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FUNDAMENTAL DATA FOR CONTACT BINARIES: RZ COMAE BERENICES, RZ TAURI, AND AW URSAE MAJORIS

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ABSTRACT

Differential corrections analyses of three binaries of the W UMa type show that RZ Tau and AW UMa have common envelopes and the relatively small gravity darkening predicted by Lucy, while RZ Com seems to have a larger gravity effect and is only marginally in contact. Some details of our method for computing contact-binary light curves are given. We suggest that W UMa binaries which have occultation primary eclipses may, in general, have large amplitudes for gravity darkening or a photometric surface-brightness effect which simulates large gravity darkening.
Subject headings: eclipsing binaries — W Ursae Majoris stars

I. INTRODUCTION

The term "contact binary" has been in use for more than two decades, but until recently there has been some uncertainty as to how literally this designation should be taken. That is, the traditional methods for light-curve analysis, which are based on the Russell model of similar ellipsoids, break down for very close binaries and cannot be used to compute the light variation for components which are fully in contact.[1] Recently Lucy (1967, 1968*a*, *b*), Rucinski (1969), Whelan (1972), Biermann and Thomas (1972), and others have put forward interesting ideas on the nature of contact binaries which are subject to quantitative test against the available observations, provided that suitable means are developed to compute the light curves. Such tests have been made by Lucy (1971) and by Mochnacki and Doughty (1972*a*, *b*) who found that (*a*) at least some of the W UMa-type binaries are fully in contact (i.e., they have common envelopes whose "photospheres" follow a single equipotential surface); and (*b*) the light variation for some contact binaries is compatible with the gravity-darkening law predicted by Lucy for convective envelopes, which has an amplitude about one-third that of von Zeipel (1924) radiative gravity darkening.

Sixteen systems were analyzed by Lucy (differential corrections in i and Ω), and six by Mochnacki and Doughty (subjective parameter adjustment). In this paper we give results of our analyses of only three systems (RZ Com, RZ Tau, and AW UMa), but these have been fitted by a procedure which has significant advantages over those by Lucy and by Mochnacki and Doughty. In the major points, our conclusions are similar to those by the other authors, but we have found the gravity darkening law

[1] Mauder (1972) has developed a means of correcting the usual rectification procedure, which is based on a model of similar ellipsoids, for tidal distortion. Mauder's solutions are in relatively good agreement with solutions of the type discussed here. His results should be useful in providing starting parameters for the fitting of common envelope models, although they were not so used in the present work.

TABLE 1

PARAMETERS FOR THREE BINARIES

Parameter	RZ Com (I)	RZ Com (II)	RZ Tau	AW UMa
i	86°04 ± 0°51	85°72 ± 0°31	82°88 ± 0°35	79°10 ± 0°36
g	1.13 ± 0.04	1.51 ± 0.02	0.29 ± 0.06	0.45 ± 0.06
T_1 (pole) [° K]	5500	5500	7200	7000
T_2 (pole) [° K]	5564	5552	7146	6737
A	1.00	1.00	1.00	1.00
Ω	5.618 ± 0.054	5.869 ± 0.040	2.4957 ± 0.0048	1.8321 ± 0.0018
q	2.292 ± 0.030	2.394 ± 0.020	0.3721 ± 0.0047	0.0716 ± 0.0005
L_1 (green)	0.3052 ± 0.0024	0.2976 ± 0.0011	0.7046 ± 0.0057	0.9148 ± 0.0099
L_1 (blue)	...	...	0.7061 ± 0.0062	0.9176 ± 0.0125
x (green)	0.50 ± 0.03	0.62 ± 0.02	0.51 ± 0.03	0.63 ± 0.01
x (blue)	...	...	0.57 ± 0.03	0.69 ± 0.01
r_1 (pole)	0.2924 ± 0.0044	0.2805 ± 0.0030	0.4632 ± 0.0010	0.5650 ± 0.0006
r_1 (side)	0.3056 ± 0.0052	0.2918 ± 0.0035	0.5024 ± 0.0014	0.6457 ± 0.0011
r_1 (back)	0.3403 ± 0.0082	0.3211 ± 0.0052	0.5392 ± 0.0019	0.6644 ± 0.0012
r_2 (pole)	0.4287 ± 0.0042	0.4240 ± 0.0029	0 3035 ± 0.0011	0.1860 ± 0.0008
r_2 (side)	0.4574 ± 0.0055	0.4509 ± 0.0037	0.3209 ± 0.0014	0.1956 ± 0.0010
r_2 (back)	0.4859 ± 0.0071	0.4761 ± 0.0047	0.3802 ± 0.0030	0.2537 ± 0.0039
R_1 (side)	0.63 × 10^6 km	0.60 × 10^6 km	...	1.6 × 10^6 km
R_2 (side)	0.95 × 10^6 km	0.93 × 10^6 km	...	0.5 × 10^6 km
m_1	$0.9m_\odot$	$0.9m_\odot$	...	$3.1m_\odot$
m_2	$2.0m_\odot$	$2.1m_\odot$	...	$0.2m_\odot$
		Theoretical Values		
x (green)	0.73	0.73	0.66	0.65
x (blue)	...	...	0.77	0.78
Ω (inner contact)	5.6599	5.8003	2.6206	1.8670
Ω (outer contact)	5.0546	5.1926	2.3923	1.8200

