NEUTRINO EMISSION FROM PLASMONS IN STRONG MAGNETIC FIELDS

HSING-HEN CHEN*† and MALVIN A. RUDERMAN*
Dept. of Physics, Columbia University, New York, N.Y., U.S.A.
and
VITTORIA CANUTO
Institute for Space Studies, Goddard Space Flight Center, NASA, New York, N.Y., U.S.A.

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Abstract. We show, contrary to previously published results, that magnetic fields of astrophysical interest have negligible effects on the emission rate of neutrinos from plasmons.

1. Introduction

Neutrino pair emission from stellar interiors has important effects on certain stages of stellar evolutions. Canuto *et al.* (1970a, b, 1971) had calculated the energy loss rate for these processes, including the plasmon decay in the presence of a very strong magnetic field. They proposed a new longitudinal plasmon mode depending only on magnetic field, which greatly enhances the neutrino energy loss rate at high densities. We point out below that the proposed enhanced emission mode is actually part of a transverse mode spectrum with refraction index n in the limit of infinity. A plasmon can decay into neutrino pairs only when $0 \leqslant n \leqslant 1$. It is therefore clear that the proposed mode cannot decay.

2. The Plasma Modes

Following the notation of Stix (1962) we have the dielectric tensor for a magnetized cold plasma

$$\begin{aligned}
&\varepsilon = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix}, \\
&S = \tfrac{1}{2}(R + L), \qquad D = \tfrac{1}{2}(R - L), \\
&R = 1 - \frac{\omega_p^2}{\omega^2} \frac{\omega}{\omega - \omega_c}, \\
&L = 1 - \frac{\omega_p^2}{\omega^2} \frac{\omega}{\omega + \omega_c}, \\
&P = 1 - \omega_p^2/\omega^2, \\
&\omega_p^2 = 4\pi\, Ne^2/m, \\
&\omega_c = eB/mc;
\end{aligned} \tag{1}$$

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† Now at School of Natural Science, Institute for Advanced Study, Princeton, New Jersey, 08540.

and plasma waves satisfy Maxwell's equations

$$A_{ij}E_j = 0 \tag{2}$$

with

$$A_{ij} = \frac{c^2k^2}{\omega^2}\left(\frac{k_ik_j}{k^2} - \delta_{ij}\right) + \varepsilon_{ij}. \tag{3}$$

Equation (2) has nontrivial solutions if and only if

$$\det|A_{ij}| = 0 \tag{4}$$

that is,

$$An^4 - Bn^2 - C = 0, \tag{5}$$

where

$$\begin{aligned} n &= \frac{ck}{\omega}, \\ A &= S\sin^2\theta + P\cos^2\theta, \\ B &= RL\sin^2\theta + SP(1+\cos^2\theta), \\ C &= RPL, \end{aligned} \tag{6}$$

and θ is the angle between **k** and **B**.

Equation (5) contains the necessary condition

$$\tan^2\theta = \frac{-P(n^2-R)(n^2-L)}{(Sn^2-RL)(n^2-P)}. \tag{7}$$

There are three modes along $\theta=0$

$$P = 0, \qquad n_0^2 = R, \qquad n_x^2 = L. \tag{8}$$

The first mode $P=0$ is a longitudinal mode. It is the familiar Langmuir static oscillation $\omega=\omega_p$.

The next two modes are transverse modes. They are plotted in Figures 1, 2, 3 and 4 respectively. We notice that

$$n_0^2 \to \pm\infty \quad \text{as} \quad \omega^2 \to \omega_c^2 \tag{9}$$

in the ordinary mode. This is indeed the mode proposed by Canuto *et al.* (1970b), which is clearly part of a transverse spectrum.

There are also three modes along $\theta=\pi/2$. One mode corresponds to

$$n_0^2 = P. \tag{10}$$

This is purely transverse. The other two modes correspond to

$$n_x^2 = RL/S, \tag{11}$$

which can also be written as

$$\begin{aligned}\omega_\pm^2 = {} & \tfrac{1}{2}(k^2c^2 + \omega_c^2 + 2\omega_p^2) \pm \\ & \pm \tfrac{1}{2}\sqrt{k^4c^4 - 2k^2c^2\omega_c^2 + \omega_c^4 + 4\omega_p^2\omega_c^2}. \end{aligned} \tag{12}$$

