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A.C. CONDUCTIVITY OF A PLASMA

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Abstract—This paper computes the electrical conductivity of a fully ionized, spatially homogeneous plasma under the influence of a uniform, periodically alternating electric field. The velocity distribution of the electrons is determined by solving the linearized Fokker–Planck equations. All the terms in the collision integral are retained, including those representing electron–electron interactions. The resultant values of conductivity is expected to be valid in the range of frequencies from zero to below the plasma frequency.

1. INTRODUCTION

The purpose of this paper is to calculate the a.c. conductivity of a spatially homogeneous plasma using the Fokker–Planck equation. The d.c. conductivity of a plasma has been calculated in the well-known works of COHEN *et al.* (1950) and SPITZER Jr. and HÄRM (1953). Their results are in good agreement with the later experimental works of LIN *et al.* (1955). BERNSTEIN and TREHAN compute the a.c. conductivity assuming a Lorentz gas model (1960). The a.c. conductivity of a real gas should approach that of a Lorentz gas at high frequencies (see detailed discussions in Section 4). Toward lower frequencies their departure is expected to increase so that their ratio becomes nearly 2 in the d.c. limit, in accordance with COHEN *et al.* and SPITZER and HÄRM. The recent works on a.c. conductivity by DAWSON and OBERMAN (1962) consider the time variation of the two-particle distribution, which is necessary when dealing with a.c. currents of ultra-high frequencies. However, the domain of applicability of their work is limited to frequencies much higher than the collision frequency. Thus, a more precise calculation for the low and intermediate range of ω appears desirable and we proceed to do this in accordance with methods to be described in the next section. After completion of most of the numerical work, a paper by ROBINSON and BERNSTEIN (1962) came to our attention. They computed the a.c. conductivity using a variational technique. Our results obtained by direct integration of the Fokker–Planck equation will be compared to theirs in Section 4.

We begin with the Boltzman equation:

$$\frac{\partial f_i}{\partial t} + \mathbf{v} \cdot \frac{\partial f_i}{\partial \mathbf{r}} = \frac{F}{m} \cdot \frac{\partial f_i}{\partial \mathbf{v}} + \left(\frac{\delta f_i}{\delta t}\right)_c \tag{1}$$

where f_i is the distribution function of particles of type i, $\left(\frac{\delta f_i}{\delta t}\right)_c$ is the change of f_i produced by collisions.

Equation (1) is deduced from Liouville theorem to describe a many-particle system under two assumptions:

(i) That the characteristic dimensions of the inhomogeneities are much larger than the average impact parameter for the particles participating in the collision.

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(ii) That the characteristic time variation of the process is much longer than the duration of an average collision, or in other words, a collision is completed and the correlation function is 'relaxed' before the distribution function itself makes any appreciable change.

It should be noted here that the term 'duration of collision' is different from the so-called 'collision time'; collision time is the time between two collisions. For particles interacting through long-range forces, this time may be regarded as the time in which deflexions gradually deflected the considered particle by 90°. Duration of collision is the time during which an interaction takes place. In a plasma it is of the

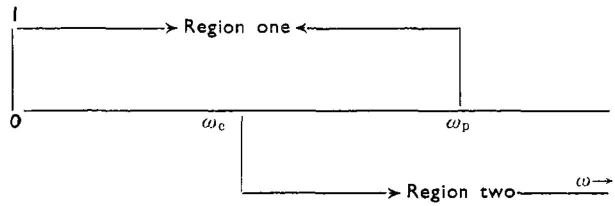

FIG. 1.—The ranges of validity computed a.c. conductivities.
The values of a.c. conductivity obtained in this paper is valid in region one. When ω exceeds ω_c—region two—the values calculated by DAWSON and OBERMAN begin to be valid.
Here ω_c is the collision frequency, ω_p is the plasma frequency.

order ω_p^{-1}. In Fig. 1 a time-scale diagram is drawn, and the validity of our calculation and those of DAWSON and OBERMAN are indicated.

