identical to the corresponding ranges observed in NGC 1866, a rich cluster of similar age in the Large Magellanic Cloud (Arp 1967; Robertson 1974*a*, *b*). Hence we do not confirm the conclusion of Arp (1967) and of Hagen and van den Bergh (1974) that the giants in the Galaxy are sensibly redder than their analogs in NGC 1866 (however, they could be *slightly* redder). The explanation for this seems to be that Arp employed the observed *red limit* of all the nearby field giants in the Galaxy, while Hagen and van den Bergh used a sample of giants from the Galactic open clusters that is statistically too small (they apparently used only three "blue" and seven "red" giants). Arp's (1959) earlier result that the giants in the Small Cloud cluster NGC 458 are much bluer than their counterparts in the Galaxy still stands, however.

The relative numbers of "blue" and "red" giants can be counted from Figure 2, and are listed in Table 2. In this table are also listed the corresponding numbers for giants in the clusters of the Large Magellanic Cloud (Robertson 1974*a*, *b*) and Small Magellanic Cloud (Arp 1959). The differences observed among the three galaxies for the ratio n_b/n_r in the middle-age group of clusters may not be significant in view of the small number of stars involved. However, the data do suggest that n_b/n_r in the Galaxy is less than half that in the Large Cloud. The statistical significance may actually be greater when one realizes that there is a bias in favor of the "blue" giants in the Galactic clusters, because the motivation to study some of these clusters was provided solely by their containing a Cepheid variable. Nevertheless, n_b/n_r is found to be in no case greater than 0.8, and therefore the test for stellar opacity proposed above cannot, unfortunately, be made within the present uncertainties of the theoretical models. We should add, in this connection, that the paucity of extremely luminous red giants in Figure 2 implies that the red-giant counts refer almost exclusively to stars burning core helium, as we have assumed (see Stothers 1969 for the theoretical implications of this).

In the old group of Galactic clusters, n_b/n_r is ~ 0. This low value implies that the blue loop during core helium burning must be small or absent for masses less than about 4 or 5 $M_\odot$, according to Figure 2. Field giants show the same effect, though much more crudely (Kraft 1966). The effect is also seen in the observed "cutoff" of short periods among Cepheids in the general field (Iben 1966; Kraft 1966; Hofmeister 1967*b*). Ultimately, theoretical models must be able to reproduce the observed pattern of giants in Figure 2.

IV. PULSATION CONSTANTS

Pulsational Q-values have been determined for a number of Cepheid envelope models by following the same prescription as was used earlier by Baker

TABLE 2

OBSERVED NUMBERS OF "BLUE" AND "RED" GIANTS IN MODERATELY YOUNG CLUSTERS OF THE MILKY WAY AND MAGELLANIC CLOUDS

Galaxy	Clusters*	Initial $M/M_\odot$ of the Giants	n_b	n_r	n_b/n_r
MW.....	Young group	~ 7	2	2	...
	Middle group	~ 5	12	36	0.33
	Old group	~ 4	1	26	0.04
LMC.....	Young group	~ 7	~ 130	~ 200	0.65
	Middle group	~ 5	46	57	0.81
SMC.....	Middle group	~ 5	11	22	0.50

* MW young group: NGC 2533, 2571; α Per; δ Lyr. MW middle group: NGC 129, 2343, 2345, 2546, 6067, 6087, 6242, 6405, 6664, 7654, 7790; M25; Cr 140; Pleiades group (low luminosity). MW old group: NGC 1778, 2287, 2516, 2669, 3114, 5460, 5662, 6885. LMC young group: NGC 1850, 1854, 2136, 2157, 2164, 2214. LMC middle group: NGC 1866. SMC middle group: NGC 458.

