Light Illuminance and Color in the Earth's Shadow

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Abstract. The illuminance in the earth's shadow at three wavelengths is computed by performing a double numerical integration over the atmosphere. As sources of light attenuation, ozone absorption and extinction by dust and clouds are taken into account in addition to the usual Rayleigh scattering and the beam divergence due to refraction. Light inserted into the shadow through scattering by molecules and aerosol particles is also included in the calculations. The solar limb darkening is accounted for. Special emphasis is placed on finding an explanation for the unusual eclipse of December 30, 1963, in which an extremely low illuminance and nonreddening were observed. Calculations are carried out for various models of the earth's atmosphere, and it is shown that the dust particles may account for large changes in the illuminance from eclipse to eclipse. The dust and clouds appear to cause the nonreddening by enhancing the importance of the diffused light in the central region of the earth's shadow. The observation of the 1963 eclipse is explained by assuming an abnormally large extinction, presumably caused by dust spread by the eruption of Mount Agung on March 17, 1963. The required additional extinction amounts to about 14% of the incident light per air mass in the visible region, which agrees with direct measurements of the photometric extinction made at various observatories during the year 1963. The theory also provides a conclusive means of distinguishing between extinction due to dust particles and that due to clouds. An approximate value for the amount of ozone in the atmosphere can be determined. The results suggest useful photometric observations of future eclipses.

INTRODUCTION

An unusually dark eclipse of the moon was observed on December 30, 1963. Three-color photometric observations of *Matsushima and Zink* [1964] showed that at mid-totality the moon was darkened by the factor 2.5×10^{-7} in all three colors, indicating an absence of the usually conspicuous reddening in the umbral region. The cause of this low illuminance, which was only 1/40 times that of a normal eclipse, has been attributed to extinction due to dust particles spread through the upper atmosphere by the volcanic explosions of Mount Agung on the island of Bali in March 1963. We have investigated the effect of dust extinction by calculating the illuminance of the umbral light for three colors using various models of the earth's atmosphere. These computations were made first with the IBM 7040 and later with the 7044 at the Computer Center of the University of Iowa. The illuminance was calculated for blue ($\lambda = 0.46\ \mu$), green ($\lambda = 0.54\ \mu$), and red ($\lambda = 0.62\ \mu$) regions.

The principal causes of changes in the light intensity as it passes through the atmosphere are refraction, which simply spreads the light over a wider area, and extinction, which actually removes a fraction of the light from the incident beam. In extinction are included both the scattering and absorption of light by various particles in the atmosphere. *Link* [1963] has derived equations giving the intensity of light in the umbra for light emitted from a point source, accounting for refraction and Rayleigh scattering, as a function of the minimum altitude of the light ray in the earth's atmosphere. The calculations here are based on Link's method with modifications made to take into account ozone absorption and scattering by dust and water particles. The brightness distribution over the sun's disk is expressed in terms of the limb-darkening coefficients, and the desired illuminance is then obtained by performing a double numerical integration over the sun's surface. The work of *Svestka* [1948] is used to take account of the light inserted into the shadow by molecular scattering, and calculations are also

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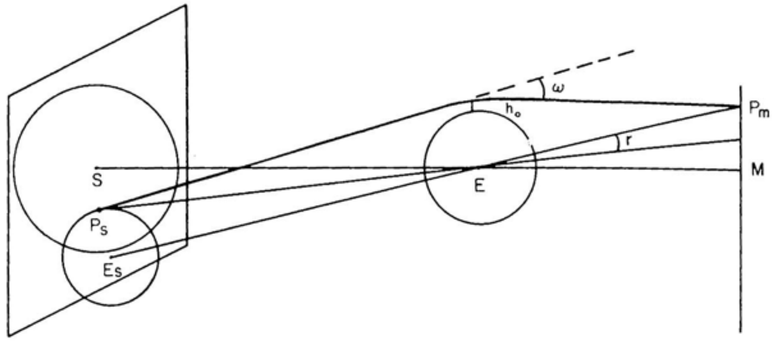

Fig. 1. Geometric relations between various integration variables.

made to include the same process for dust particles.

