The Effect of Large-Scale Eddies on Climatic Change

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(Manuscript received 11 January 1973)

ABSTRACT

A parameterization for the fluxes of sensible heat by large-scale eddies developed in an earlier paper is incorporated into a model for the mean temperature structure of an atmosphere including only these fluxes and the radiative fluxes. The climatic changes in this simple model are then studied in order to assess the strength of the dynamical feedback and to gain insight into how dynamical parameters may change in more sophisticated climatic models. The model shows the following qualitative changes: 1) an increase in the solar constant leads to increased static stability, decreased dynamic stability, and stronger horizontal and vertical winds; 2) an increase in the amount of atmospheric absorption leads to decreased static and dynamic stability, and stronger horizontal and vertical winds; and 3) an increase in rotation rate leads to greater static and dynamic stability, weaker horizontal winds, and stronger vertical winds. The quantitative results provide support for the common assumption that the static stability remains constant during climatic changes. Twenty-five percent changes in the external parameters cause changes in the static stability of the order of only a few tenths of a degree per kilometer. The results also show that the assumption that the horizontal eddy flux can be represented by a diffusion law with a constant eddy coefficient is a bad one, because of the strong negative feedback in the eddy fluxes.

1. Introduction

In a recent paper (Stone, 1972b; hereafter referred to as I) we derived expressions for the fluxes of sensible heat due to large-scale atmospheric eddies and demonstrated that these fluxes have a very strong negative feedback. This negative feedback will tend to limit the atmosphere's response to changes in external parameters and tend to inhibit climatic change. In this paper we present a study of how the equilibrium state of a very simple model atmosphere that includes these eddy fluxes and their feedback will respond to changes in the solar constant, in the amount of absorbing material, and in the planet's rate of rotation. The temperature structure of this model atmosphere will be determined in the same way as in I, i.e., the only fluxes included will be the fluxes of sensible heat due to radiation and the large-scale eddies. Clearly, then, we will not be calculating true climatic changes. Rather, our goal is to assess the strength of the negative feedback in the eddy fluxes and to gain some insight into how the important dynamical parameters might change in a more sophisticated model.

In our study we will be particularly interested in checking two assumptions commonly made in studies of climatic change: 1) the assumption that the static stability is constant [as, for example, in Manabe and Wetherald (1967), Saltzman (1968), Wiin-Nielsen (1970), and Sela and Wiin-Nielsen (1971)]; and 2) the assumption that the horizontal eddy transport of sensible heat can be represented by a simple diffusion law with a constant eddy coefficient [as in Sellers (1969), Wiin-Nielsen (1970), and Sela and Wiin-Nielsen (1971)]. The first assumption is equivalent to assuming that the vertical eddy flux of heat has an infinite negative feedback, and the second to assuming that the horizontal eddy flux has zero feedback.

2. Model for the eddy fluxes

The basic equation we will solve for the equilibrium temperature structure is the temperature equation for a flat, Boussinesq atmosphere, averaged over time and longitude:

$$\frac{\partial}{\partial y}(\overline{v\theta})+\frac{\partial}{\partial z}(\overline{w\theta})=\bar{Q}, \tag{2.1}$$

where Q is the radiative flux divergence, y and z the meridional and vertical coordinates, v and w the respective components of velocity, and θ the potential temperature,

$$\theta \approx T+\Gamma z. \tag{2.2}$$

Here T is the temperature and Γ the adiabatic lapse rate. The bar over a variable denotes the average over time and longitude.

The expressions we will use for the eddy fluxes, $\overline{v\theta}$ and $\overline{w\theta}$, are those derived theoretically in I [Eqs. (2.22) and (2.23)]:

$$\overline{v\theta}=-0.86\frac{gH^2}{\langle \bar{T}\rangle f}\left\langle\frac{\partial\bar{\theta}}{\partial z}\right\rangle\left\langle\frac{\partial\bar{\theta}}{\partial y}\right\rangle\frac{(1+\mathrm{Ri})^{\frac{1}{2}}}{\mathrm{Ri}}\frac{y}{L}\left(1-\frac{y}{L}\right), \quad (2.3)$$

$$\overline{w\theta}=+0.36fH^2\left\langle\frac{\partial\bar{\theta}}{\partial z}\right\rangle\frac{1}{\mathrm{Ri}(1+\mathrm{Ri})^{\frac{1}{2}}}\frac{z}{H}\left(1-\frac{z}{H}\right). \quad (2.4)$$

Here Ri is the Richardson number for the zonal thermal wind,

$$\mathrm{Ri}=\frac{f^2\langle\bar{T}\rangle\left\langle\dfrac{\partial\bar{\theta}}{\partial z}\right\rangle}{g\left\langle\dfrac{\partial\bar{\theta}}{\partial y}\right\rangle^2}. \quad (2.5)$$

In these equations angle brackets denote averages over all $y(0\leqslant y\leqslant L)$ and all $z(0\leqslant z\leqslant H)$, L is the equator to pole distance, H the scale height,

$$H=\frac{R\langle\bar{T}\rangle}{g}, \quad (2.6)$$

R the gas constant, g the acceleration of gravity, and f the Coriolis parameter (which we will evaluate at 45° latitude).

The specific y and z dependences that appear in (2.3) and (2.4) are not crucial, since it is the mean values of the fluxes which determines the mean gradients of $\bar{\theta}$, and it is these mean gradients that we will concentrate on in this study. Since changes in these mean gradients will be affected by the parameter dependences which appear in (2.3) and (2.4), we will summarize the evidence for the validity of these expressions. A number of indirect checks were made in I. There these same expressions were used for calculating static stabilities and horizontal temperature gradients under a variety of conditions and were found to give good results when compared with numerical calculations and observations. Specifically, the results were good not only for Earth, but also for Mars where many of the parameters have substantially different values (e.g., L and g). In addition, the same expressions were used in I to deduce characteristic response times for the adjustment of the temperature structures of the atmospheres of Earth and Mars when perturbations from equilibrium are introduced. Again the results were in good agreement with numerical calculations.