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TABLE 3

THEORETICAL PULSATION CONSTANTS AND PERIOD RATIOS FOR CEPHEIDS BASED ON TWO DIFFERENT SETS OF OPACITIES

			Carson			Cox-Stewart		
$M/M_\odot$	log $(L/L_\odot)$	log T_e	Q_0(day)	P_1/P_0	P_2/P_0	Q_0(day)	P_1/P_0	P_2/P_0
5.....	2.9	3.75	0.0374	0.741	0.581	0.0373	0.745	0.587
		3.85	0.0357	0.748	0.590	0.0347	0.756	0.610
5.....	3.3	3.75	0.0410	0.719	0.541	0.0404	0.729	0.552
		3.85	0.0384	0.738	0.573	0.0366	0.748	0.593
7.....	3.3	3.75	0.0387	0.738	0.573	0.0382	0.744	0.581
		3.85	0.0369	0.743	0.583	0.0354	0.753	0.607
7.....	3.7	3.75	0.0433	0.706	0.520	0.0421	0.719	0.536
		3.85	0.0403	0.735	0.557	0.0376	0.746	0.586

and Kippenhahn (1965). This included: the use of the linear wave equation for small adiabatic radial pulsations, the outer boundary conditions taken at the photosphere, the neglect of any interaction between the equilibrium convective flux and the pulsations, and a convective mixing length equal to 1.5 times the pressure scale height. Table 3 presents the results for a sample of models constructed with (*a*) the new opacities and (*b*) the Cox-Stewart opacities, for $(X, Z) = (0.739, 0.021)$. Masses of 5 and 7 $M_\odot$ have been chosen as spanning the range appropriate to most of the observed Cepheids with luminosities determined from cluster membership (see Fig. 2 and Sandage and Tammann 1969). The range of luminosities listed for each mass covers values that are typical of the various crossings of the Cepheid instability strip, as determined from theoretical evolutionary tracks. The first and second effective temperatures listed for each luminosity are values characteristic of the general vicinity of the red and blue edges of the Cepheid instability strip, as determined observationally.

Q-values based on the Cox-Stewart opacities have already been extensively discussed in the literature (see § I for references), and it is reassuring that our values are, in those cases that can be compared, close to the values determined by others. We confirm, once again, the important result that any realistic change in luminosity or effective temperature at a fixed mass produces only a small change in the Q-value. Convection in the (mostly radiative) envelopes is also relatively unimportant, as are the choices of axial-rotation rate (§ V) and of surface boundary conditions.

Turning now to the Q-values based on the new opacities, we find a small increase over the Q-values determined with the Cox-Stewart opacities. The increase becomes greater for higher masses, brighter luminosities, and hotter effective temperatures. However, it never exceeds 7 percent for the fundamental mode, and is usually much smaller, whereas an increase of 15 percent would be needed to eliminate the mass discrepancy. Thus the new opacities help, but do not completely solve the problem. The reason why the changes are so small is that the Q-values for Cepheids are determined largely by conditions in the envelope within the temperature range log $T = 3.9$–5.4; the Carson and Cox-Stewart opacities just do not differ sufficiently in this range, where hydrogen and helium are the main contributors to the opacity. It is, however, barely possible that further work on opacities may entirely remove the mass discrepancy, since the necessary increase of opacity at low temperatures seems to be much less than was estimated originally by Fricke *et al.* (1971).

The ratios of periods of the first overtone and fundamental mode in the models based on the new opacities are in no instance greatly different from the values determined for the Cox-Stewart opacities, although the new opacities do lower P_1/P_0 somewhat—certainly in the direction required by the observed values of $P_1/P_0 = 0.70$–0.71 (see Petersen 1973). Petersen's (1974) arbitrary increments of the Cox-Stewart opacities also produce a slight decrease of P_1/P_0. However, it is doubtful whether future changes in opacity will lower the theoretical values of P_1/P_0 much more.

More sensitive to opacity is the phase of the secondary bump on the radial-velocity curve predicted from nonlinear models for some Cepheids. Although no nonlinear models are at present available in the case of the new opacities, Fricke *et al.* (1971) have shown that an arbitrary scale-factor change, K, in the Cox-Stewart opacities will lead to a change in the inferred mass of any Cepheid showing a prescribed secondary bump by an amount $K^{1.12}$. Since these authors have also shown that the Q-value for the fundamental mode will change like $K^{0.03}$, and since the present Q-values based on the new opacities are, on the average, 2–4 percent larger than the Q-values based on the Cox-Stewart opacities, we may expect, very crudely, that recomputed masses of "bump" Cepheids will be ~ 3 times larger than the masses inferred by using the Cox-Stewart opacities. Alternatively, we may compare directly the new opacities with the Cox-Stewart opacities, and we find, roughly, $K \approx 2$ in the relevant region of temperature and density; this implies that revised masses of "bump" Cepheids will be ~ 2 times larger than those inferred from the Cox-Stewart opacities. Of course, these estimates are very rough, because the new opacities are not exact multiples of the Cox-Stewart values. Nevertheless, it is interesting to note that the masses based on the Cox-Stewart opacities have turned out to be, on the