INTEGRATION OF THE LIGHT ILLUMINANCE

To find the illuminance at a single point in the earth's shadow it is necessary to integrate over the surface of the sun. As will be shown, this process is equivalent to an integration over the atmosphere of the earth. In Figure 1, P_m is an arbitrary point on the plane perpendicular to the shadow axis SEM and containing the center of the moon. If the earth's atmosphere is assumed to be spherically symmetric, light which is emitted from any point on the circle containing the point P_s, and which passes through the earth's atmosphere with minimum altitude h_0, must strike the point P_m on the plane of the moon. ω is the total angle of refraction for the light ray with minimum altitude h_0. If the illuminance at the arbitrary point P_m is denoted by e_λ, an element dS of the sun's surface at P_s produces an illuminance at P_m given by

$$de_\lambda = k b_\lambda T_\lambda \, dS \tag{1}$$

where b_λ is the light intensity of the sun at the point P_s, T_λ is the transmission coefficient for the ray which begins at P_s and ends at P_m, including all changes in intensity such as that due to refraction, ozone absorption, dust extinction, and Rayleigh scattering, and k is a function of the relative positions of the three bodies being considered. The total illuminance at P_m is then calculated by the following integration over the surface of the sun, as illustrated in Figure 2.

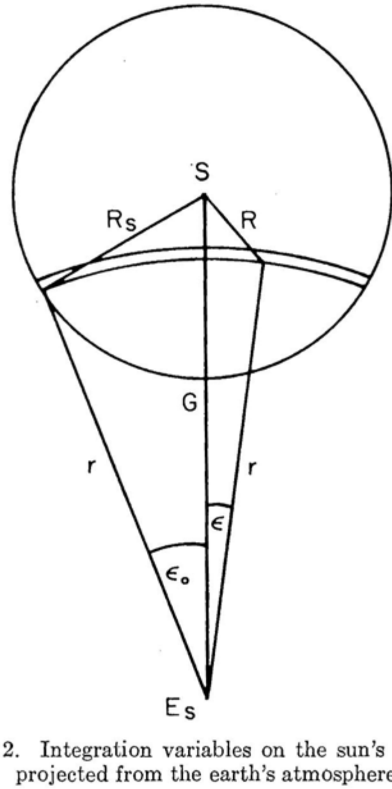

Fig. 2. Integration variables on the sun's disk as projected from the earth's atmosphere.

$$e_\lambda(G) = 2k \int_{r=G-R_s}^{G+R_s} \int_{\epsilon=0}^{\epsilon_0} b_\lambda T_\lambda(r) r \, dr \, d\epsilon \tag{2}$$

G is the angular distance of the point E_s from the center of the sun and is also equal to the angular distance of P_m from the center of the umbra, M (Figure 1). R_s is the angular radius of the sun. The factor 2 is due to the integration parameter ϵ as defined in Figure 2.

When the earth is not eclipsing the sun,

$T_\lambda(r) = 1$ and the illuminance is

$$E_\lambda = 2k \int_{G-R_s}^{G+R_s} \int_0^{\epsilon_0} b_\lambda r \, dr \, d\epsilon \qquad (3)$$

For convenience in the following calculations we consider the shadow illuminance relative to that of the full moon, D_λ, such that

$$D_\lambda = e_\lambda / E_\lambda \qquad (4)$$

The relative light intensity on the sun may be expressed in terms of the integration variables by using the definition of the limb-darkening coefficients

$$b_\lambda = I_\lambda(\theta)/I_\lambda(0)$$
$$= A_\lambda + B_\lambda \sin\theta + C_\lambda \sin^2\theta \qquad (5)$$

and the law of the cosines

$$G^2 + r^2 - 2rG\cos\theta = \sin^2\theta \qquad (6)$$

where θ is the angle between vectors drawn from the center of the sun to the observer and from the center of the sun to the position on the sun at which $I_\lambda(\theta)$ is evaluated.