However, one can also obtain more direct checks on (2.3) and (2.4) by comparing the values they give for the fluxes with actual observations of the eddy fluxes in the earth's atmosphere. It is convenient to compare the horizontal eddy coefficient K predicted by Eq. (2.3) with the observations, rather than $\overline{v\theta}$ itself; K is defined by

$$\overline{v\theta}=-K\frac{\partial\bar{\theta}}{\partial y}. \quad (2.7)$$

To obtain an equation comparable to (2.3), we identify

$$\frac{\partial\bar{\theta}}{\partial y}=6\left\langle\frac{\partial\bar{\theta}}{\partial y}\right\rangle\frac{y}{L}\left(1-\frac{y}{L}\right) \quad (2.8)$$

(so that the mean value of $\partial\bar{\theta}/\partial y$ is conserved). Substituting (2.8) into (2.7) and comparing the result with (2.3), we identify

$$K=0.144\frac{gH^2}{\langle\bar{T}\rangle f}\left\langle\frac{\partial\bar{\theta}}{\partial z}\right\rangle\frac{(1+\mathrm{Ri})^{\frac{1}{2}}}{\mathrm{Ri}}. \quad (2.9)$$

If we take as typical values $g=980$ cm^2 sec^{-1}, $H=8$ km, $\langle\bar{T}\rangle=250$K, $f=1.03\times10^{-4}$ sec^{-1}, $\langle\partial\bar{\theta}/\partial z\rangle=3.3$K km^{-1}, Ri$=56$ [corresponding to $\langle\partial\bar{\theta}/\partial y\rangle=-0.4$K (100 km)$^{-1}$], we use (2.9) to determine $K=1.5\times10^{10}$ cm^2 sec^{-1}. This is in good agreement with observed atmospheric values. For example, Wiin-Nielsen and Sela (1971) reported an average value of K for the year 1963 of 1.7×10^{10} cm^2 sec^{-1}. In addition, Eq. (2.9) predicts qualitatively correct seasonal changes in K. If we approximate Ri$\gg1$, substituting (2.5) into (2.9), we obtain

$$K=0.144\frac{gH^2}{\langle\bar{T}\rangle f^2}\left|\left\langle\frac{\partial\bar{\theta}}{\partial y}\right\rangle\right|\left(\frac{g}{\langle\bar{T}\rangle}\left\langle\frac{\partial\bar{\theta}}{\partial z}\right\rangle\right)^{\frac{1}{2}}. \quad (2.10)$$

The main quantity in this expression which varies seasonally is $\langle\partial\bar{\theta}/\partial y\rangle$, and one would consequently expect seasonal changes of $\overline{v\theta}$ to be proportional to $\langle\partial\bar{\theta}/\partial y\rangle^2$. Clapp (1970) has summarized the observations of $\overline{v\theta}$ in different seasons and correlated them with the horizontal temperature gradient. His data show that this square law is, in fact, closely obeyed. The same square law occurs in Green's (1970) parameterization of the eddy heat flux.

Finally, we can check the vertical flux given by (2.4). Palmén and Newton (1969, p. 53) have estimated the mean vertical eddy flux necessary to achieve equilibrium north of 32° latitude in winter. This eddy flux includes contributions from both large-scale and small-scale eddies, but since in the mid-troposphere the dominant contribution will be from the large-scale eddies, Palmén and Newton's mid-troposphere estimate should be comparable to values calculated from (2.4) with $z=\frac{1}{2}H$. Their value is 2 ly min^{-1} (1.3×10^4 ergs cm^{-2} sec^{-1}). To get a flux from (2.4) in comparable units we multiply $\overline{w\theta}$ by ρC_p and take for the density, $\rho=0.7\times10^{-3}$ gm cm^{-3} (a typical mid-troposphere value) and for the specific heat, $C_p=1.0\times10^7$ ergs (°K)$^{-1}$ gm^{-1}. For the other parameters in (2.3) we use the same values as in estimating K above, except for Ri, for which we adopt as a typical winter value Ri$=25$ [corresponding to $\langle\partial\bar{\theta}/\partial y\rangle=-0.6$K (100 km)$^{-1}$]. The result is $\rho C_p\overline{w\theta}=1.2\times10^4$ ergs cm^{-2} sec^{-1}, in good agreement with Palmén and Newton's value.

The checks described above show that there is good justification for using the above expressions for the eddy fluxes in climatic calculations. However, it is clear that more definitive checks are still desirable. Such checks might be obtained by studying the seasonal variations in the atmosphere's eddy transports, or by using the data available from numerical simulations of the atmosphere.

3. Model for the radiative fluxes

We will model the radiative flux divergence in the same way as in I, i.e., by the simple linearized model introduced by Speigel (1957) and generalized by Goody (1964). Assuming that the deviations of the temperature from the radiative equilibrium state, $\bar{T}_r(y,z)$, are small and that the deviations are characterized by a single dominant scale, we have

$$\bar{Q}=\frac{T_r-T}{\tau}, \tag{3.1}$$

where τ is the characteristic radiative relaxation time for the system, and it depends on the characteristic scale. The temperature deviations from the radiative state throughout most of the earth's atmosphere are less than 20%, so this approximation is adequate for our parameter study.