The explicit expression of $\left(\frac{\delta f_i}{\delta t}\right)_c$ depends on the nature of the interaction force. In a fully ionized plasma, the particles interact through the long-range Coulomb forces. The cumulative effect of 'weak' deflexions resulting from the relatively distant collisions outweighs the effect of occasional large deflexions due to relatively close collisions, so one may neglect the contribution by those very close encounters (COHEN *et al.*)—encounters which result in deflexions of 90° or larger.

Also, the effect of distant particles lying outside the Debye length λ_D may be neglected because of the shielding of inner particles. Thus, in the computations of $\left(\frac{\delta f_i}{\delta t}\right)_c$, it is only necessary to consider the collisions with impact distance intermediate between λ_D and b_0, where $b_0 = \frac{e^2}{KT}$ is the impact parameter yielding a 90° deflexion. The effects of these collisions are cumulative, and the total deflexion produced in an interval of time is similar to that of the Brownian motion; hence, one may expand $\left(\frac{\delta f_i}{\delta t}\right)_c$ in powers of $\langle\Delta \mathbf{v}\rangle$, where $\langle\Delta \mathbf{v}\rangle$ is the average velocity change due to collisions (COHEN *et al.*, CHANDRASEKHAR, 1943). This procedure leads to the following Fokker–Planck collision integral (ROSENBLUTH *et al.*, 1957):

$$\left(\frac{\delta f_i}{\delta t}\right) = \Gamma_i\left\{-\frac{\partial}{\partial \mathbf{v}}\left(f_i\frac{\partial h_i}{\partial \mathbf{v}}\right) + \frac{1}{2}\frac{\partial^2}{\partial \mathbf{v}\,\partial \mathbf{v}}\left(f_i\frac{\partial^2 g}{\partial \mathbf{v}\,\partial \mathbf{v}}\right)\right\} \tag{2}$$

where

$$h_i = \sum_j \frac{m_i + m_j}{m_j} \int d\mathbf{v}' f_j(\mathbf{v}'_j) \, |\mathbf{v} - \mathbf{v}'_j|^{-1} \tag{3}$$

$$g = \sum_j \int d\mathbf{v}' f_j(\mathbf{v}') \, |\mathbf{v} - \mathbf{v}'| \tag{4}$$

and

$$\Gamma_i = \frac{4\pi z^2 e^4}{m_i^2} \ln \frac{m_i m_j v_{\text{th}}^2 \lambda D}{2(m_i + m_j) e^2}. \tag{5}$$

The summation in h_i and g sums over all species, m_i is the mass of the i-th species, e is the electronic charge, $\lambda_D = \left(\frac{KT}{4\pi n e^2}\right)^{\frac{1}{2}}$ is the Debye length, and v_{th} is the relative thermal velocity.

In this paper we consider only plasma with singly-charged ions. The extension of the present method to those with multiply-charged ions is straightforward.

2. DERIVATION OF EQUATIONS AND FORMULAE

If the distribution function f has an azimuthal symmetry about a certain axis, then, following ROSENBLUTH *et al.*, the collision term may be written down explicitly in spherical polar co-ordinates in velocity space:

$$\begin{aligned}
\left(\frac{\delta f_i}{\delta t}\right)_c = \Gamma_i \Bigg\{ & -v^{-2} \frac{\partial}{\partial v}\left[f_i v^2 \frac{\partial h_i}{\partial v}\right] - v^{-2} \frac{\partial}{\partial \mu}\left[f_i (1 - \mu^2) \frac{\partial h_i}{\partial \mu}\right] \\
& + (2v^2)^{-1} \frac{\partial^2}{\partial v^2}\left[f_i v^2 \frac{\partial^2 g}{\partial v^2}\right] \\
& + (2v^2)^{-1} \frac{\partial^2}{\partial \mu^2}\left[f_i \left\{v^{-2}(1 - \mu^2)^2 \frac{\partial g^2}{\partial \mu^2}\right.\right. \\
& \left.\left. + v^{-1}(1 - \mu^2) \frac{\partial g}{\partial v} - v^{-2}\mu(1 - \mu^2) \frac{\partial g}{\partial \mu}\right\}\right] \\
& + v^{-2} \frac{\partial^2}{\partial \mu \, \partial v}\left[f_i (1 - \mu^2) \left\{\frac{\partial^2 g}{\partial \mu \, \partial v} - v^{-1} \frac{\partial g}{\partial \mu}\right\}\right] \\
& + (2v^2)^{-1} \frac{\partial}{\partial v}\left[f_i \left\{-v^{-1}(1 - \mu^2) \frac{\partial^2 g}{\partial \mu^2} - 2 \frac{\partial g}{\partial v} + 2\mu v^{-1} \frac{\partial g}{\partial \mu}\right\}\right] \\
& + (2v^2)^{-1} \frac{\partial}{\partial \mu}\left[f_i \left\{v^{-2}\mu(1 - \mu^2) \frac{\partial^2 g}{\partial \mu^2} + 2\mu v^{-1} \frac{\partial g}{\partial v}\right.\right. \\
& \left.\left. + 2v^{-1}(1 - \mu^2) \frac{\partial^2 g}{\partial \mu \, \partial v} - 2v^{-2} \frac{\partial g}{\partial \mu}\right\}\right] \Bigg\}
\end{aligned} \tag{6}$$

where $\mu = \cos\theta$ is the direction cosine between $\mathbf{v}$ and $\mathbf{E}$. Equation (6) is an exact expression of the Fokker–Planck equation in spherical co-ordinates for a distribution function with azimuthal symmetry.

We assume that the system is subject to a weak electric field $\mathbf{E}_0 e^{i\omega t}$ whose direction lies along z-axis. Then following CHAPMAN and COWLING (1939) and SPITZER Jr. we expand f_i in a power series of E:

$$f_i(\mathbf{v}, t) = f_i^{(0)}(v) + E_0 f_i^{(1)}(\mathbf{v}, t) + E_0^2 f_i^{(2)}(\mathbf{v}, t) + \ldots\ldots \tag{7}$$

where $f_i^{(0)}(v)$ is a time-independent Maxwellian distribution and $f^{(1)}(\mathbf{v}, t)$, $f^{(2)}(\mathbf{v}, t)$, ... are the perturbed part due to applied electric field. When a steady state has been reached and no transient current exists, the time-dependent part of $f_i^{(j)}(\mathbf{v}, t)$ must be proportional to $e^{i\omega t}$. Since the average energy imparted to the electrons between encounters is small compared with their kinetic energy, the velocity-dependent part of $f_i^{(j)}(\mathbf{v}, t)$ can be written as $e^{-[m_i v^2/2kt]} D_i^{(j)}(v)\mu$. Therefore, we have

$$f_i^{(j)}(\mathbf{v}, t) = \frac{m^{3/2}}{(2\pi KT)^{3/2}} e^{-m_i v^2/2KT} D_i^{(j)}(v)\mu e^{i\omega t}. \tag{8}$$

Combining equations (1), (6) and (7) keeping only terms linear in E_0, we obtain

$$\frac{\partial_{fi}^{(1)}}{\partial t} + \frac{e}{m_i} e^{i\omega t}\mathbf{E}_0 \frac{\partial}{\partial \mathbf{v}} f_i^{(0)} = \left(\frac{\delta f_i^{(1)}}{\delta t}\right)_c \tag{9}$$

where $(\delta f_i/\delta t)_c^{(1)}$ is the linearized Fokker–Planck collision integral.

Since the ions' contribution to electric current is negligible compared to electrons, we will consider only electron distributions and drop the subscript i in the distribution function hereafter.