The principal advantage of the above method of integration lies in the form of the transmission factor T_λ for the ray traveling from P_s to P_m. Under the assumption of a spherically symmetric atmosphere T_λ is a function of the minimum altitude h_0 and there is a one-to-one correspondence between h_0 and the angle r [*Link*, 1963]. Hence

$$T_\lambda = T_\lambda(r) = T_\lambda(h_0) \qquad (7)$$

For this reason it is equally valid to say that the integration is over the surface of the sun or that it is over the earth's atmosphere. It is convenient to write

$$T_\lambda(r) = T_\lambda^R(r) T_\lambda^r(r) T_\lambda^O(r) T_\lambda^A(r) T_\lambda^W(r) \qquad (8)$$

where the superscripted T's represent the contributions to the transmission factor from each of the five dominant types of extinction—Rayleigh, refraction, ozone, dust, and water particle (cloud), respectively.

Refraction and Air Mass for Light Passing Parallel to the Earth's Surface

To compute the change in intensity of light due to refraction and the decrease in intensity due to extinction of all kinds, we computed three quantities numerically as a function of the minimum altitude h_0 of a light ray passing through the earth's atmosphere. These are the refraction of the ray, ω, the gradient of ω, $d\omega/dh_0'$ (h_0' being the minimum altitude the light ray would have if it were not refracted), and the air mass passed through by the light ray, M. (The air mass in the vertical direction above $h = 0$ is defined as unity.) The formulas for these quantities are given by *Link* [1963] and were numerically integrated by taking the atmospheric density for each ½ km as tabulated in the *United States Standard Atmosphere* [*Champion et al.*, 1962].

Sources of Attenuation of Light

Rayleigh scattering. The Rayleigh extinction is given by the factor

$$T_\lambda^R = e^{-A_\lambda M} \qquad (9)$$

where A_λ is the Rayleigh extinction coefficient for a unit air mass. M is calculated above. A_λ is the product of the reduced equivalent thickness of the atmosphere (7.995 km) and the usual Rayleigh scattering coefficient, σ_λ, which is tabulated by *Penndorf* [1957].

Refraction. The expression for the refraction factor is obtained from the geometry. *Link* [1963] derives the form

$$T_\lambda^r = \left[1 - \frac{\omega}{\pi_m + \pi_s}\left(1 - \frac{h_0'}{R_0}\right)\right] \cdot \left[1 - R_0 \frac{d\omega}{dh_0'} \frac{1}{\pi_m + \pi_s}\right] \qquad (10)$$

where R_0 is the radius of the earth and π_m and π_s are the geocentric parallaxes of the sun and the moon. Thus T_λ^r for a given minimum altitude h_0 may be computed from the above equation, using the values of ω and $d\omega/dh_0'$ tabulated in the second and third columns of Table 1.

Ozone absorption. The ozone absorption, T_λ^O, can be calculated if the vertical distribution of ozone, $o(h)$, is known. The function $o(h)$ is defined as the equivalent path length of pure ozone at 0°C and 760 mm which is contained in 1 km of air at the altitude h. Then the total amount of ozone passed through by a ray with minimum altitude h_0, which is designated by

TABLE 1. Refraction, Refraction Gradient, and and Tangential Air Mass and Ozone Quantity as a Function of the Minimum Altitude of the Light Ray