Substituting (3.1) and (2.2) into (2.1), we obtain

$$\frac{\partial}{\partial y}(\overline{v\theta})+\frac{\partial}{\partial z}(\overline{w\theta})=\frac{\bar{\theta}_r-\bar{\theta}}{\tau}, \tag{3.2}$$

where $\bar{\theta}_r$ is the potential temperature of the radiative equilibrium state. Substituting (2.3) and (2.4) into (3.2), we obtain an equation for $\bar{\theta}$ in terms of $\langle\partial\bar{\theta}/\partial z\rangle$, $\langle\partial\bar{\theta}/\partial y\rangle$, $\bar{\theta}_r$ and τ. We find two equations for $\langle\partial\bar{\theta}/\partial z\rangle$ and $\langle\partial\bar{\theta}/\theta y\rangle$ by differentiating this equation separately with respect to y and z, and then integrating over all y and z, i.e.,

$$\left\langle\frac{\partial\bar{\theta}}{\partial z}\right\rangle=\left\langle\frac{\partial\bar{\theta}_r}{\partial z}\right\rangle+\frac{0.72f\tau}{\mathrm{Ri}(1+\mathrm{Ri})^{\frac{1}{2}}}\left\langle\frac{\partial\bar{\theta}}{\partial z}\right\rangle, \tag{3.3}$$

$$\left\langle\frac{\partial\bar{\theta}}{\partial y}\right\rangle=\left\langle\frac{\partial\bar{\theta}_r}{\partial y}\right\rangle-\frac{1.73\tau gH^2(1+\mathrm{Ri})^{\frac{1}{2}}}{\langle\bar{T}\rangle fL^2\,\mathrm{Ri}}\left\langle\frac{\partial\bar{\theta}}{\partial z}\right\rangle\left\langle\frac{\partial\bar{\theta}}{\partial y}\right\rangle. \tag{3.4}$$

The radiative transfer problem is now reduced to specifying the mean gradients of the radiative state, $\langle\partial\bar{\theta}_r/\partial z\rangle$ and $\langle\partial\bar{\theta}_r/\partial y\rangle$, and τ. Once these are given the above two equations can be solved simultaneously for the two mean gradients in radiative-dynamical equilibrium, $\langle\partial\bar{\theta}/\partial z\rangle$ and $\langle\partial\bar{\theta}/\partial y\rangle$. These equations are highly nonlinear since Ri itself depends on these gradients [see Eq. (2.5)]. The simplest way to solve these equations is to treat (3.3), (3.4) and (2.5) as three equations for the two mean gradients and for Ri, and to derive a single equation for Ri. The result is

$$\mathrm{Ri}=\frac{\mathrm{Ri}'}{1-\delta g_1(\mathrm{Ri})}\left[1+\frac{\delta B' g_2(\mathrm{Ri})}{1-\delta g_1(\mathrm{Ri})}\right]^2, \tag{3.5}$$

where

$$g_1=\frac{0.72}{\mathrm{Ri}(1+\mathrm{Ri})^{\frac{1}{2}}}, \tag{3.6}$$

$$g_2=\frac{1.73(1+\mathrm{Ri})^{\frac{1}{2}}}{\mathrm{Ri}}, \tag{3.7}$$

$$\delta=f\tau, \tag{3.8}$$

$$\mathrm{Ri}'=\frac{f^2\langle\bar{T}\rangle\left\langle\dfrac{\partial\bar{\theta}_r}{\partial z}\right\rangle}{g\left\langle\dfrac{\partial\bar{\theta}_r}{\partial y}\right\rangle^2}, \tag{3.9}$$

$$B'=\frac{gH^2}{\langle\bar{T}\rangle f^2L^2}\left\langle\frac{\partial\bar{\theta}_r}{\partial z}\right\rangle. \tag{3.10}$$

Once Eq. (3.5) is solved for Ri, the mean gradients can be found from (3.3) and (3.4).

If we replace Eq. (2.3) by the simple eddy diffusion law [Eqs. (2.7) plus (2.8)], then Eq. (3.4) is replaced by

$$\left\langle\frac{\partial\bar{\theta}}{\partial y}\right\rangle=\left\langle\frac{\partial\bar{\theta}_r}{\partial y}\right\rangle-\frac{12K\tau}{L^2}\left\langle\frac{\partial\bar{\theta}}{\partial y}\right\rangle. \tag{3.11}$$

This equation can be solved directly for $\langle\partial\bar{\theta}/\partial y\rangle$, without solving for Ri, if K is specified.

To determine the gradients of the radiative equilibrium state in terms of the basic external parameters, we will use the approximate radiative equilibrium solution given by Goody (1964, Section 8.4). This solution, which assumes a grey atmosphere and uses the Eddington approximation, is

$$\bar{T}_r=\frac{1}{2^{\frac{1}{4}}}T_e(1+\tfrac{3}{2}\tau^*e^{-z/h})^{\frac{1}{4}},\quad z\neq 0, \tag{3.12}$$

$$\bar{T}_r=\frac{1}{2^{\frac{1}{4}}}T_e(2+\tfrac{3}{2}\tau^*)^{\frac{1}{4}},\qquad z=0. \tag{3.13}$$

Here τ^* is the total optical depth of the atmosphere in the infrared ($\tau^*\approx 4$), h the scale height of the absorbing material ($h\approx 2$ km), and $T_e(y)$ the effective temperature of the atmosphere. The global average value of T_e is related to the solar constant F and albedo a by

$$\sigma\langle\bar{T}_e\rangle^4=F\frac{(1-a)}{4}, \tag{3.14}$$

where σ is the Stefan-Boltzmann constant.