Substituting equation (8) into equation (9) we find, after some algebraic manipulations, the following second-order linear integral-differential equation:

$$D''(x) + P(x)D'(x) + Q(x)D(x) = R(x) + S(x) \tag{10}$$

where

$$P(x) = -2x - \frac{1}{x} + \frac{2x^2\Phi'(x)}{H(x)} \tag{11}$$

$$Q(x) = \frac{-iBx^3 + 2(1 + \Phi = 2x^3\Phi')}{H(x)} + \frac{1}{x^2} \tag{12}$$

$$R(x) = -\frac{2\,dx^4}{H(x)} - \frac{8(2{\cdot}4x^6 - 2x^4)}{3\pi^{1/2}H(x)} I_0(\infty) \tag{13}$$

$$S(x) = \frac{16}{3\pi^{1/2}H(x)}\{xI_3(x) - 1{\cdot}2xI_5(x) - x^4I_0(x)(1 - 1{\cdot}2x^2)\} \tag{14}$$

$$\Phi(x) = \int_0^x e^{-y^2}\,dy \tag{15}$$

$$H(x) = \Phi(x) - x\Phi'(x) \tag{16}$$

$$I_n(x) = \int_0^x y^n D(y) e^{-y^2}\,dy \tag{17}$$

$$I_0(\infty) = \int_0^\infty D(y) e^{-y^2}\,dy \tag{18}$$

with

$$\alpha = -E_0KT/\pi e^3 n \ln\lambda \qquad \lambda = \lambda_D/b_0$$

$$x = v\Big/\left(\frac{2KT}{m}\right)^{1/2} \qquad B = \frac{4\sqrt{2}\lambda\omega}{\omega_p \ln\lambda} = \frac{\omega}{\omega_c},$$

where ω_c is approximately the 90° deflexion time of a particle with thermal velocity. When $\omega = 0$, equation (10)* reduces to equation (8) of SPITZER Jr. which considers d.c. electric conductivity.

In a d.c. electric field, the electrons are not accelerated in a steady state. Hence, the inertia force term is zero and $I_0(\infty) = \frac{3\pi^{\frac{1}{2}}}{8}\alpha$.

3. SOLUTION OF EQUATION

Equation (10) is a linear integral-differential equation whose unknown $D(x)$ is a complex function of a real variable. The present section will discuss the method of its solution. As will be evident in what follows, the procedure for numerical integration is far from straightforward.

On the one hand, we encounter the problem of the instability of the solution at small and at large x. Because of the existence of singularities in equation (10) at $x = 0$ and at $x = \infty$, a slight deviation of $D(x)$ at either small or large x, tends to be built up quite rapidly. In order to obtain a physically acceptable solution, it is required that $D(x)$ does not approach infinity too fast, leading to infinite conductivities. The starting value of D at small x can be obtained by means of a series solution. Because of the instability, we cannot proceed to integrate in a step-wise manner. To overcome this difficulty, we adopted a scheme used by COHEN *et al.* We shall refer to their paper for full details.

On the other hand, we note that $I_0(\infty)$ is no longer a known quantity as it is in the case of d.c. conductivity; it depends on the solution $D(x)$ itself. We proceed as follows: Since we want conductivities at different frequencies, it is necessary to obtain solutions for different values of the parameter B. We begin with a small value $B = 0{\cdot}05$. Using an $I_0(\infty)$ taken from the d.c. case, i.e. $I_0(\infty) = 0{\cdot}655$, we obtain a solution to equation (10) from which we get a new $I_0(\infty)$. Next, we pass on to $B = 0{\cdot}1$ using the $I_0(\infty)$ obtained for the previous B. In this way, we proceed to ever-increasing values of B, until the initial adopted $I_0(\infty)$ and the final calculated $I_0(\infty)$ differ by no more than 2 per cent. This occurs at $B = 1{\cdot}37$. From this point on, we resort to a method of systematic trials. The initial and final $I_0(\infty)$ for all values of B agree to within 2 per cent, which is considered sufficiently accurate for the present purposes.