h_0	ω	$\frac{R_0 d\omega}{dh_0'}$	M	$O(h_0)$ (0.26 cm STP)	$O(h_0)$ (0.36 cm STP)
1	63.4	45,230	73.7	6.23	8.62
2	57.8	41,230	65.5	6.40	8.85
3	52.6	37,600	58.2	6.57	9.10
4	47.7	34,260	51.5	6.76	9.36
5	43.2	31,220	45.5	6.96	9.64
6	39.1	28,380	40.1	7.18	9.94
7	35.3	25,730	35.2	7.41	10.25
8	31.8	23,190	30.8	7.64	10.58
9	28.7	20,520	26.9	7.89	10.93
10	26.2	14,700	23.3	8.15	11.28
11	23.8	18,600	20.2	8.41	11.64
12	20.8	23,210	17.2	8.66	11.99
13	17.7	19,390	14.6	8.91	12.34
14	15.0	16,290	12.4	9.14	12.66
15	12.8	13,660	10.6	9.34	12.94
16	10.9	11,540	9.03	9.50	13.16
17	9.31	9,738	7.69	9.60	13.29
18	7.94	8,215	6.56	9.62	13.33
19	6.79	6,848	5.60	9.56	13.24
20	5.81	5,944	4.77	9.40	13.01
21	4.95	5,266	4.07	9.12	12.63
22	4.20	4,429	3.47	8.74	12.10
23	3.57	3,734	2.96	8.26	11.43
24	3.04	3,151	2.53	7.68	10.64
25	2.59	2,661	2.16	7.04	9.75
26	2.20	2,254	1.85	6.36	8.81
27	1.88	1,906	1.58	5.67	7.85
28	1.61	1,618	1.36	4.98	6.90
29	1.37	1,371	1.16	4.32	5.98
30	1.17	1,160	0.998	3.71	5.13
35	0.536	548	0.466	1.52	2.11
40	0.245	237	0.226	0.56	0.77
45	0.117	108	0.114	0.19	0.27
50	0.059	47	0.061	0.07	0.09
55	0.032	23	0.033	0.02	0.03
60	0.018	14	0.018	0.01	0.01
65	0.010	7	0.009	0.00	0.00
70	0.005	4	0.005	0.00	0.00
75	0.003	2	0.002	0.00	0.00
100	0.000	0	0.000	0.00	0.00

$O(h_0)$, is given by

$$O(h_0) = 2 \int_{h_0}^{\infty} o(h) \sec z \, dh \qquad (11)$$

where z is the angle between the light ray's path at the altitude h and a radial element from the earth's center intersecting the path at that point. Since the refraction of the light ray in the high atmospheric region of the ozone layer is very small, the integration can be simplified by taking a straight line for the ray's path. Then

$$\sec z = [R_0/2(h - h_0)]^{1/2} \qquad (12)$$

For the vertical distribution of ozone, $o(h)$, the empirical formula of *Green* [1964] was used:

$$o(h) = \frac{P_1}{P_2} \frac{e^{P(h)}}{[1 + e^{P(h)}]^2} \qquad (13)$$

where

$$P(h) = (h - P_3)/P_2 \qquad (14)$$

and P_1, P_2, and P_3 are parameters adjusted to fit the desired distribution. The suggested standard distribution of *Green* [1964] was used.

The calculations were carried out with the following two values for the vertical ozone content:

$$\int_0^{\infty} o(h) \, dh = 0.26 \text{ cm STP}$$

and

$$= 0.36 \text{ cm STP}$$

The results for $O(h_0)$ are tabulated in the last two columns of Table 1. T_λ^O follows from the tabulated values of $O(h_0)$, since it may be expressed as

$$T_\lambda^O = e^{-k_\lambda O(h_0)} \qquad (15)$$

where k_λ is the ozone absorption coefficient. The experimental values of k_λ at normal pressure and at a temperature of 18°C have been found by *Inn and Tanaka* [1953]. Although k_λ is a function of temperature, no corrections are necessary because, as *Vigroux* [1953] has shown, the variation is only 1% between −92°C and 50°C for the wavelengths considered here.

Dust extinction. The extinction cross section of a single spherical particle of radius a may be written as

$$C_{ext}(a) = \pi a^2 Q_{ext}(a) \qquad (16)$$

in which the efficiency factor $Q_{ext}(a)$ is defined. Q_{ext} for spherical particles were computed by *van de Hulst* [1957] in terms of the parameter $x = 2\pi a/\lambda$ and the refractive index of the particles. If the refractive index is the same for all the particles, Q_{ext} is a function of x only, and the color dependency in the aerosol extinction enters through this parameter. For particles with $x < 0.1$, Q is negligible, and for $x > 10$, Q is nearly constant at 2.0. For intermediate values Q increases with x but not monotonically. The possible direct color effect, however, largely cancels in an integration over a continuous range of particle sizes, and the assumption of neutral extinction is made here. The assumption that volcanic dust is a neutral scattering agent was borne out by extensive measurements of extinction coefficients and photoelectric scanning observations made by *Moreno et al.* [1965] for about 18 months after the eruption of Mount Agung. Under the assumption of neutral extinction

