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THE ASTROPHYSICAL JOURNAL
Vol. 137, No. 4, May 15, 1963

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THE STRUCTURE OF STARS OF VERY LOW MASS

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Received October 20, 1962; revised November 27, 1962

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ABSTRACT

Completely convective models have been constructed for stars of masses 0.09, 0.08, 0.07, 0.06, 0.05, and 0.04 (solar units), taking into account the non-relativistic degeneracy of the stellar material. It is shown that there is a lower limit to the mass of a main-sequence star. The stars with mass less than this limit become completely degenerate stars or "black" dwarfs as a consequence of gravitational contraction, and, therefore, they never go through the normal stellar evolution.

I. INTRODUCTION

Very little observational or theoretical information is available concerning the structure of stars of very low mass. In this paper an attempt will be made to study the internal structure of stars of mass $M < 0.1\, M_\odot$ by using theoretical models. In particular, we shall be concerned with the effects of degeneracy on the structure of stars having masses between $0.09 M_\odot$ and $0.04 M_\odot$.

In order to evaluate physical quantities such as the central temperature and the central density, we have to make use of certain models for the stars under study. We are primarily concerned with low-mass stars when they are contracting and nuclear reactions involving the destruction of H^2, Li^6, Li^7, Be^9, B^{10}, and B^{11} are taking place. Under these circumstances, we can assume that the models are completely convective, as has recently been shown by Hayashi (1962). Therefore, we study the structure of stars of low mass by assuming that they can be represented by spheres of polytropic index 1.5. Limber (1958) has used such models in studying the structure of main-sequence stars of spectral type M. Although we are concerned with the contracting stars and not the main-sequence stars, the structure equations given by Limber are applicable to our stars. Therefore, we shall not derive them here. For the derivations of these equations, Limber's paper or a paper by the author (Kumar 1962) should be consulted.

II. COMPUTATION OF THE PHYSICAL STRUCTURE

We compute the physical structure for stars having the following two chemical compositions: (I) $X = 0.90$, $Y = 0.09$, $Z = 0.01$; (II) $X = 0.62$, $Y = 0.35$, $Z = 0.03$. For each chemical composition and a given mass, we compute P_c, T_c, and ρ_c at several values of the radius. The variation of T_c and ρ_c with the radius R is shown in Figure 1 for a star of mass 0.07. The T_c-R relation for the first composition is given by the solid curve, whereas the broken curve gives the same relation for the second composition.

No models have been computed for those radii at which electron conduction becomes important, and consequently the assumption of convective equilibrium does not hold. For each mass, the computation of models was stopped when the degeneracy parameter

$$y\left[=\frac{N_e h^3}{2(2\pi m_e kT)^{3/2}}\right]$$

reached a value close to 15. When y reaches this value, the material becomes appreciably degenerate, and only then does the electron conduction become an efficient process for heat transport. However, to obtain a rough estimate of the central density and central temperature at a smaller radius, one more model was computed for each mass by making use of the condition of convective equilibrium. When y has a value of 30 or 40, electron

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conduction may destroy convection equilibrium in the central regions; still the convective model should give numerical results which have the correct order of magnitude. The dotted parts of the curves for the T_c-R relation are approximately correct, because in these regions the value of y is greater than 15.

Figure 1 shows that, as the radius of the star is decreased, the central density keeps on increasing, while the temperature T_c shows an interesting variation. At first, it increases and then reaches a maximum value. As the radius is further decreased, the temperature begins to decrease. This behavior of the temperature is due to the effects of degeneracy. We can visualize it physically in this way: when the radius changes from a large value to smaller ones, the star of a given mass can be pictured as a contracting star. In the case of a contracting star composed of perfect gas, a part of the energy released by contraction takes the form of thermal energy, and consequently the central temperature of the star increases. However, when we are dealing with a contracting star composed of partially degenerate matter, the energy absorbed by the stellar material does