From (3.12) and (3.13) we calculate for the mean static stability of the radiative equilibrium state

$$\left\langle\frac{\partial\bar{\theta}_r}{\partial z}\right\rangle=\Gamma-\frac{\langle\bar{T}_e\rangle}{2^{\frac{1}{4}}H}[(2+\tfrac{3}{2}\tau^*)^{\frac{1}{4}}-(1+\tfrac{3}{2}\tau^*e^{-H/h})^{\frac{1}{4}}]. \tag{3.15}$$

In calculating the horizontal gradient of the radiative equilibrium state, we will follow I in using the gradient at 45° latitude as typical. Then from Eq. (3.8) in I we have

$$\left\langle\frac{\partial\bar{\theta}_r}{\partial y}\right\rangle=-0.38\frac{\langle\bar{T}_r\rangle}{L}. \tag{3.16}$$

We note from Eqs. (3.2) and (2.2), making use of the forms of (2.3) and (2.4), that

$$\langle\bar{T}\rangle=\langle\bar{T}_r\rangle, \tag{3.17}$$

i.e., the mean temperature of our model atmosphere is unaffected by the eddy fluxes (which only re-distribute heat) and is completely determined by the radiative fluxes. From (3.12) we have

$$\langle\bar{T}_r\rangle=\frac{1}{H}\int_0^H\bar{T}_r dz=\frac{\langle\bar{T}_e\rangle}{2^{\frac{1}{4}}H}\int_0^H(1+\tfrac{3}{2}\tau^* e^{-z/h})^{\frac{1}{4}}dz.$$

This integral may be evaluated by breaking the integral into two ranges, where $x=\frac{3}{2}\tau^* e^{-z/h}$ is respectively greater or less than unity, and expanding $(1+x)^{\frac{1}{4}}$ in appropriate power series in the two ranges. The result is

$$\langle\bar{T}_r\rangle=\frac{\langle\bar{T}_e\rangle}{2^{\frac{1}{4}}}\left\{\frac{4h}{H}[(\tfrac{3}{2}\tau^*)^{\frac{1}{4}}-1]+\frac{h}{3H}\left[1-\frac{1}{(\frac{3}{2}\tau^*)^{\frac{3}{4}}}\right]+\cdots\right.$$
$$\left.+1-\frac{h}{H}\log\tfrac{3}{2}\tau^*-\frac{1}{4}\frac{h}{H}[\tfrac{3}{2}\tau^* e^{-H/h}-1]+\cdots\right\}.$$

The second terms in these expansions are only about 10% as large as the first, so it will be consistent with our other approximations and adequate for our parameter study to neglect them. Retaining only the first terms from the expansions, we find for the mean temperature

$$\langle\bar{T}_r\rangle=\frac{\langle\bar{T}_e\rangle}{2^{\frac{1}{4}}}\left\{\left[\frac{4h}{H}(\tfrac{3}{2}\tau^*)^{\frac{1}{4}}-1\right]+1-\frac{h}{H}\log\tfrac{3}{2}\tau^*\right\}. \tag{3.18}$$

Typically, $\langle\bar{T}_r\rangle\approx\langle\bar{T}_e\rangle$.

Finally, we need an expression for the radiative relaxation time τ. Gierasch *et al.* (1970) have shown that an adequate approximation for a parameter study, when the characteristic space scale is H, is

$$\tau\approx\frac{C_pP_0}{\sigma g\langle\bar{T}\rangle^3}, \tag{3.19}$$

where P_0 is the surface pressure of the atmosphere.

4. Climatic changes

Excluding the universal constant σ, there are nine parameters which determine an equilibrium state in our model. They are: the gas constant, R (i.e., the molecular weight of the atmosphere); the specific heat of the atmosphere, C_p; the surface pressure, P_0 (i.e., the total mass of the atmosphere); the equator-to-pole distance, L (i.e., the radius of the planet); the acceleration of gravity, g (i.e., the mass of the planet); the Coriolis parameter at 45° latitude, f (i.e., the rate of rotation of the planet); the amount of solar radiation absorbed at the ground, $(F/4)(1-a)$; the relative scale height of the absorbing material in the atmosphere, h/H; and the total optical depth in the atmosphere, τ^*. Once these nine parameters are specified, calculating the equilibrium state is straightforward: Γ is given by g/C_p; $\langle\bar{T}_e\rangle$ by Eq. (3.14); $\langle\bar{T}\rangle$ by (3.17) and (3.18); τ by (3.19); H by (2.6); $\langle\partial\bar{\theta}_r/\partial z\rangle$ and $\langle\partial\bar{\theta}_r/\partial y\rangle$ by (3.15) and (3.16); Ri by (3.5) [see I]; and finally $\langle\partial\bar{\theta}/\partial z\rangle$ by (3.3) and $\langle\partial\bar{\theta}/\partial y\rangle$ by (3.4). If we use an eddy diffusion law for the horizontal eddy flux, then (3.4) is replaced by (3.11), and K must be specified independently.

We will adopt the following values as standard values for the terrestrial atmosphere:

$$R=2.9\times10^6 \text{ ergs } (^\circ\text{K})^{-1}\text{ gm}^{-1}$$
$$C_p=1.0\times10^7 \text{ ergs } (^\circ\text{K})^{-1}\text{ gm}^{-1}$$
$$P_0=10^6 \text{ dyn cm}^{-2}$$
$$L=10^9 \text{ cm}$$
$$g=980 \text{ cm sec}^{-2}$$
$$f=1.03\times10^{-4}\text{ sec}^{-1}$$
$$F(1-a)=8.86\times10^5 \text{ ergs cm}^{-2}\text{ sec}^{-1}$$
$$(\text{e.g., } F=2 \text{ cal cm}^{-2}\text{ min}^{-1},\ a=0.365)$$
$$h/H=\tfrac{1}{4}$$
$$\tau^*=4.$$

We then calculate in the manner outlined above

$$\langle\bar{T}_e\rangle=250\text{K}$$
$$\langle\bar{T}\rangle=235\text{K}$$
$$\tau=1.41\times10^7 \text{ sec}$$
$$H=6.86 \text{ km}$$
$$\left\langle\frac{\partial\bar{\theta}_r}{\partial z}\right\rangle=-10.0\text{K km}^{-1}$$
$$\left\langle\frac{\partial\bar{\theta}_r}{\partial y}\right\rangle=-0.892\text{K }(100\text{ km})^{-1}$$
$$\text{Ri}=25.7$$
$$\left\langle\frac{\partial\bar{\theta}}{\partial z}\right\rangle=+1.45\text{K km}^{-1}$$
$$\left\langle\frac{\partial\bar{\theta}}{\partial y}\right\rangle=-0.378\text{K }(100\text{ km})^{-1}.$$

These values differ slightly from the standard terrestrial atmosphere calculated in I (Table 1, Model no. 1) mainly because we have used a grey atmosphere solution for the radiative equilibrium solution, whereas in I we used non-grey calculations.