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TABLE 4

THEORETICAL BLUE EDGES OF THE CEPHEID INSTABILITY STRIP BASED ON TWO DIFFERENT SETS OF OPACITY TABLES WITH LINEAR INTERPOLATION

			Carson			Cox-Stewart		
$M/M_\odot$	log $(L/L_\odot)$	Convection	$(\log T_e)_0$	$(\log T_e)_1$	$(\log T_e)_2$	$(\log T_e)_0$	$(\log T_e)_1$	$(\log T_e)_2$
4.....	2.5	Yes	3.828	3.844	3.859	3.811	3.819	3.825
		No	3.817	3.838	3.857	3.774	3.785	3.786
5.....	3.1	Yes	3.817	3.838	3.843	3.789	3.795	3.798
		No	3.816	3.837	3.839	3.757	3.761	3.765
7.....	3.3	Yes	3.812	3.830	3.838	3.784	3.790	3.794
		No	3.810	3.829	3.837	3.750	3.758	3.763
7.....	3.5	Yes	3.816	3.831	3.808*	3.775	3.782	3.786
		No	3.815	3.830	3.802*	3.736	3.749	3.760
7.....	3.7	Yes	3.823	3.834	stable	3.772	3.776	3.780
		No	3.821	3.833	stable	3.727	3.738	3.744
9.....	4.0	Yes	3.82:	3.80:*	stable	3.76:	3.77:	3.77:
		No	3.82:	3.80:*	stable	3.72:	3.73:	3.73:*

* Marginally unstable.

average, 1.8 times smaller than the expected "evolutionary" masses (Fricke *et al.* 1972).

V. PULSATIONAL INSTABILITY RESULTS

It is important to determine how the new opacities affect the theoretical edges of the Cepheid instability strip. Linear nonadiabatic theory has been used here to calculate the stability coefficients for the three lowest modes of radial pulsation in Cepheid envelope models. Unlike Baker and Kippenhahn (1965), we have solved the nonadiabatic pulsation equations by the inward-integration method rather than by the relaxation method. For comparative purposes, a set of purely radiative envelope models has been computed in addition to the set of partially convective ones. In a few additional models, uniform rotation has been introduced, by using the simple approximations of mean spherical shells and pseudo-radial pulsations (Stothers 1974).

Results of our calculations for four stellar masses (4, 5, 7, and 9 $M_\odot$) with $(X, Z) = (0.739, 0.021)$ are given in Table 4, which lists the effective temperatures of the theoretical blue edges of the instability strip. The luminosity adopted for each mass corresponds approximately to the second crossing of the instability strip, which is usually the slowest crossing; for 7 $M_\odot$, luminosities for both the first crossing and a hypothetical later crossing are also included. Since linear interpolation in the new opacity tables turns out to be too crude to determine sufficiently accurately the location of the blue edges in some cases, we have recalculated a second set of blue edges by using quadratic interpolation. The results are presented in Table 5. However, there is reason to suspect that the opacity tables do not have a fine enough grid to fix securely the blue edges at the brightest luminosities (for both sets of opacities), and therefore the results at 9 $M_\odot$, in particular, are rather uncertain.

As other authors have already found in the case of the Cox-Stewart opacities, the red edge of the instability strip for the fundamental mode apparently does not exist at all, according to the theoretical computations, or else occurs at an effective temperature that is far cooler than is actually observed. This problem of the red edge, first identified by Baker and Kippenhahn, is still unsolved for *both* sets of opacities, although its resolution probably lies with the strong convection that develops at cool effective temperatures.