4. RESULTS AND DISCUSSION

The current is given by

$$\mathbf{J} = -e\int d\mathbf{v}\mathbf{v}f_1(\mathbf{v}, t) = AE_0 e^{i\omega t} I_3(\infty) \tag{19}$$

where

$$I_3(\infty) = \int_0^\infty x^3 D(x) e^{-x^2}\, dx$$

$$A = \frac{2}{3}\frac{(2KT)^{3/2}}{\pi^{3/2}m^{1/2}e^2 \ln \lambda}. \tag{20}$$

* Note that $I_0(\infty)$ is essentially the total change of momentum of electrons arising from electron–ion interactions. Since the mutual electronic interaction cannot change the total momentum of the electrons, $I_0(\infty)$, by Newton's second law, must equal the total force exerted on the electrons by the applied field minus the inertia force of electrons. This relation gives us:

$$I_0(\infty) = \frac{3\pi^{\frac{1}{2}}}{8}\alpha - \frac{iB}{2}I_3(\infty).$$

Since $\mathscr{J} = \sigma\mathscr{E}$, we have the complex conductivity

$$\sigma = AI_3(\infty), \tag{21}$$

the impedance

$$Z = \frac{1}{AI_3(\infty)} = \frac{c}{I_3(\infty)}, \tag{22}$$

the resistance

$$R = \frac{1}{A}\frac{ReI_3(\infty)}{|I_3(\infty)|^2} = \frac{cReI_3(\infty)}{|I_3(\infty)|^2}, \tag{23}$$

and the reactance

$$X = \frac{1}{A}\frac{ImI_3(\infty)}{|I_3(\infty)|^2} = \frac{cImI_3(\infty)}{|I_3(\infty)|^2}, \tag{24}$$

with

$$c = \frac{1}{A} = \frac{3}{2}\frac{1}{(2KT)^{3/2}}\pi^{3/2}m^{1/2}e^2 \ln \lambda.$$

It may be remarked here that the a.c. conductivity depends on three factors:

(i) The inertia of the conducting electrons.

(ii) The mutual interaction among electrons and ions.

(iii) The mutual interaction among electrons themselves.

The mutual electronic interactions have no direct effect on conductivity since the total change of momentum due to such interactions is zero. Nevertheless, they alter the distribution of electrons and thereby modify the effect which electron–ion collisions and electron inertia have in impeding the current. When ω is small, the conductivity is primarily determined by collisions. The inclusion of electron–electron interactions reduces the conductivity by a factor of approximately two. As ω increases and becomes of order ω_c, this effect becomes less and less important because there is then insufficient time in each a.c. cycle to allow an effective modification of the distribution by electron–electron interactions. When ω well exceeds ω_c, we may neglect this effect and $D(x)$ reduces to

$$D_l(x) = \frac{x^4}{1 + \frac{iB}{2}x^3} \tag{25}$$

and the corresponding conductivity becomes

$$\sigma_l(x) = A\int_0^\infty \frac{x^7 e^{-x^2}}{1 + \frac{iB}{2}x^3}\,dx \tag{26}$$

which is just the a.c. conductivity of a Lorentz gas (BERNSTEIN and TREHAN).

If we further increase ω, the inertia of electrons become dominant. Then we may treat collision effect as a perturbation and obtain

$$\sigma_\infty = A\left(-\frac{3\pi^{1/2}}{4}i + \frac{2}{B^2}\right)\frac{1}{B} \tag{27}$$

$$Z_\infty = \frac{1}{A}\frac{4B^2}{64 + 9\pi B^2}(8 + 3\pi^{1/2}iB) \tag{28}$$

$$R_\infty = \frac{1}{A}\frac{32}{9\pi}\left(1 - \frac{64}{9\pi B^2}\right) \tag{29}$$

$$X_\infty = \frac{1}{A}12\pi^{1/2}B\left(1 - \frac{64}{9\pi B^2}\right). \tag{30}$$