$$T_\lambda{}^A(h_0) = 10^{-\beta L(h_0)} \tag{17}$$

where β is the vertical extinction coefficient due to dust particles and $L(h_0)$ is the ratio of the extinction coefficient for a horizontal path with minimum height h_0 to the extinction coefficient in the zenith direction from $h = 0$. Let $n(a, h)$ be the number of particles per unit volume and unit radius interval. If the size distribution of particles is independent of the altitude distribution, i.e., $n(a, h) = n_1(a)\ n_2(h)$, then

$$L(h_0) = \frac{2\int_{h_0}^{\infty}\int_0^{\infty} n(a, h)\pi a^2 Q(a) \sec z\ da\ dh}{\int_{h_0}^{\infty}\int_0^{\infty} n(a, h)\pi a^2 Q(a)\ da\ dh}$$

$$= \frac{2\int_{h_0}^{\infty} n_2(h) \sec z\ dh}{\int_0^{\infty} n_2(h)\ dh} \tag{18}$$

β can be calculated if the number of particles above unit area and the size distribution are known. In the present paper, the total β due to both dust and water particles can be determined by fitting the theoretical illuminance curves to the observed curves. The value of β determined directly from standard photometric observations provides an independent means of checking the adopted model for the dust layer.

Cloud extinction. In addition to the dust and haze, a varying content of water particles (clouds) generally contributes to the atmospheric extinction. Since the central region of the earth's shadow is illuminated at any point by a wide area of the terminator region around the earth, bad weather conditions at various local areas may cause a considerable degree of extinction. The extinction due to clouds may be accounted for in an approximate manner if it is assumed to be neutral. This is a reasonable assumption because most of the extinction in clouds is due to particles with $a \geq 1\ \mu$ which is equivalent to $x \geq 10$. The water particles are then accounted for in the same manner as the dust.

Illuminance by scattered light. Scattering of light by the molecules and dust particles in the atmosphere reinserts light quanta into the region of interest. In some cases, such as the dark eclipse of December 1963, this contribution to the illuminance is significant because the refracted light may be attenuated so that its illuminance is of the same order as that of the scattered light. *Svestka* [1948] calculated this additional illuminance in the shadow due to scattering by molecules. His results for a lunar parallax of 61′ were 2.5×10^{-7} and 2.1×10^{-7} times the illumination of the full moon for the wavelengths 4500 A and 7000 A, respectively. Because the difference for these extreme wavelengths is small, and because it is obvious from Svestka's equations that the change is continuous, it is permissible to interpolate for the intermediate wavelengths.

Because the 1963 eclipse was unusually dark, presumably owing to dust in the atmosphere, it is important to consider the similar contribution from aerosol particles. With only a slight modification to an equation of Svestka, we can find the illuminance due to dust scattering:

$$\frac{3\pi_m{}^2}{4R_0}\int_{H_1}^{H_2} 10^{-\beta L(h_0)}\beta L(h_0)\ dh_0$$

where H_1 and H_2 are the lower and upper limits of the dust layer. The exact result depends upon the model used for the dust layer, but a typical result is found to be 1.3×10^{-7} and in all models less than 1.6×10^{-7} times the illuminance of the full moon. Thus the additional illuminance by dust is comparable to that by molecules.

Numerical Results and Discussion

Rayleigh and ozone atmospheres. To investigate the contribution of Rayleigh scattering as compared with the other sources of extinction, we first computed the illuminance for a simple Rayleigh atmosphere by setting $T_\lambda^O = T_\lambda^A = T_\lambda^W = 1.0$ in (8). The result is shown in Figure 3. The vertical ozone content is variable in both season and latitude, ranging between about 0.21 cm STP and 0.38 cm STP [*Albright*, 1939]. For the December 1963 eclipse observations of *Matsushima and Zink* [1964] the effective part of the earth's terminator was located in the South Pacific Ocean, where both the season and latitude would supposedly cause a relatively high ozone content. Therefore, calculations were made for both the normal content of 0.26 cm STP and a high content of 0.36 cm STP. The results are plotted in Figure 4. The effect of increasing the ozone content is appreciable in the red and green regions but insignificant in the blue region. It appears, therefore, that accurate measurements in the red color near the outer region of the umbra should provide a means of determining the amount of ozone in the atmosphere. The two models in Figure 4 give the illuminance ratio of green (or visible) to blue light at a point 10 minutes from the umbra center to be 29 and 35 for the high and low ozone content, respectively. Since Matsushima and Zink determined the $B - V$ color (see Table 2) near mid-totality to be slightly negative (that is, the color was slightly bluish), it is apparent that the aerosol extinction must be taken into account for that eclipse.