Compared with the new opacities, the Cox-Stewart opacities lead to more extensive convection in the hydrogen ionization zone. This is a result both of the cooler effective temperatures of the models that are pulsationally unstable and of the larger values of the hydrogen opacity (see § II). Allowance for convection

TABLE 5

THEORETICAL BLUE EDGES OF THE CEPHEID INSTABILITY STRIP BASED ON THE CARSON OPACITY TABLES WITH TWO DIFFERENT METHODS OF INTERPOLATION

			Linear Interpolation			Quadratic Interpolation		
$M/M_\odot$	log $(L/L_\odot)$	Convection	$(\log T_e)_0$	$(\log T_e)_1$	$(\log T_e)_2$	$(\log T_e)_0$	$(\log T_e)_1$	$(\log T_e)_2$
4.....	2.5	Yes	3.828	3.844	3.859	3.845	3.849	3.860
5.....	3.1	Yes	3.817	3.838	3.843	3.838	3.840	3.845
7.....	3.3	Yes	3.812	3.830	3.838	3.837	3.839	3.842
7.....	3.5	Yes	3.816	3.831	3.808*	3.834	3.838	3.839*
7.....	3.7	Yes	3.823	3.834	stable	3.831	3.837	3.836*
9.....	4.0	Yes	3.82:	3.80:*	stable	3.825:	3.82:*	stable

* Marginally unstable.

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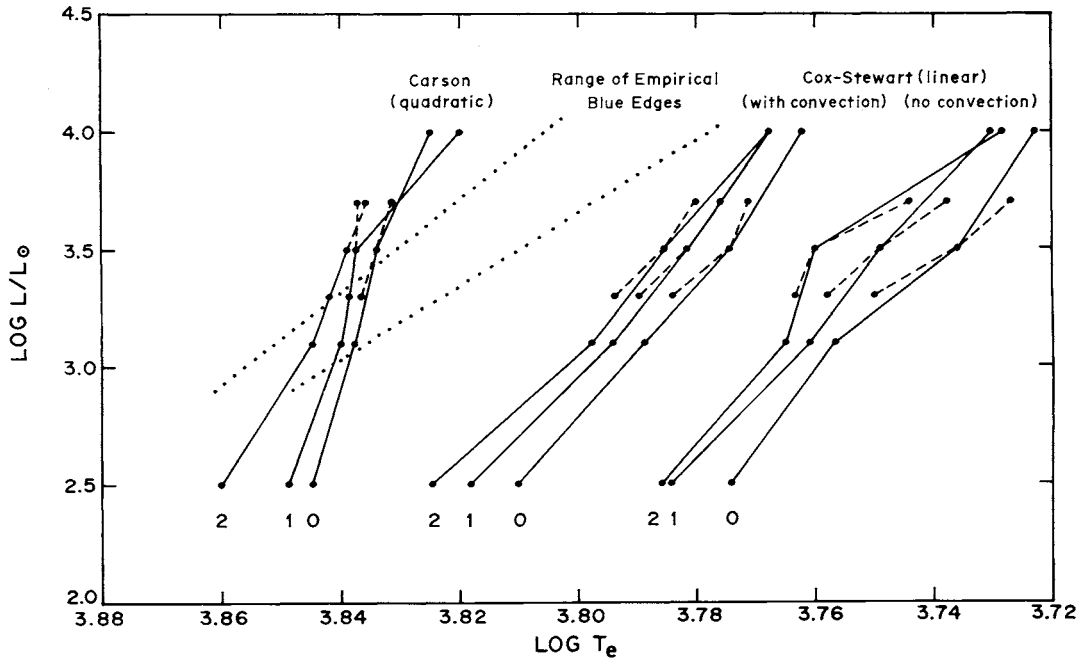

FIG. 3.—Blue edges of the Cepheid instability strip in the theoretical H-R diagram: *solid lines*, theoretical blue edges for the most characteristic luminosities of Cepheids of various masses; *dashed lines*, theoretical blue edges for the various possible luminosities of Cepheids of 7 $M_\odot$; *dotted lines*, hottest and coolest empirical blue edges quoted in the literature. A number referring to the pulsational mode identifies the theoretical edges, which are based on the chemical composition $(X, Z) = (0.739, 0.021)$.