TABLE 1.—VALUES OF $D(x)$ FOR $\omega = \omega_c$ AND $\omega = 0$ $\omega_c = \dfrac{\omega_p \ln \lambda}{4\sqrt{2}\,\lambda}$

	$\omega = \omega_c$		$\omega = 0$
x	$Re[D(x)]$	$Im[D(x)]$	$D(x)$
0·10	0·0005887	−0·0002029	0·0008093
0·11	0·0009252	−0·0003438	0·001300
0·12	0·001376	−0·000542	0·001970
0·13	0·001956	−0·0008068	0·002847
0·14	0·00268	−0·001149	0·003955
0·15	0·00356	−0·00158	0·005317
0·16	0·00461	−0·00210	0·006955
0·17	0·00583	−0·00272	0·008886
0·18	0·00724	−0·00346	0·01113
0·19	0·00884	−0·00431	0·01370
0·20	0·01063	−0·00528	0·01660
0·22	0·01483	−0·00761	0·02347
0·24	0·01985	−0·01048	0·03180
0·26	0·0257	−0·0139	0·04165
0·28	0·0324	−0·0180	0·05304
0·30	0·0400	−0·0226	0·06601
0·32	0·0483	−0·0279	0·08057
0·34	0·0575	−0·0339	0·09672
0·36	0·0675	−0·0405	0·1145
0·38	0·0783	−0·0478	0·1338
0·40	0·0899	−0·0557	0·1548
0·44	0·1153	−0·07366	0·2015
0·48	0·1435	−0·09436	0·2545
0·52	0·1744	−0·1179	0·3137
0·56	0·2080	−0·1442	0·3792
0·60	0·2439	−0·1734	0·4508
0·64	0·2882	−0·2055	0·5285
0·68	0·3227	−0·2405	0·6123
0·72	0·3652	−0·2785	0·7023
0·76	0·4096	−0·3196	0·7983
0·80	0·4559	−0·3637	0·9005
0·88	0·5535	−0·4813	1·123
0·96	0·6570	−0·5718	1·371
1·04	0·7656	−0·6957	1·645
1·12	0·8782	−0·8335	1·945
1·20	0·9937	−0·9858	2·273
1·28	1·111	−1·1531	2·630
1·36	1·2290	−1·3359	3·017
1·44	1·3457	−1·5347	3·435
1·52	1·4598	−1·7500	3·887
1·60	1·5693	−1·9820	4·375
1·76	1·7657	−2·4962	5·465
1·92	1·1915	−3·0739	6·728
2·08	1·9973	−3·7049	8·190
2·24	1·9913	−4·3694	9·880
2·40	1·8852	−5·0382	11·83
2·72	1·4266	−6·2407	16·62
2·88	1·2198	−6·6809	19·53
3·04	1·3105	−6·8066	22·74
3·20	2·1113	−5·6758	26·00