Aerosol and cloud extinction. The vertical distribution of aerosols shows considerable variation depending on the place and time of observation. However, a series of recent high-altitude balloon flights indicates that the stratospheric aerosols extend up to 30 km, with a maximum at an altitude between 15 and 20 km [*Rosen,* 1964, and private communication, 1965]. Furthermore, from observations of the unusual reddening of the sky at sunset, *Meinel and Meinel* [1964] and others estimated that the dust of assumed volcanic origin extended to an altitude of approximately 20 km. Below the maximum density the aerosol content decreases, reaching a minimum between 5 and 10 km [*Rosen,* 1964]; this sink is apparently due to rain washout. However, the effect of the sink may have been largely canceled by the partial cloud cover existing in the effective part of the terminator at the time of the 1963 eclipse [*Brooks,* 1964]. The cloudy regions are confirmed by Tiros photographs on that date. Since the clouds were located at the approximate altitude of the aerosol sink, and since the vertical distributions of both aerosols and water particles are very imperfectly known, it was assumed that the combined particle density was uniform up to a maximum altitude.

TABLE 2. Comparison of Shadow Illuminances at a Point 10 Minutes from the Shadow Center for Different Values of the Vertical Extinction Coefficient

β	$-\log D_{4600}(=B)$	$-\log D_{5400}(=V)$	$B - V$
0.00	15.197	11.353	3.844
0.03	16.098	13.945	2.153
0.04	16.199	14.896	1.303
0.08	16.255	16.244	0.011
0.09	16.256	16.292	−0.036

The visual extinction in the vertical direction due to the normal content of aerosol particles is about $\beta = 0.028$ [*Allen,* 1963]. This value yields the total visual extinction (including Rayleigh scattering and ozone absorption) of 0.084 [*Allen,* 1963] which corresponds to the average clear sky condition. Figure 5 shows the shadow illuminance computed for a model of aerosols extending to 25 km with $\beta = 0.028$ and with molecular and aerosol scattering into the shadow included. As compared with Figure 4, the red and green curves are lowered by at least a factor of 10 near the central region, and these decreases are much greater throughout the umbra than those due to changing the ozone content. It appears, therefore, that accurate measurements of the shadow density provide a means of determining the aerosol content of the atmosphere. The ozone and aerosol extinctions may be distinguished because the aerosols have little or no color effect. In Figure 5 it is seen that the reddening is still distinctive through the umbra, indicating that the assumed aerosol extinction is still too small to

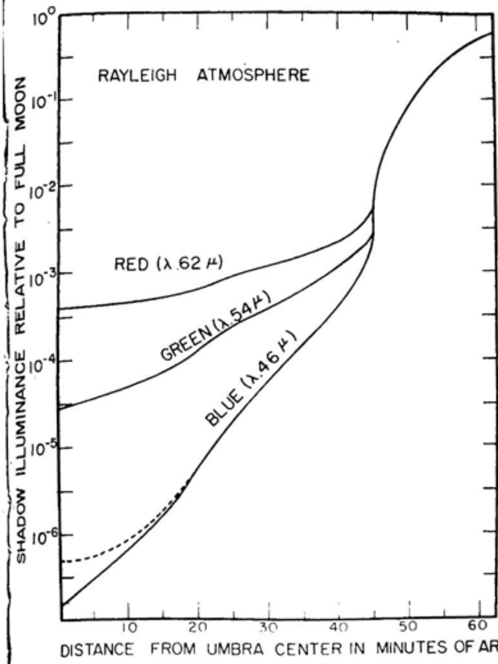

Fig. 3. Shadow illuminance in three colors for a pure Rayleigh atmosphere. The contribution of molecular scattering into the shadow is appreciable only in the blue color, as indicated by the broken line.