by least squares, while they have *assumed* the "convective" law to hold. We list below the reasons we consider quantitative results based on our approach (Wilson and Devinney 1971) to be relatively well determined.

a) The accuracy of the basic light-curve program is at least one order of magnitude better than that of Lucy or Mochnacki and Doughty, who claim accuracy of only about ± 0.01 mag. This is important for differential-corrections solutions since the required numerical derivatives are computed by differencing adjacent values of the dependent variable. The light curves differenced in our solutions have a precision of one part in several thousand in the worst case or, more typically, one part in 10^4.

b) The results, which we give in table 1, follow from least-squares solutions which make use of observations in two passbands (for RZ Tau and AW UMa, but not for RZ Com) fitted simultaneously. In fact, an arbitrary number of passbands can be treated simultaneously, but we presently avoid ultraviolet light curves because of blanketing effects. Observation points are weighted by the program according to the source of noise (variable sky transparency, shot noise, instrumental noise); according to the relative scatter of the several light curves; and according to the intrinsic weight of individual points (e.g., number of individual observations per normal point). For all three binaries discussed here we assumed variable transparency to be the main source of scatter.

c) The fit to the observations seems quite good for all three binaries, with some minor qualifications, which will be noted.

d) A very stringent test of the degree to which the model represents a real binary is afforded by including third light as an adjustable parameter. Models which are physically unrealistic can be fitted in many cases by "trading-off" their deficiencies for third light. When third light was included as a free parameter, we found it to be of the order of its probable error [i.e., about $10^{-2}(l_1 + l_2)$].

e) Before applying the differential-corrections program, reasonably extensive trial-and-error experiments were carried out with the light-curve program. For each binary, therefore, even the first iteration of the differential-corrections program began from a relatively good fit.

Incidentally, the differential-corrections program is now faster than that used for MR Cyg (Wilson and Devinney 1971) by a factor of 8.

II. MODEL CONSTRAINTS FOR CONTACT BINARIES

The basic model and computing scheme used here has been described by Wilson and Devinney (1971), and some additional details of the reflection effect have been given by Wilson *et al.* (1972), so we need only state some constraints imposed when operating in the contact binary mode. Since our program treats the two binary components separately, we specify that there be no physical discontinuities across the boundary (on the common envelope) which separates, for computing purposes, one component from the other. To some extent the placement of this boundary is arbitrary, and we define it to be the intersection of the common envelope with a plane which passes through the inner Lagrangian point normal to the line of centers, or x-axis. This boundary curve is therefore a ring around the narrow part of the neck connecting the components, and integration of the light of a component begins and ends at this ring for the lower latitude rows on each component. The parameter list for contact binaries includes: i, inclination of the orbit plane to the plane of the sky; g, exponent in the gravity-darkening law; T_1, polar effective temperature for the component covered at primary eclipse (component 1); A, bolometric albedo; Ω, Roche modified surface potential; q, mass ratio, $\mathfrak{m}_2/\mathfrak{m}_1$; L_1, relative luminosity of component 1; x, limb darkening coefficient; and l_3, third light. Additional details on our conventions regarding g, Ω, q, L_1, and x were given by Wilson and Devinney (1971). Complete definitions for