shifts the blue edge of the instability strip toward somewhat hotter effective temperatures than occur when convection is neglected; the amount of shift found here is in generally good agreement with the amount found by Iben and Tuggle (1972*a*, Tables A13 and A14). However, in order to achieve agreement with the observed phase lag of 90° between the light and radial-velocity variations of typical Cepheids, Castor (1971) showed that convection has to be neglected. Therefore, it is a great advantage of the new opacities that convection is unimportant in a purely natural way. For these opacities, the predicted phase lag at the stellar surface runs approximately from 45° to 135° across the instability strip. Contributions to the phase lag arise in both the hydrogen and second helium ionization zones, provided that these zones are nonadiabatic; thus, in the cooler models the hydrogen ionization zone is the more important in determining the phase lag at the stellar surface (since the second helium ionization zone lies too deep to be significantly nonadiabatic), while in the hotter models the second helium ionization zone is the more important (since the hydrogen ionization zone lies too close to the surface to affect significantly the luminosity variations).

A second important result obtained here is the significant increase of the effective temperature of the blue edge of the instability strip when the new opacities are used in place of the old ones. This increase results from additional excitation in the inner part of the second helium ionization zone, caused by a greater pronouncement of the well-known "bump" in the opacity curve there. A comparison of the predicted blue edges is shown in Figure 3; the difference of effective temperatures induced by the different opacities (with convection included in the models) is $\Delta \log T_e \approx 0.04$.[1] The minor irregularities in the blue edges are due to numerical inaccuracies, arising from the crude method of interpolation in the opacity tables and from the use of a mean luminosity for each mass (the luminosity should actually be somewhat different for each set of opacities).

Observationally, the blue edge for Cepheids is not as well determined as one would like. The 13 calibrating Cepheids that are members of Galactic clusters and associations define only a ridge band for Cepheids in the color-magnitude diagram (see Fig. 2 and Sandage and Tammann 1969). The full width of the Cepheid instability strip, however, has been estimated for Galactic Cepheids by Schmidt (1972*a*) and by Sandage and Tammann (1969), who included some extragalactic Cepheids. Since the two proposed blue edges are very close to each other, we shall adopt a mean of the two. To plot this mean on the theoretical H-R diagram, it is useful to employ both the hottest and coolest relations between intrinsic color and effective temperature that have been proposed in the literature (Oke 1961; Johnson 1966; Rodgers 1970;

[1] A special calculation for 0.7 $M_\odot$ with $\log (L/L_\odot) = 1.7$ indicates that a similar difference ($\Delta \log T_e \approx 0.03$) persists into the region of RR Lyrae stars. In this case, the blue edges based on the new opacity tables with quadratic interpolation lie at $(\log T_e)_0 = 3.855$, $(\log T_e)_1 = 3.872$, and $(\log T_e)_2 = 3.879$.

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Parsons 1971; Böhm-Vitense 1972; Schmidt 1972*a*; Bell and Parsons 1974). The results are displayed in Figure 3.

Comparison of observed and theoretical blue edges in Figure 3 shows remarkably good agreement in the case where the new opacities are adopted. Although at luminosities corresponding to 9 $M_\odot$ the predicted blue edges for the new opacities seem to be rather too hot, the stellar models for both the new and the old opacities are less reliable here. Certainly, the Cox-Stewart opacities predict blue edges that are everywhere significantly too cool, particularly if convection must be neglected (or reduced) in order to avoid a phase-lag problem at the stellar surface. A change in the adopted metals abundance is already known to have little effect on the location of blue edges, since the opacity and mean molecular weight of material in the pulsating layers are mostly due to hydrogen and helium. Inclusion of possible energy losses due to waves running off the surface will only exacerbate the problem of blue edges that are already too cool. Although adoption of a different standing-wave boundary condition for the luminosity amplitudes at the surface or of a more accurate method of interpolation in the opacity tables could raise the effective temperatures of the blue edges somewhat (Iben and Tuggle 1972*a*, *b*), nearly all authors concur that a basic discrepancy with observations is obtained in the case of the Cox-Stewart opacities (see the references in § I).