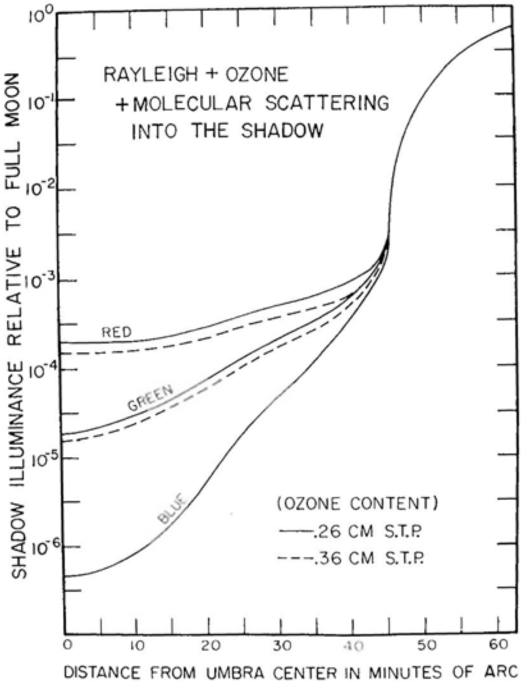

Fig. 4. Shadow illuminance with the atmosphere including Rayleigh scattering and ozone absorption.

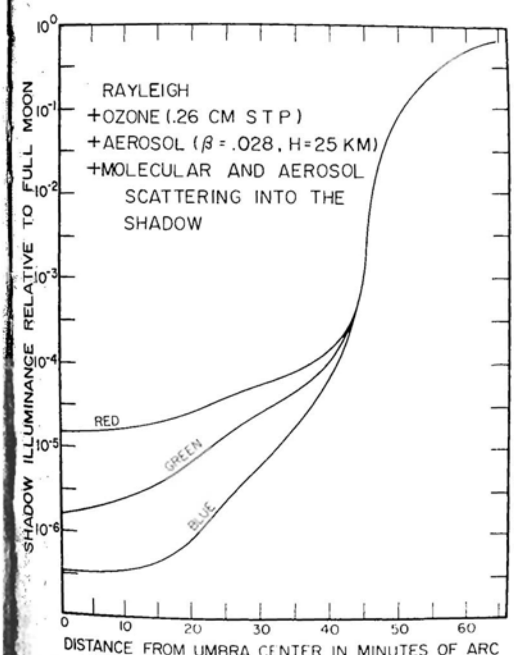

Fig. 5. Shadow illuminance with the atmosphere including a normal content of aerosol particles.

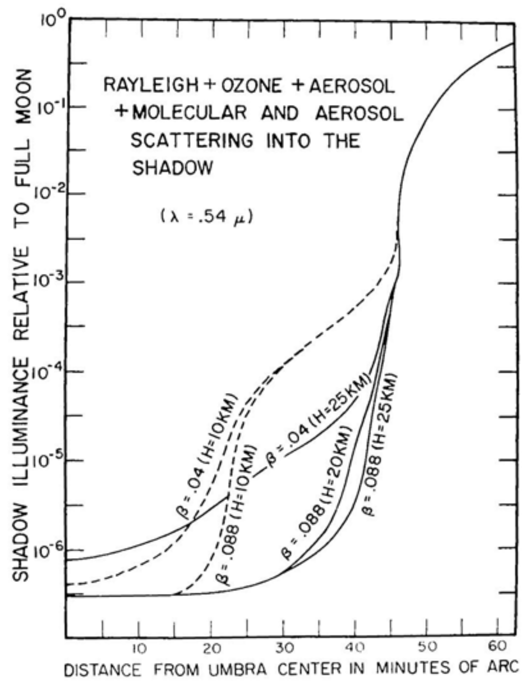

Fig. 6. Comparison of the illuminance for atmospheric models with different vertical contents of aerosol particles.

explain the observed colors of the December 1963 eclipse.