A and l_3 are to be found in Wilson *et al.* (1972). When these parameters have been specified, the polar temperature of the secondary is constrained to have that value which causes the boundary discontinuity in local surface flux to vanish. This temperature may be computed from the gravity darkening law as

$$T_2 = T_1(a_2/a_1)^{0.25g}\,, \tag{1}$$

where a_1, a_2 are quantities proportional to the acceleration due to gravity at the poles of the two components (this notation is unconventional, but we have been using g for the gravity-darkening exponent). The luminosity of the secondary component, L_2, is now determined. L_2 is computed by numerical integration but is not, of course, a free parameter, so we list only L_1 (normalized) $= L_1/(L_1 + L_2)$. Indeed, L_1 is a free parameter only in the sense of being a convenient scale factor (before normalization), since the luminosity ratio is entirely determined for our contact-binary model when i, g, T_1, Ω, and q have been fixed. To complete the list of constraints, we set $\Omega_2 = \Omega_1$, $g_2 = g_1$, $A_2 = A_1$, and $x_2 = x_1$ automatically when running in the contact mode. It may be desirable in the future to replace the last three conditions by more sophisticated constraints based on improved physical models for contact binaries and their energy exchange.

Our approach differs from that of Mochnacki and Doughty and of Lucy in that we treat the limb and gravity darkenings as adjustable parameters. It would seem that both approaches can be defended. We feel that, at this early stage, departures of these parameters from expected values may offer some of the best clues regarding necessary modifications for theoretical models. Also, in view of the demonstrated strong determinacy of photometric mass ratios, Lucy's fixing of q at the (sometimes decidedly incorrect) spectroscopic value might vitiate some solutions to the extent that the deduced degree of contact might be seriously affected.

III. RESULTS

The systems analyzed here were selected on the basis of having accurate observations, complete eclipses, and absence of light-curve asymmetries. AW UMa does, in fact, have a light curve which is asymmetric about phase zero, but it is extremely interesting from certain points of view and was therefore included. All three binaries were analyzed by Lucy (1971), while RZ Tau and AW UMa were studied also by