The only likely way to gain a significant increase in the effective temperature of the blue edges is to increase the helium abundance, Y. This would mean raising Y from an expected value of 0.2–0.3, which is the directly observed value for main-sequence stars that are precursors of Cepheids, to about 0.40 (see also Cox *et al.* 1973; King *et al.* 1973). Normal evolutionary processes during the red-giant phase immediately preceding the main Cepheid phase do convect helium up from the deep interior, but the resulting increase in the envelope value of Y is at most 0.03. Hypothetical mass loss down to a layer in the interior where $Y = 0.40$ would require a loss of about 80 percent of the star's mass—an unacceptably large amount (§ I)—and would even produce a further discrepancy with the observed blue edge of the instability strip (see the models of Stobie 1969 and of King *et al.* 1973). On the other hand, the new opacities require no arbitrary increase in Y and no mass loss to produce agreement with the observed blue edge.

Uniform rotation affects the models based on both sets of opacities very slightly. It shifts the blue edge to a hotter effective temperature by an amount $\Delta \log T_e \approx 0.01\lambda$, where $\lambda = \frac{2}{3}v^2R/GM$. According to Kraft (1966), the observed surface rotational velocities ($v \sin i$) of Cepheids are less than 25 km s^{-1}, so that λ is at most about 0.02. Hence $\Delta \log T_e \leq 0.0002$. The reason for the growth (however slight) of pulsational instability is that the pulsation amplitudes drop off more steeply below the surface in the presence of rotation; therefore, the amplitudes are smaller in the strong damping region below the second helium ionization zone.

For Cepheids of sufficiently high luminosity, only the fundamental mode is predicted to be unstable, according to the results for *both* sets of opacities. This was shown earlier for the Cox-Stewart opacities by Iben and Tuggle (1972*a*) and by King *et al.* (1973), and is shown in Figure 3 for the new opacities. The transition luminosity is $\log (L/L_\odot) \approx 4.0$ in the case of the new opacities. Although it appears to be considerably higher for the Cox-Stewart opacities (on the basis of the present models), a change of the surface boundary condition for the luminosity amplitudes can lower it considerably (see the models of Iben and Tuggle 1972*a* and of King *et al.* 1973). Therefore, the two sets of opacities do not make unambiguously different predictions in this case. We do remark, however, that for periods longer than, say, $\sim$10 days a growing percentage of Cepheids ought to be purely fundamental-mode pulsators. Perhaps it is relevant that a growing percentage of Cepheids show very large light amplitudes when the period exceeds 10 days (e.g., van Genderen 1969).

For fainter luminosities, both the fundamental mode and the first overtone are predicted to be unstable; and, at still lower luminosities, the second overtone should also be unstable. (However, the *reddest* Cepheids at any luminosity, except possibly at the lowest luminosities, are expected to be unstable only in the fundamental mode.) The locations of the theoretical blue edges in Figure 3 suggest that the bluest Cepheids ought to be purely first or second overtone pulsators. Since the second overtone is easier to discriminate observationally because of its shorter period, we shall focus attention on the faintest luminosities of all. The region of unstable second overtones begins at $\log (L/L_\odot) \approx 3.5$ for the new opacities. Although it begins at much higher luminosities for the Cox-Stewart opacities (on the basis of the present models), a change in the surface boundary condition for the luminosity amplitudes can lower it to the level just quoted (see the models of Iben and Tuggle 1972*a* and of King *et al.* 1973). Again, the two sets of opacities do not lead to unambiguously different predictions. But we shall use the present results based on the new opacities in predicting the following transition quantities: $\log (L/L_\odot) \approx 3.5$, $P_2 \approx 3$ days, and $(P_2/P_0) \approx 0.58$.

Observationally, it has been suspected for a long time (Arp 1960) that a small group of Cepheids in the Small Magellanic Cloud having sinusoidal light curves and low light amplitudes are pulsating in the second overtone. These Cepheids are bluer than other Cepheids of the same apparent magnitude, and have luminosities that range up to $\log (L/L_\odot) \approx 3.4$ and periods up to 4 days, with the average periods being a factor $\sim$0.60 too short compared with "normal" periods at the same luminosity (Payne-Gaposchkin and Gaposchkin 1966). We find that these observational data are in excellent agreement with our theoretical predictions for the second overtone, although, unfortunately, they do not allow a definite

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decision to be made about the question of stellar opacity.