TABLE 2.—THE CONDUCTIVITY, THE RESISTANCE AND THE REACTANCE OF a.c. CURRENT

	σ/A		R/c		X/c	
ω/ω_c	Real gas	Lorentz gas	Real gas	Lorentz gas	Real gas	Lorentz gas
0·0	1·734	3·0	0·577	0·333	0·0	0·0
0·05	1·729	2·880	0·577	0·340	0·045	0·70
0·1	1·713	2·653	0·577	0·354	0·089	0·130
0·15	1·687	2·430	0·578	0·367	0·134	0·185
0·2	1·651	2·233	0·579	0·380	0·178	0·236
0·25	1·608	2·061	0·580	0·393	0·223	0·285
0·3	1·561	1·913	0·582	0·404	0·267	0·332
0·35	1·510	1·784	0·584	0·415	0·311	0·377
0·4	1·458	1·671	0·587	0·425	0·354	0·421
0·45	1·406	1·572	0·590	0·434	0·397	0·465
0·5	1·354	1·483	0·593	0·443	0·440	0·508
0·55	1·303	1·405	0·597	0·452	0·482	0·550
0·6	1·255	1·334	0·600	0·460	0·525	0·592
0·65	1·208	1·270	0·603	0·467	0·567	0·634
0·7	1·164	1·212	0·607	0·475	0·608	0·675
0·75	1·122	1·159	0·610	0·482	0·650	0·716
0·8	1·082	1·111	0·614	0·488	0·691	0·756
0·85	1·045	1·066	0·617	0·495	0·732	0·797
0·9	1·009	1·025	0·620	0·501	0·773	0·807
0·95	0·976	0·987	0·624	0·507	0·813	0·877
1·0	0·944	0·952	0·627	0·513	0·853	0·916
1·1	0·886	0·889	0·634	0·524	0·934	0·996
1·2	0·834	0·835	0·640	0·534	1·013	1·074
1·3	0·786	0·785	0·645	0·544	1·094	1·159
1·4	0·743	0·741	0·650	0·553	1·172	1·230
1·5	0·721	0·702	0·657	0·561	1·222	1·308
2·0	0·560	0·557	0·672	0·599	1·588	1·693
3·0	0·408	0·398	0·711	0·655	2·335	2·453
4·0	0·313	0·305	0·734	0·696	3·086	3·207
5·0	0·255	0·248	0·766	0·728	3·842	3·960
6·0	0·214	0·211	0·784	0·754	4·601	4·710
7·0	0·183	0·182	0·790	0·776	5·380	5·460
8·0	0·162	0·161	0·796	0·794	6·08	6·21
9·0	0·144	0·143	0·814	0·810	6·69	6·81
10·0	0·130	0·129	0·830	0·824	7·51	7·6

TABLE 3.—COMPARISON OF THE a.c. CONDUCTIVITIES OBTAINED IN THIS PAPER (DIRECT INTEGRATION) WITH THOSE OBTAINED BY BERSTEIN AND ROBINSON (VARIATIONAL CALCULATION)

		$Re\sigma/A$		$-Im\sigma/A$	
ω/ω_c	Log X ($X = 4\sqrt{2}\,\omega/\omega_c$)	BERNSTEIN–ROBINSON	SHEN–CHEN	BERNSTEIN–ROBINSON	SHEN–CHEN
0·0	$-\infty$	1·734	1·734	0·000	0·000
0·0057	−3·0	1·734	1·734	0·004	0·002
0·0179	−2·5	1·730	1·732	0·049	0·408
0·0565	−2·0	1·705	1·722	0·154	0·151
0·179	−1·5	1·605	1·608	0·447	0·443
0·565	−1·0	0·992	0·987	0·816	0·813
0·901	−0·8	0·637	0·632	0·785	0·787
1·425	−0·6	0·357	0·353	0·645	0·649
2·261	−0·4	0·183	0·179	0·477	0·482
3·559	−0·2	0·087	0·084	0·329	0·336
5·65	0·0	0·040	0·038	0·201	0·204

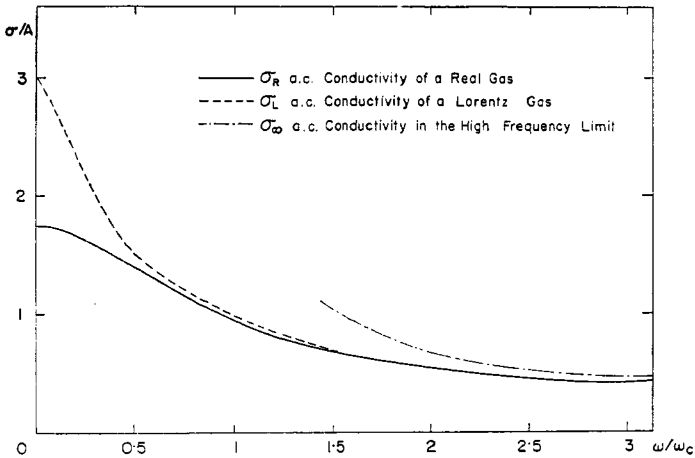

FIG. 2.—A.C. conductivities.