For the purpose of comparing the effects of a dust layer extending up to 25 km with those of a cloud layer which has a maximum altitude of 10 km, we computed the shadow illuminance in both of these cases with the same β. The results for green light for the above two cases with $\beta = 0.04$ are compared in Figure 6. In the same figure the models with $\beta = 0.088$ are compared. The difference between the models differing only in the altitude of the particles is extremely large, reaching about factors of 1.0×10^{-1} and 5.0×10^{-3} for the respective value of β at 30 minutes from the umbra center.

Finally, it was attempted to find a model that would agree exactly with the measurements of Matsushima and Zink for the 1963 eclipse, which consisted of a decrease of illuminance by a factor 3.17×10^{-7} in the visual region, with no reddening or an even slightly bluish color near 10 minutes from the umbra center. An average ozone content of 0.26 cm STP was taken for all models. The values obtained for the shadow illuminance at a point 10 minutes from the umbra center are listed in Table 2. The small change in the blue illuminance as β is changed is due to the blue light in this region of the shadow being mainly scattered light rather than refracted light. From comparison with the results of Matsushima and Zink, the value $\beta = 0.084$ is found to give the best agreement with observation. The computed illuminances in three colors are plotted in Figure 7.

The derived value of $\beta = 0.084$ is higher than normal by 0.056. This is in good agreement with the increase in the visual extinction as determined by *Moreno et al.* [1964, 1965] and *Przybylski* [1965]. In Figure 4 the effect of changing the ozone content is small in comparison with the other sources of attenuation of light. Consequently, the determination of the ozone content from the eclipse observation requires accurate photometry near the limb of the umbra. In Figure 6 the effects of aerosol extinction and weather conditions are easily distinguishable in the outer half of the umbra. The same figure also indicates that data for this region would serve as a means of determining the upper limit of the aerosol layer. Unfortunately, the only quantitative measurement

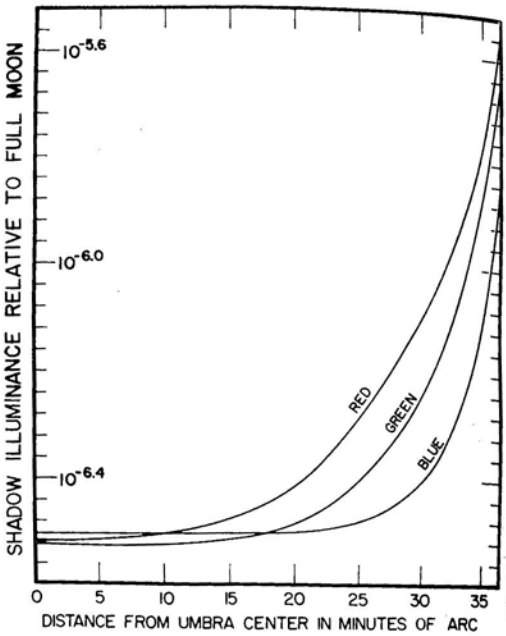

Fig. 7. Shadow illuminance in the central region of the umbra for the atmospheric model adopted for the December 1963 eclipse. Atmospheric particles extend uniformly to 20 km and the value of β is 0.084.

of the December 1963 eclipse was by Matsushima and Zink, and it was interrupted during the time the observed point was passing through this region.

The above results indicate that the primary cause of the unusually dark and nonreddened eclipse on December 30, 1963, was the additional extinction due to the volcanic dust ejected into the atmosphere by the eruption of Mount Agung. Since the eclipse was unusually dark, the scattered light was important in the central region of the shadow; in fact, it dominated the results in the blue and green wavelengths. It appears that in future eclipse observations accurate measurements near the limb of the umbra will be extremely valuable in distinguishing the effects of different sources of light attenuation. Above all, photometric measurements in the red color near the umbra limb should be most useful in determining the structure of the atmosphere.

The most serious approximations in the above theory may be eliminated at the expense of greater computer time. With the present type of observational evidence these refinements a not warranted, but frequent observations from artificial satellites would make improved calculations worth while. The assumption of a spherically symmetric atmosphere could be eliminated by an additional integration over all latitudes. The use of a single effective wavelength for each color when actually a broad bandwidth filter is used in the observations can be eliminated by making the calculations for a number of wavelengths and integrating over the filter sensitivity function.

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