VI. CONCLUSION

New evolutionary tracks, pulsation constants, and pulsational stability coefficients have been computed for stellar models of interest in connection with the problems of classical Cepheids, by employing (*a*) the standard Cox-Stewart opacities and (*b*) the new opacities recently computed by one of us (Carson). These new opacities contain larger helium and metals contributions, and a smaller hydrogen contribution, than do the Cox-Stewart opacities for the same chemical composition (X, Z). The difference in *metals* contribution affects chiefly the location and shape of the evolutionary tracks on the H-R diagram, while the difference in *hydrogen* and *helium* contributions has its greatest effect on the pulsational properties of the Cepheid models.

Our conclusions based on a comparison of evolutionary tracks for the mass range 5–7 $M_\odot$ are as follows. First, the theoretical mass-luminosity relation for evolved giants is changed very little by substituting the new opacities for the older ones, and is unlikely to be changed significantly by future opacity revisions if, as seems likely, the "hydrogenic" (Cox-Stewart) and "Thomas-Fermi" (Carson) opacities bracket, on the average, the "true" atomic opacities for the deep stellar interior. Therefore, the "evolutionary" masses determined for Cepheids remain about the same as before. Second, the derived properties of the blue loop on the H-R diagram during core helium burning are not yet certain enough, with either set of opacities, to make firm predictions about observable differences due to opacity. This is confirmed by the observational data for giant stars in moderately young clusters of the Galaxy, which we have collected in this paper and have supplemented by other data for the Magellanic Clouds.

Q-values for the fundamental mode of radial pulsation in Cepheid envelope models are found to be increased by 1–7 percent if the new opacities are used in place of the older ones. The resulting slight increase of the inferred "pulsational" masses of Cepheids is, however, almost exactly canceled by a slight increase of the inferred "evolutionary" masses, and therefore the classical mass discrepancy remains.[2] The period ratios P_1/P_0 are also not lowered significantly (less than 2 percent), and the disagreement with the observed period ratios for "beat" Cepheids still exists if these stars have "evolutionary" masses. But the new opacities are expected to raise the previously deficient pulsational masses of the "bump" Cepheids to the values predicted from evolutionary theory.

The pulsational instability of the Cepheid envelope models is relatively sensitive to small changes in the main features of the opacity curves. In contrast to the older opacities, we find by using the new opacities that convection becomes an unimportant factor in the hotter envelopes, that the phase lag between luminosity and radial velocity at the stellar surface is approximately 90° as observed, and that the blue edge of the theoretical instability strip on the H-R diagram is bluer and steeper than for the older opacities and is in better agreement with the general location (but not the slope) of the observed blue edge, if, as seems appropriate, a normal helium abundance and no mass loss are assumed. However, the problem of the theoretically unlimited red edge remains. The predicted transition between second-overtone instability and fundamental-mode instability is beautifully confirmed by observational data for the Small Magellanic Cloud in the case of the new opacities (and possibly for the older opacities, if the surface boundary conditions for the pulsation amplitudes are suitably altered). Axial rotation is found to increase the effective temperature of the theoretical blue edges by a very slight amount.

Crucial to a further understanding of the instability of Cepheids is a correct knowledge of the pulsational boundary conditions in the atmosphere and of convection in the hydrogen ionization zone. Only then can the present observational evidence, which tentatively favors the new hydrogen and helium opacities, be used in a definitive test of the two sets of opacities.

We are very grateful to Dr. Chao-wen Chin for calculating the model sequences shown in Figure 1, and to the referee for inducing us to improve and to extend the series of Cepheid models. R. S. thanks Dr. Gretchen L. Hagen for communicating a preliminary version of her own composite H-R diagram for cluster giants. T. R. C. thanks the National Academy of Sciences–National Research Council for the award of a senior postdoctoral research associateship at the Institute for Space Studies, and Dr. Robert Jastrow for his hospitality at the Institute.

[2] The mass discrepancy may have been removed by the recent (Hanson 1975) redetermination of the distance modulus of the Hyades, on which rests the empirical luminosity scale for Cepheids. If this redetermination is correct, the luminosities of Cepheids will be about 30 percent brighter than before. Since M (pulsational) $\propto L^{1.5}$ while M (evolutionary) $\propto L^{0.3}$, the "pulsational" masses will be raised to values close to the "evolutionary" masses.

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