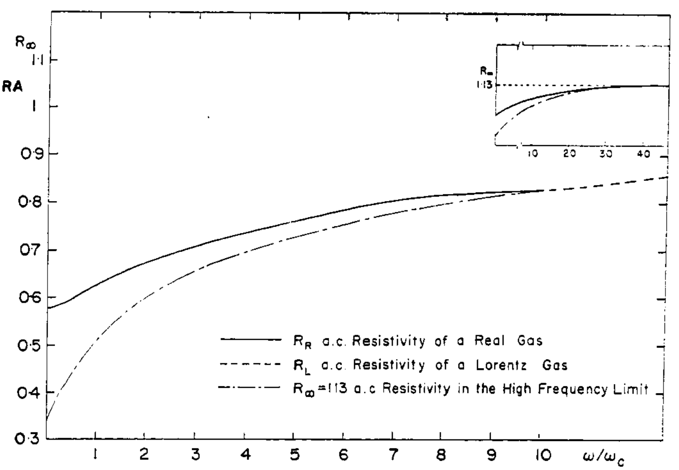

FIG. 3.—A.C. resistivities.

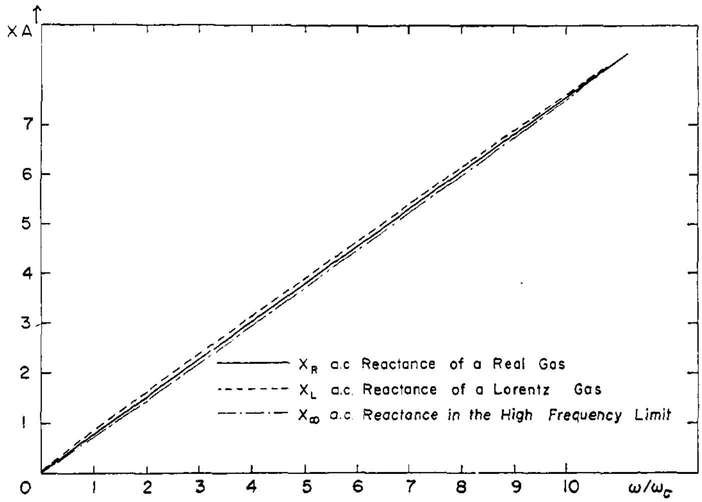

FIG. 4.—A.C. reactances.

In Table 1 the values of $D(x)$ for $B = 1$ are given and compared with the corresponding values for the d.c. case obtained by SPITZER Jr. and HÄRM.

In Table 2 the resistance, the reactance and the absolute value of conductivity are given for various B from 0 to 10. For $B > 10$, one may use equation (26) to compute them. The error will be within 2 per cent. For $B > 50$ the collisions become unimportant and equations (27)–(30) will give the correct values to within 2 per cent. However, there the validity of the Fokker–Planck equation already becomes questionable and one should use DAWSON–OBERMAN'S values instead of ours.

In Table 3, the complex conductivity calculated in this paper are compared with those obtained by ROBINSON and BERNSTEIN. They showed that transport coefficients obtained from the Fokker–Planck equation should possess an extremal nature, and proceed to calculate conductivities using the variational technique. Their Table 6 gives conductivities for various values of the logarithm of ω/ω_c including very large values of ω_c. In our Table 3 only those values are included for comparison which fall within the range of validity of the Fokker–Planck equation. The discrepancy between the two results are generally within 5 per cent.

We should add that our results can be readily applied to the case of conductivity in the presence of a uniform magnetic field. The addition of the magnetic field leads to equations which are entirely similar to (10). If the electric field is parallel to the magnetic field, the conductivity is not affected. If it is perpendicular to the magnetic field, the conductivity becomes $\sigma_H(\omega) \equiv \sigma(\omega + \omega_H)$, where σ is the function obtained in this paper and $\omega_H = eH/mc$.

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