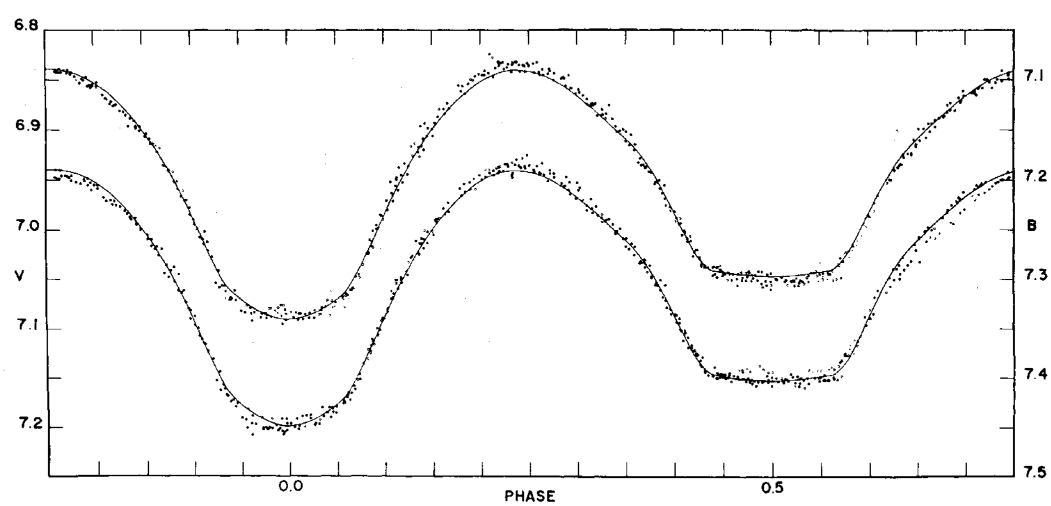

FIG. 1.—Paczyński's B, V observations of AW UMa and our theoretical curves

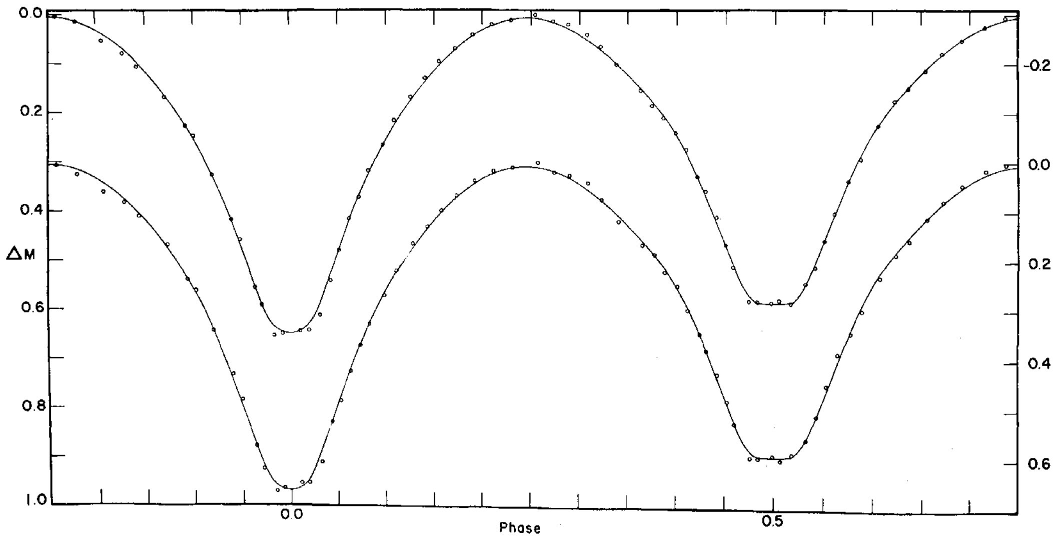

FIG. 2.—Binnendijk's λ4420 (blue) and λ5300 (green) observations of RZ Tau with our fitted curves. Green is above.

Mochnacki and Doughty (1972*a*, *b*). The final adjusted parameters from our differential-corrections program are listed in table 1, and the resulting theoretical curves are graphed among the observations in figures 1, 2, and 3. Also given in table 1 are absolute dimensions and masses found by incorporating radial-velocity information (Paczyński 1964; Struve and Gratton 1948). Absolute dimensions are not listed for RZ Tau because the spectroscopic observations (Struve *et al.* 1950) are not sufficiently numerous to yield meaningful results. In computing these quantities we used our photometric mass ratios in combination with spectroscopic information from the radial velocity curve of the more massive component.

For both RZ Com and AW UMa we find masses which seem incompatible with their positions in the H-R diagram. The more massive component of each binary is

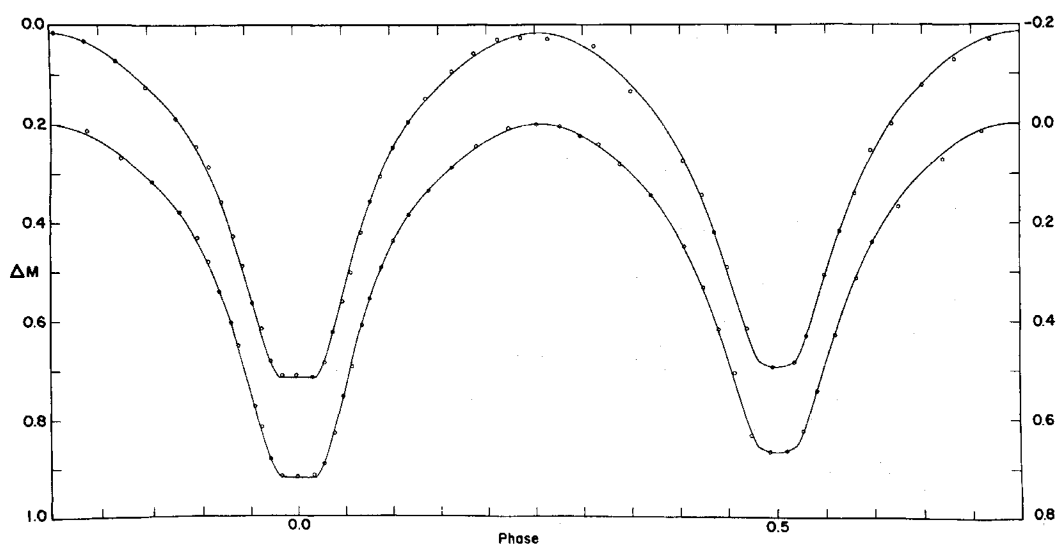

FIG. 3.—Broglia's λ5300 observations of RZ Com at his epochs I (*above*) and II, with our theoretical curves.