The largest error in our model occurs in the mean static stability, which is $+3.3\mathrm{K\ km^{-1}}$ in the real atmosphere. This error is caused mainly by our neglect of large-scale latent heat fluxes (Palmén and Newton, 1969, Section 2.6). It is this discrepancy which makes our model inadequate for calculating realistic climatic changes in the mean gradients. The effective value of the eddy diffusion coefficient in the above equilibrium state, calculated from Eq. (2.9), is

$$K=0.803\times10^{10}\ \mathrm{cm^2\ sec^{-1}}. \tag{4.1}$$

This is the value which, when used in Eq. (3.11), would give the same value for $\langle\partial\bar{\theta}/\partial y\rangle$ as that given above. It is smaller than typical values in the real atmosphere primarily because of the smaller static stabilities in our model atmosphere [cf. Eq. (2.9)].

We will now describe how the above equilibrium state responds to changes in some of the input parameters. Specifically, we will vary τ^*, $\beta=F(1-a)$, and f. Changes in the first two are commonly studied as potential sources of climatic change (e.g., Manabe and Wetherald, 1967; Sellers, 1969). Changes in f are of interest because rotation is so fundamental to the baroclinic instability process which gives rise to the large-scale eddy fluxes.

Fig. 1 illustrates the response of the static stability to changes in the above three parameters. The abscissa is a generalized climatic variable, φ, and the three curves show how $\langle\partial\bar{\theta}/\partial z\rangle$ changes when $\varphi=\tau^*$, β or f. In all the figures the zero subscripts refer to values in the standard atmosphere given above, and parameters normalized to their standard values are plotted along the abscissa. Quantitatively, the changes in $\langle\partial\bar{\theta}/\partial z\rangle$ are small. The largest plotted change occurs for $\tau^*=2$ and is an increase of $0.63\mathrm{K\ km^{-1}}$. In this same case $\langle\partial\bar{\theta}_r/\partial z\rangle$ increased by $4.3\mathrm{K\ km^{-1}}$. The great reduction in the magnitude of the increase in the static stability when the dynamical fluxes are included illustrates their very strong negative feedback.

Qualitatively, the static stability is decreased by an increase in absorbing material, τ^*. This decrease is caused directly by an increase in the greenhouse effect, which causes the radiative equilibrium state to be more unstable [Eq. (3.15)]. The static stability is increased by an increase in rotation rate f. An increase in f tends to suppress the horizontal eddy flux [Eq. (2.3)], and thus the horizontal temperature gradient is increased. This, in turn, decreases the Richardson number [Eq. (2.5)] and this leads to an increase in the static stability [Eq. (3.3)]. Finally, increases in the solar constant β have very little effect on the static stability. This is because of two competing effects. Increasing β increases $\langle\bar{T}\rangle$, and this affects both $\langle\partial\bar{\theta}_r/\partial y\rangle$ [Eq. (3.16)] and τ [Eq. (3.19)]. The increased differential solar heating leads to larger horizontal temperature gradients, thence to smaller Ri and increased static stability. On the other hand, the higher temperatures lead to a shorter radiative relaxation time, which makes the radiation more efficient at destabilizing the atmosphere and decreases the static stability. The two effects almost cancel each other.

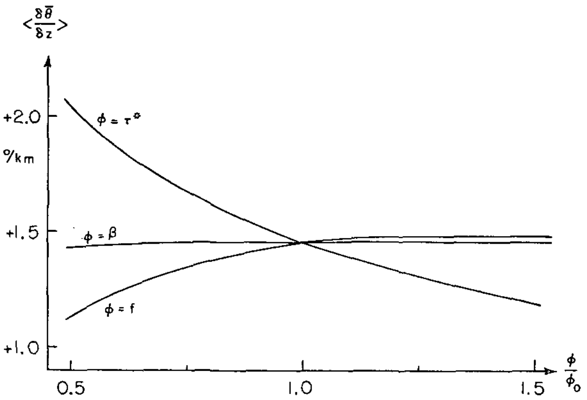

FIG. 1. Mean static stability vs normalized values of τ^*, β and f.

Fig. 2. illustrates how the mean horizontal temperature gradient responds to changes in the amount of absorbing material. The solid line shows the response according to our model, i.e., Eq. (3.4), and the dotted line shows the response according to the eddy diffusion law with constant K, i.e., Eq. (3.11), using the value of K given by (4.1). The two models give nearly the same results. The horizontal gradients are increased by an increased amount of absorbing material, because the increased greenhouse effect causes a rise in the mean temperature [cf. Eq. (3.18)] and therefore an increase in the differential solar heating [cf. Eq. (3.16)].

Fig. 3 shows the response of the mean horizontal temperature gradient to changes in the solar constant.

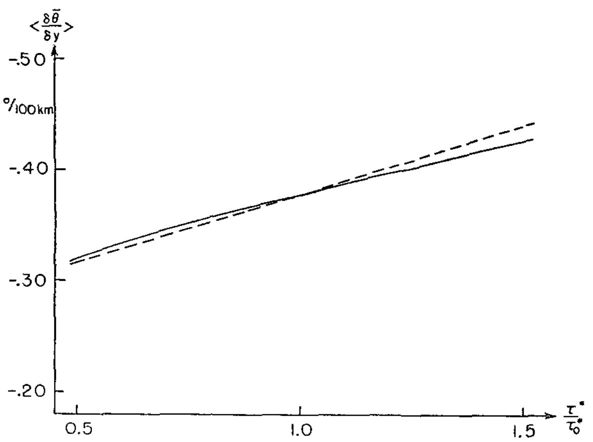

FIG. 2. Mean horizontal temperature gradient vs normalized values of the amount of absorbing material. The solid and dotted lines refer to calculations with non-constant and constant eddy coefficients, respectively.