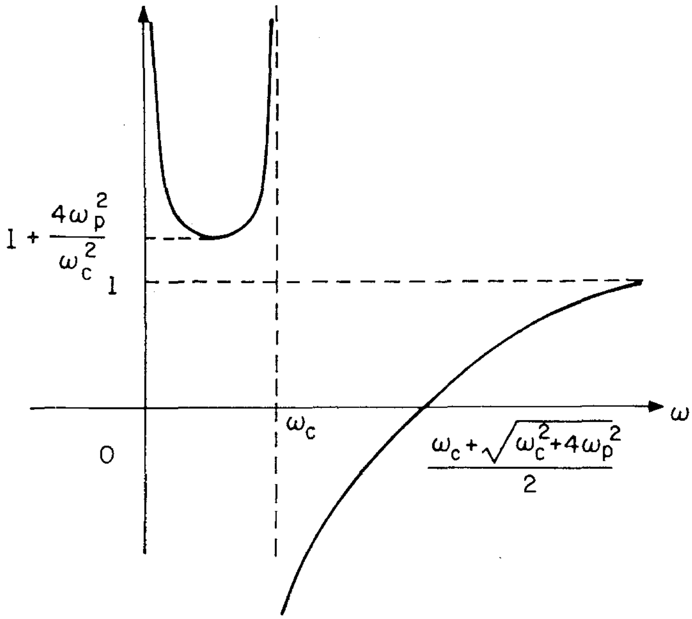

FIG. 1 $n^2 = R$

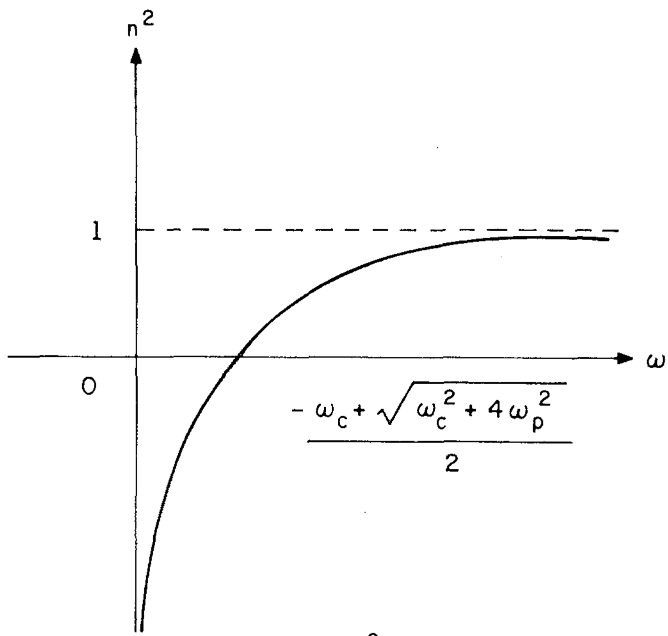

FIG. 2 $n^2 = L$

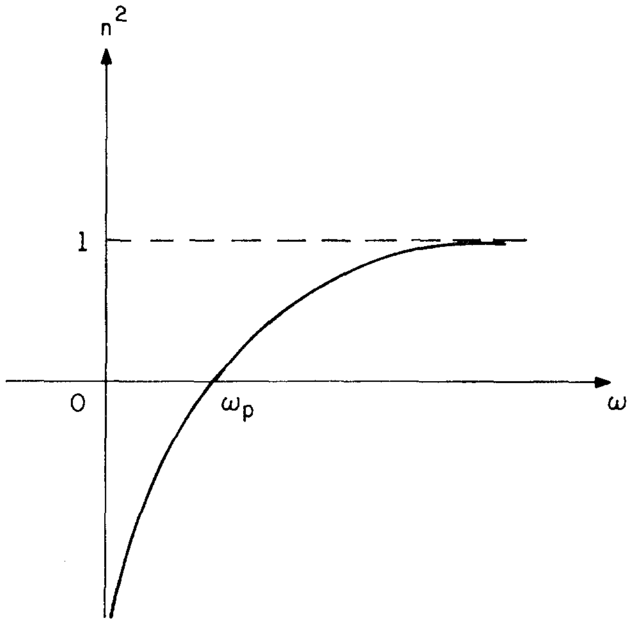

FIG. 3 $n^2 = P$

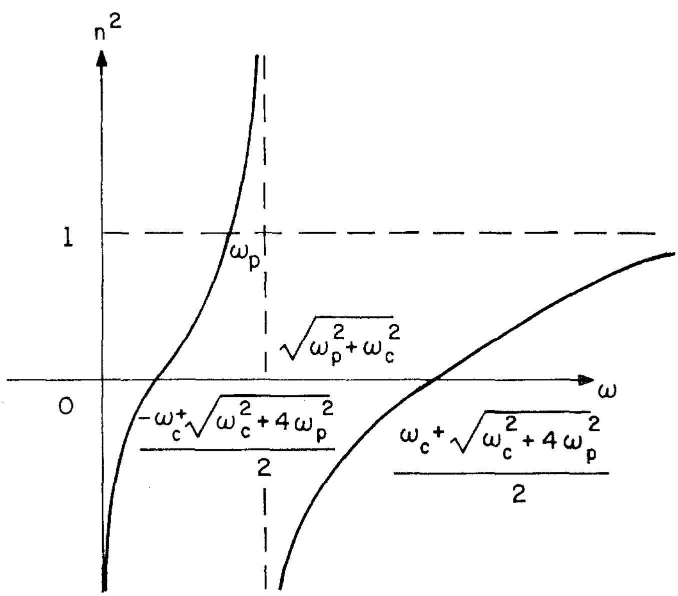

FIG. 4 $n^2 = RL/S$

The polarizations of these two modes are mixed transverse and longitudinal. Only when we approach to the electrostatic limits ($n \to \infty$, $ck \gg \omega$), do we get purely transverse and purely longitudinal mode

$$\begin{aligned} &\omega_+^2 \to c^2k^2 \text{ (transverse)}, \\ &\omega_-^2 \to \omega_p^2 + \omega_c^2 \text{ (longitudinal)}. \end{aligned} \tag{13}$$

3. Discussion

A plasmon can decay into neutrino pairs without violating energy momentum conservation law only when the plasmon has an effective 'mass' term in its dispersion relation, e.g., $\omega^2 = c^2k^2 + \omega_p^2$.

This condition can be met if and only if

$$0 \leqslant n^2 \leqslant 1. \tag{14}$$

We have shown above in Section 2 that the previously proposed plasma modes $\omega = \omega_c$ exists only in the electrostatic limit $n \to \infty$. It violates the criterion (14), and therefore cannot decay. According to the calculations of Canuto (1970b) (otherwise correct) we conclude that very strong magnetic field (in the astrophysical sense, nevertheless $\omega_c \ll \omega_p$) does not have important effect on the energy loss rate of stars by plasmon neutrino pair emissions.

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