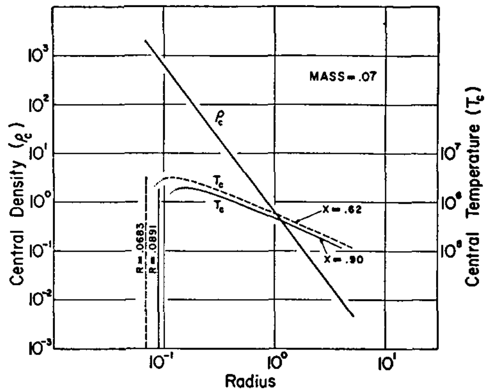

FIG. 1.—The central temperature and the central density in completely convective models

not manifest itself as thermal energy. When partially degenerate matter is compressed, energy is needed to bring the degenerate electrons closer because they obey the Fermi-Dirac distribution rather than the Maxwell-Boltzmann distribution. Therefore, the central temperature remains constant for a while, and later it begins to decrease as a result of further contraction. For a given mass and chemical composition, there exists a limiting value of the radius below which there exists no model, for the material has become completely degenerate. The solid vertical line in Figure 1 gives the limiting radius for the first composition, whereas the broken vertical line gives the same quantity for the second composition.

In Table 1 are given the limiting radii and the limiting densities for the masses 0.09, 0.08, 0.07, 0.06, 0.05, and 0.04. Figure 2 shows the temperature and density distributions in a population I star of mass 0.07 and radius 0.5. The quantity x is defined by $x = 0.2737\xi$. To show the effects of non-relativistic degeneracy on the temperature distribution, we have also plotted the corresponding temperature distribution in a model composed of perfect gas. In Figure 3 are plotted the central temperatures and the central densities of all the models with the population I composition. For a given mass, the plot of various models in the temperature-density diagram gives an evolutionary path for a contracting star. The dividing line between the degenerate and non-degenerate regions in this diagram is obtained by equating the electron pressure from the complete degeneracy law with that obtained from the equation of state for a perfect gas. The limiting densities for each mass are shown in this diagram by the vertical lines. Figure 4 gives the temperature-density diagram for the models with the population II composition. A more detailed discussion of the models for stars of very low mass has been presented by the author in a separate paper (Kumar 1962).

III. DISCUSSION

The numerical results presented here show clearly that, for a given chemical composition, there exists a limiting mass below which a contracting star cannot reach the main-sequence stage because the temperature and density at the center are too low for hydrogen-burning to start. Instead, the star becomes a degenerate star as a consequence of the contraction. After the star has evolved beyond the stage of maximum central temperature, further contraction will take it toward the stage of complete degeneracy. Thus all stars having a mass less than a certain limiting mass ultimately become completely

TABLE 1

PROPERTIES OF THE COMPLETELY DEGENERATE MODELS

MASS	$X = 0.90$, $Y = 0.09$, $Z = 0.01$		$X = 0.62$, $Y = 0.35$, $Z = 0.03$	
	Radius	Central Density	Radius	Central Density
0.09	0.0819	1.384×10^3	0.0628	3.069×10^3
.08	.0852	1.092×10^3	.0653	2.427×10^3
.07	.0891	8.357×10^2	.0683	1.856×10^3
.06	.0938	6.141×10^2	.0719	1.363×10^3
.05	.0997	4.261×10^2	.0764	9.471×10^2
0.04	0.1074	2.726×10^2	0.0823	6.059×10^2

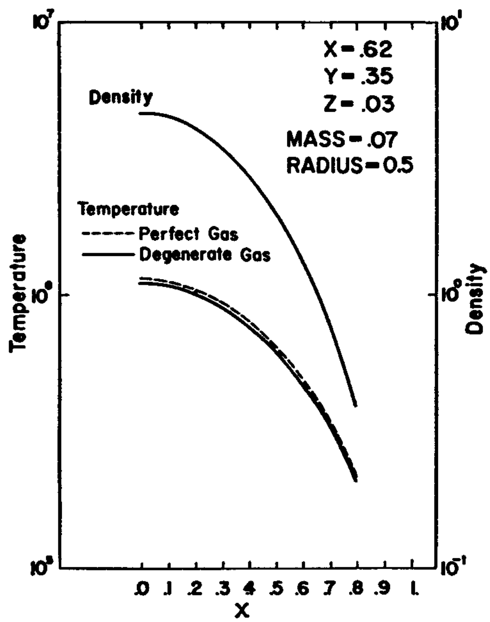

FIG. 2.—The temperature and density distributions in a completely convective model

degenerate objects or "black" dwarfs without ever going through the normal stellar evolution. The exact determination of this limiting mass for a given composition requires a knowledge of the luminosity of the contracting stars. This can be obtained if we know the atmospheric structure in addition to the interior models computed here. However, assuming reasonable luminosities for these stars (see the following paper), we find that, for stars with population I composition, the limiting mass is approximately 0.07. Similarly, for the population II stars the limiting mass is approximately 0.09.