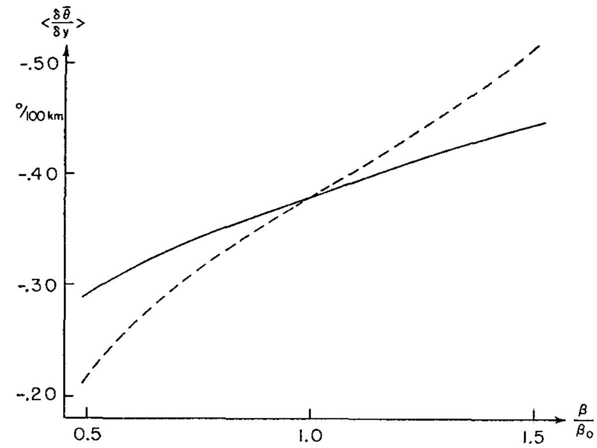

FIG. 3. As in Fig. 2 except for normalized values of the solar constant.

Again the solid and dotted lines represent the response for non-constant and constant K, respectively. In this case the assumption of a constant eddy diffusion coefficient leads to an overestimate of the changes by a factor of about 2. The horizontal temperature gradients are increased by an increase in the solar constant, because of the increased differential solar heating.

Fig. 4 illustrates the response of the mean horizontal temperature gradient to changes in the rotation rate. Again the dotted line refers to the response for a constant K, and in this case there is no change in the horizontal temperature gradient. Since the radiative equilibrium state does not depend on f, a constant K leads to a solution for $\langle \partial\bar{\theta}/\partial y \rangle$ which is also independent of f. This result emphasizes how bad the assumption of a constant eddy coefficient may be. The solid line shows the response of $\langle \partial\bar{\theta}/\partial y \rangle$ when K is not assumed to be constant. Increasing the rotation rate tends to suppress the horizontal eddy flux [cf. Eq. (2.3)], thereby increasing the horizontal temperature gradient.

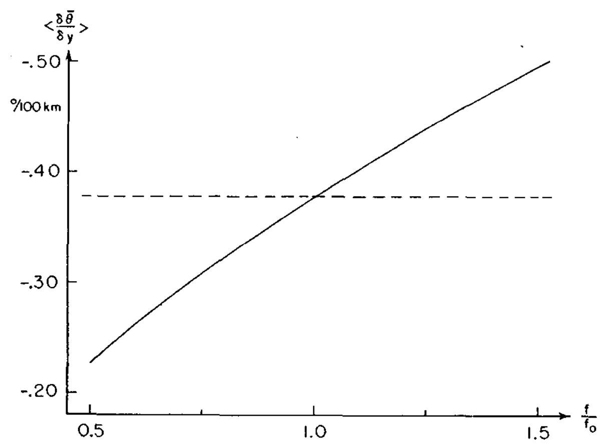

FIG. 4. As in Fig. 2 except for normalized values of the rotation rate.

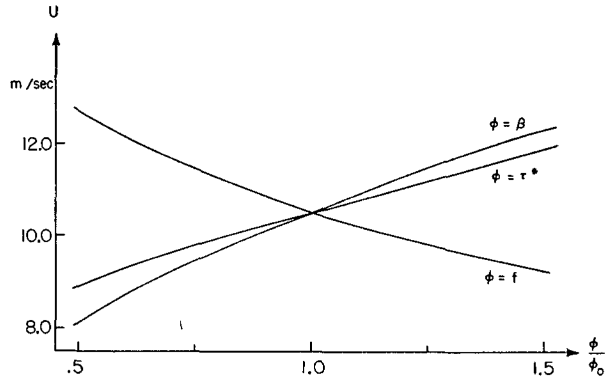

FIG. 5. Mean zonal wind vs normalized values of β, τ^* and f.

Fig. 5 shows the mean baroclinic component of the horizontal wind U, as a function of β, τ^* and f. Here U is computed from the thermal wind relation,

$$U = \frac{-gH\left\langle \dfrac{\partial\bar{\theta}}{\partial y} \right\rangle}{f\langle \bar{T} \rangle} = \frac{-R\left\langle \dfrac{\partial\bar{\theta}}{\partial y} \right\rangle}{f}. \tag{4.2}$$

The horizontal winds are increased by increasing the solar constant or the amount of absorbing material, or by decreasing the rate of rotation.

In Fig. 6 we give the response of the Richardson number to changes in τ^*, β and f, using Eq. (2.5). The atmosphere's dynamic stability is increased (i.e., Ri increased) by increasing the rate of rotation, or by decreasing the solar constant or the amount of absorbing material. The changes in Ri are primarily due to changes in $\langle \partial\bar{\theta}/\partial y \rangle$ and thus in $\langle \partial\bar{u}/\partial z \rangle$.

In Fig. 7 we show how the longitudinal wavelength λ of the large-scale eddies responds to changes in β, τ^* and f. This wavelength is calculated from the approxi-

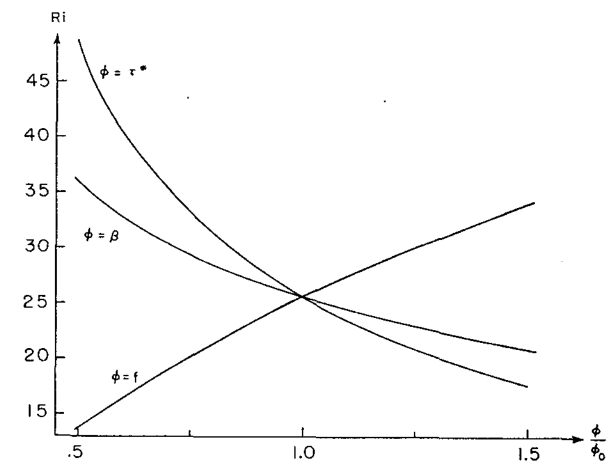

FIG. 6. Richardson number vs normalized values of τ^*, β and f.