only slightly above the main sequence but has a mass appropriate to a much earlier star. RZ Com, for example, seems to have the mass ($2.0\ \mathfrak{m}_\odot$) of an early A main-sequence star, yet is only about 1 mag above the main sequence at type K0. If the spectroscopic rather than the photometric mass ratio is used, we find $1.6\ \mathfrak{m}_\odot$ rather than $2.0\ \mathfrak{m}_\odot$, which is still too high and really does not resolve the difficulty. Taken at face value, the masses for RZ Com and AW UMa are not consistent with any normal view of pre- or post-main-sequence evolution, nor even with any reasonably simple binary star evolution. However, according to Popper (1967), radial-velocity amplitudes are measured systematically too large from plates taken at inadequate dispersion (>20 Å mm^{-1}). The dispersions for RZ Com and AW UMa were, respectively, 76 and 75 Å mm^{-1} at Hγ. Furthermore, Paczyński (1964) specifically commented that he suspected blending problems for AW UMa, so the main problem seems to be that for RZ Com, whose radial-velocity curves seem to be reasonably well defined. Probably this system should be observed again at high dispersion.

For the binaries with observations in two colors we find, as usual, one value for each of the elements i, g, Ω, and q; and two values (for the two passbands) for L_1 and x. RZ Com is represented by two significantly different 5300 Å light curves observed (Broglia 1960) at slightly different epochs, which we analyzed separately. The resulting solution differences for the two epochs will be discussed.

Table 1 also contains theoretical values for linear cosine limb-darkening coefficients, determined by graphical fitting to tables by Carbon and Gingerich (1968), for comparison with our observational values. The agreement seems satisfactory considering that these numbers are probably quite sensitive to small photometric perturbations and deficiencies of the model.

The bolometric albedo A, when allowed to adjust, came in most cases to values near unity, but with rather large probable error. Indeed, one can expect the albedo to be weakly determined because the reflection effect is fairly small for contact systems. Although Rucinski (1969) has predicted that A should lie in the range 0.4–0.5 for stars with fully adiabatic convective envelopes (and this has been verified by Hosokawa [1957, 1959], Napier [1970], and Wilson *et al.* [1972] for Algol and other binaries), we have simply set A to unity for all final iterations. We do not claim that is the ideal way to treat this element—only that it is one of several possible ways and is the one we actually followed. Since the reflection effect is small, effects on the adjustment of other parameters should be slight.

Mochnacki and Doughty (1972*b*) have pointed out an apparent inconsistency between the spectral type and color index (Eggen 1967) of RZ Tau, and have suggested that the spectral type is no earlier than F6, rather than F0 as given by Struve *et al.* (1950). We have assumed that the spectral type is correct since it is not yet established that interstellar reddening is not responsible for the discrepancy. According to Eggen (1967), the color excess and distance modulus are nearly indeterminate for this system. In any case, the solutions are very insensitive to changes in the assumed temperature of the primary component. Effective temperatures for all three systems were estimated from the Harris (1963) calibration of spectral type against T_e. These temperatures were made slightly higher than the Harris values because they are polar temperatures in the program, not mean surface temperatures.

IV. DISCUSSION

a) AW Ursae Majoris

The light curves of this system were observed by Paczyński (1964). They show a significant asymmetry, but there is no effect on the listed parameters due to this problem. We repeated the solution after having removed the asymmetry (by subtraction of a sine wave) and found essentially the same results. The final computed light curves "split the difference" between the light levels at the two quadrature points. There is a small discrepancy in secondary eclipse between the fits in B and V, but we do not consider this alarming in view of the number of constraints enforced in the simultaneous two-color solutions, and consequent small number of degrees of freedom. Our values for the inclination, degree of contact, and the rather extreme mass ratio are similar to those found by Lucy and by Mochnacki and Doughty, and the value for the gravity-darkening exponent, $g = 4\beta = 0.45 \pm 0.06$, provides impersonal evidence on the observational question posed by Lucy's predicted value of 0.32. The light-curve asymmetry might conceivably be caused by a stream of material which leaves the secondary component at its outer Lagrangian point if it could be shown that such a stream would have sufficient continuous opacity to attenuate the light of the binary by about 1 percent. However we, along with Lucy and Mochnacki and Doughty, find the photosphere to be definitely within the outer contact surface, so a positive ejection mechanism is required if this idea is to work.