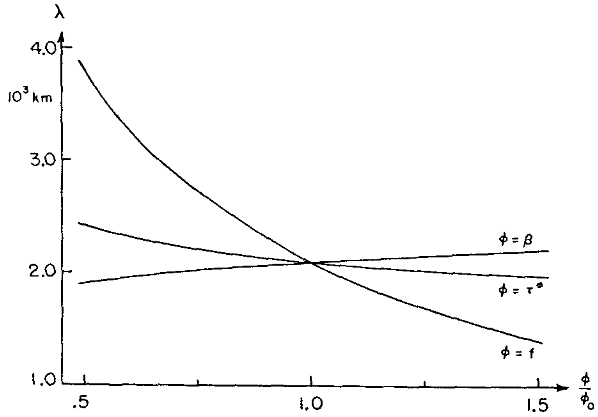

FIG. 7. Scale of the eddies vs normalized values of β, τ^* and f.

mate formula

$$\lambda=\frac{2\pi U}{f}\left(\frac{1+\mathrm{Ri}}{5/2}\right)^{\frac{1}{2}} \tag{4.3}$$

(Stone, 1966). The calculated scales are smaller than those of real atmospheric eddies because of the model's underestimate of the real atmosphere's static stability. The eddies increase in size when the amount of absorbing material or rotation rate is decreased, or when the solar constant is increased.

Fig. 8 gives the Rossby number Ro as a function of τ^*, β and f, using the relation

$$\mathrm{Ro}=\frac{U}{f(\lambda/2)}. \tag{4.4}$$

The motions become more highly geostrophic if the amount of absorbing material or solar constant is decreased or if the rotation rate is increased.

Fig. 9 shows the corresponding response for the formation time t for the eddies, i.e., the time for an eddy to increase in amplitude by a factor of e during its linear growth phase, as calculated from the approximate

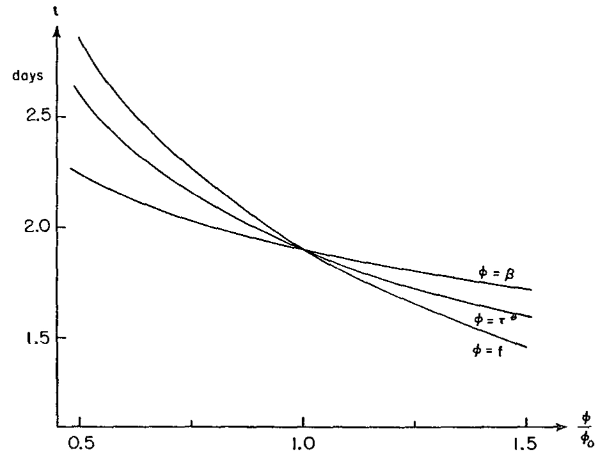

FIG. 9. Formation time of the eddies vs normalized values of β, τ^* and f.

formula (Stone, 1966)

$$t=\frac{1}{f}\left(\frac{1+\mathrm{Ri}}{5/54}\right)^{\frac{1}{2}}. \tag{4.5}$$

The formation time decreases as the solar constant, amount of absorbing material, or rotation rate are increased.

In Fig. 10 we give the response of the magnitude of the vertical velocity w accompanying the eddies to changes in β, τ^* and f. This magnitude was taken to be the maximum value of Eq. (3.5) of Stone (1972a), with the appropriate scale factors inserted, and the amplitude given by Eq. (2.19) of I. The resulting formula is

$$w=\tfrac{1}{4}(\tfrac{3}{4})^{\frac{1}{2}}\frac{fH(1.09)^{\frac{1}{2}}}{1+\mathrm{Ri}}. \tag{4.6}$$

In order of magnitude this is equivalent to $w=(H/\lambda)\,\mathrm{Ro}U$. The vertical velocities are increased by increases in the solar constant, the amount of absorbing material, or the rotation rate.

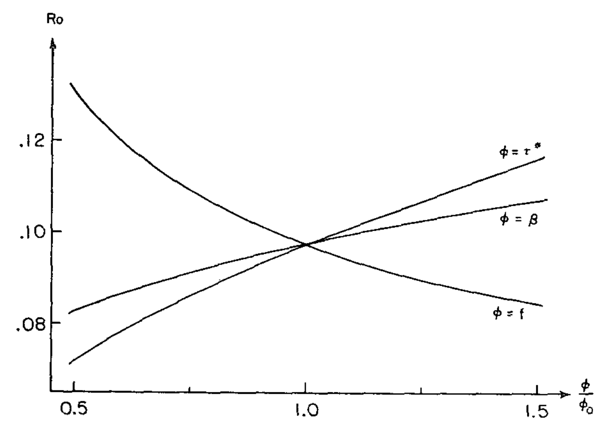

FIG. 8. Rossby number vs normalized values of τ^*, β and f.

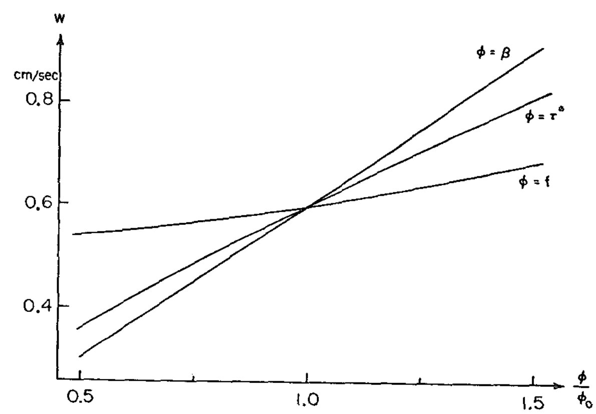

FIG. 10. Vertical velocity vs normalized values of β, τ^* and f.

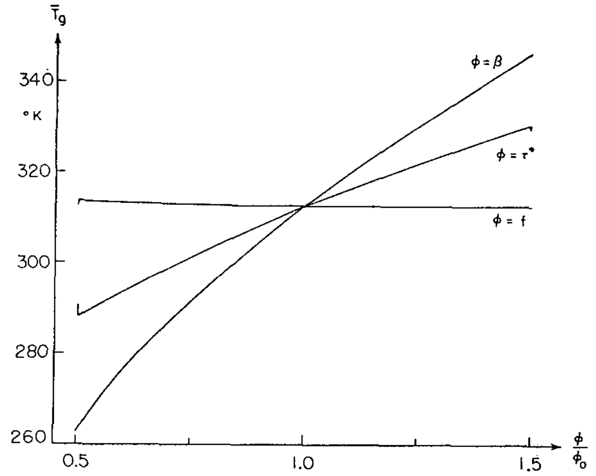

FIG. 11. Mean ground temperature vs normalized values of β, τ^* and f.

Finally, Fig. 11 shows the response of the average ground temperature $\bar{T}_g$ to changes in β, τ^* and f. This temperature is defined as

$$\bar{T}_g=\frac{1}{L}\int_0^L \bar{T}(y,0)dy, \tag{4.7}$$

and upon substituting (2.2) and (3.2) into (4.7), and making use of the properties of (2.3), (2.4) and (3.3), we find

$$\bar{T}_g=\bar{T}_r(T_e=\langle\bar{T}_e\rangle, z=0)-\frac{H}{2}\left[\left\langle\frac{\partial\bar{\theta}}{\partial z}\right\rangle-\left\langle\frac{\partial\bar{\theta}_r}{\partial z}\right\rangle\right]. \tag{4.8}$$

These ground temperatures are higher than those in the real atmosphere because in our model the motions only extend up to one scale height. In the real atmosphere the motions extend significantly higher, and the resulting mixing has a greater cooling effect on the ground temperatures. (However, quantities like Ri and $\langle\partial\bar{\theta}/\partial z\rangle$ are virtually independent of this scale in our model.) Fig. 11 shows that the average ground temperature is increased by increases in the solar constant and the amount of absorbing material and slightly decreased by increases in the rotation rate.

The ticks on the ends of the curves in Fig. 11 indicate how much the ends would be displaced if the static stability had been assumed to have the same value as in the standard model in all the calculations. These displacements are very small, which shows that the changes in the *average* ground temperature are almost completely determined by the changes in the radiative equilibrium solution. This would not be true, however, of ground temperatures at locations far from $y=\frac{1}{2}L$, since at these locations heating and cooling by the horizontal eddy flux is important, and this mechanism has a stronger response to changes in τ^*, β and f (cf. Figs. 2–4).

Since the curve in Fig. 11 for $\varphi=\beta$ is essentially unaffected by changes in static stability, we may compare it with the same curve calculated by Manabe and Wetherald (1967). They assumed a constant static stability, but had a much more sophisticated treatment of radiation. Their curve (their Fig. 7, for fixed absolute humidity) is indistinguishable from ours, except that it is systematically cooler because of the greater extent of the vertical mixing in their model. For example, both curves give a difference of 38K in $\bar{T}_g$ between the states with $F=1.5$ and 2.5 cal cm^{-2} min^{-1}. This comparison indicates that our simple treatment of radiation is more than adequate for our purposes.

5. Summary and conclusions

The results of Section 4 show how many of the important dynamical parameters will change when only the transport of sensible heat by the large-scale eddies is taken into account. These changes are likely to be a good guide to the qualitative changes in more sophisticated models since most of the physical processes neglected in our model also have negative feedback. For this same reason the quantitative changes calculated in Section 4 will in many cases represent upper bounds to true climatic changes. The qualitative changes are as follows:

1) An increase in the solar constant (or equivalently a decrease in the albedo) will lead to increased static stability but decreased dynamic stability, stronger latitudinal temperature gradients and horizontal winds, larger eddies that form more rapidly and are less geostrophic, and stronger vertical motions.

2) An increase in the amount of absorbing material in the atmosphere will lead to decreased static and dynamic stability, stronger latitudinal temperature gradients and horizontal winds, smaller eddies that form more rapidly and are less geostrophic, and stronger vertical motions.

3) An increase in rotation rate will lead to greater static and dynamic stability, stronger latitudinal temperature gradients but weaker horizontal winds, smaller eddies that form more rapidly and are more geostrophic, and stronger vertical motions.

Perhaps the most important conclusions that can be drawn from our simple model concern the assumptions that can be made in modeling the large-scale eddy fluxes in more complicated atmospheric models. Referring to Figs. 1 and 11, we see that the negative feedback in these eddy fluxes is so strong that changes in the external parameters of 25% only lead to changes in the static stability of the order of a few tenths of a degree per kilometer, and that these changes have a negligible influence on ground temperatures. This result gives considerable justification for the common assumption that the static stability remains constant during climatic changes. The negative feedback provided by the large-scale eddies will make it difficult to achieve large changes in this parameter. On the other hand, referring to Figs. 2–4, we see that the changes predicted by a model with a constant eddy coefficient law for the horizontal eddy flux may be very bad. Thus, one should treat with caution predictions based on models which assume a constant K. For example, Sellers' (1969) estimate of how large a decrease in solar constant is necessary to initiate an ice age is low because of his assumption of a constant K.

Acknowledgments. This work was supported in part by the Atmospheric Sciences Program of the National Science Foundation under Grant GA 28371 to Harvard University.

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