b) *RZ Tauri*

Fitting of these observations (Binnendijk 1963) gave little trouble after we abandoned the spectroscopic mass ratio (Struve *et al.* 1950) of 0.54. The agreement at $\lambda\lambda$4420 and 5300 seems very good except in the annular phases of primary eclipse, where we cannot reproduce the observed flat phase interval without destroying the fit at other phases. Mochnacki and Doughty had this same difficulty. We speculate that perhaps one or two unfortunate residuals are responsible for this apparent flat region, and that it may not be shown by future light curves of RZ Tau. The value of g (0.29 ± 0.06 p.e.) is in excellent agreement with the convective value. A paper by one of us (Wilson *et al.* 1972) contains a remark that classical gravity darkening was used for the secondary component of the Algol system, based on experience in fitting contact binaries. At that time we had completed our analysis of RZ Com, for which we found g greater than unity, and were only beginning our study of the other two systems. These other systems, of course, did show the convective darkening.

Insertion of the photometric mass ratio (0.37) into the mass function for the primary component gives individual masses of 4.0 and 1.5 $\mathfrak{m}_\odot$, which are almost surely too high. However, one can see by plotting the few radial-velocity points that the mass function itself is quite uncertain. A reduction in $a_1 \sin i$ of 25 percent would produce reasonable masses (about 1.6 and 0.6 $\mathfrak{m}_\odot$) with $q = 0.37$. It is probably a coincidence that the published values of $a_1 \sin i$ and $\mathfrak{m}_2/\mathfrak{m}_1$ happen to give plausible main-sequence masses, since a small change in either quantity, within the observational uncertainty, can give a result such as the above value of $\mathfrak{m}_1 = 4.0\ \mathfrak{m}_\odot$.

c) *RZ Comae Berenices*

The two 5300 Å light curves shown in figure 3 were observed two to three months apart by Broglia (1960). They are significantly different by inspection, and Broglia attributed this temporal behavior to a bright area existing on the outer facing hemisphere of the small star at epoch II (*lower curve*). Binnendijk (1964) rediscussed these observations and pointed out that a somewhat more consistent explanation can be given in terms of a dark area on the inner hemisphere of the larger star at the other epoch. The decision between these possibilities is equivalent to a decision as to which epoch (if either) corresponds to the normal light curve of the system. Since the fit we have achieved is good at both epochs, one would tend to say that the light curve which yields parameters closer to the expected (theoretical) values is the normal case. The only elements for which we have predicted values are the limb- and gravity-darkening parameters. Unfortunately, the limb darkening is closer to the expected result for light curve II, while the gravity darkening is in better agreement for light curve I. Therefore it seems that we cannot, at present, say which of the two light curves is normal, and one might suspect that transient effects are present at both epochs.

Unless these transient effects have seriously undermined the validity of our solutions for RZ Com, it seems that the components are only marginally in contact. That is, one solution (epoch I) shows the binary to be slightly over contact while the other shows it slightly under contact. Could this result be related to the fact that RZ Com shows, apparently, a radiative (von Zeipel) gravity effect, or perhaps an even larger effect? The spectral type of K0 leads us to expect deep convection zones for the components of this system, whether they are in contact or not, and we do not know of any physical reason to associate marginal contact with (apparently) radiative gravity darkening. However, we can point to an interesting photometric effect, which may eventually lead us to understand the finding of a gravity exponent of unity for a convective-envelope binary, when more systems with total primary eclipses have been studied sufficiently well. We have found, by experimenting with RZ Com–like light curves, that the total eclipse can be the deeper only if g is of the order of unity or greater. No other parameter can be changed to produce this effect and, in particular, the convective value of g always yields annular primary eclipses. If these statements prove to be valid for all combinations of contact-binary parameters, it will be easy to identify systems (they will be Binnendijk's class W) which, from the point of view of light-curve analysis, show radiative gravity darkening. One can then hope to find common physical characteristics of these systems which will indicate the reason for the observed darkening. For example, it may be found that such systems are subject to a particular photometric surface effect which simulates gravity darkening, although perhaps is physically unrelated to it. Alternatively it may be productive to bypass equation (1) and include T_2 as an adjustable parameter in fitting the RZ Com light curves, but of course the extra parameter will weaken the solution.

The last remarks serve to emphasize a point which many workers have recognized for some years—that since unexplained odd functions (sine terms) are present in the light curves of many systems, we should not be surprised to find even functions due to the same effects. An urgent present need, therefore, is for more quantitative work, both observational and theoretical, on the physical causes of asymmetries in the light curves of close binaries.

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