Suitable model atmospheres for contracting stars of low mass are being computed, which, together with the interior models presented here, will give us not only the evolutionary tracks in the H-R diagram for these stars but also the exact limiting mass which gives a lower limit to the mass of a main-sequence star and the time scale for the Helm-

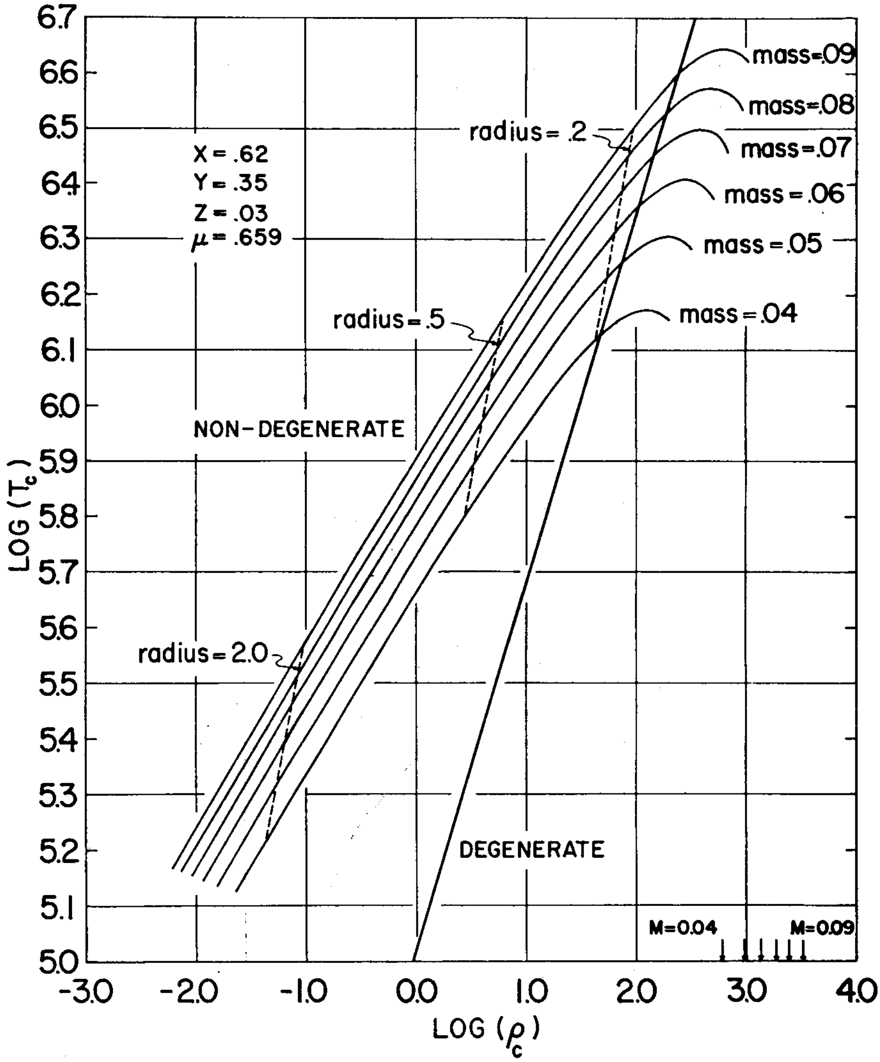

FIG. 3.—The temperature-density diagram for completely convective models. Population I

holtz-Kelvin contraction. However, it is possible to compute approximately the Helmholtz-Kelvin time scale without knowing the atmospheric structure of contracting stars. This problem is discussed in the following paper.

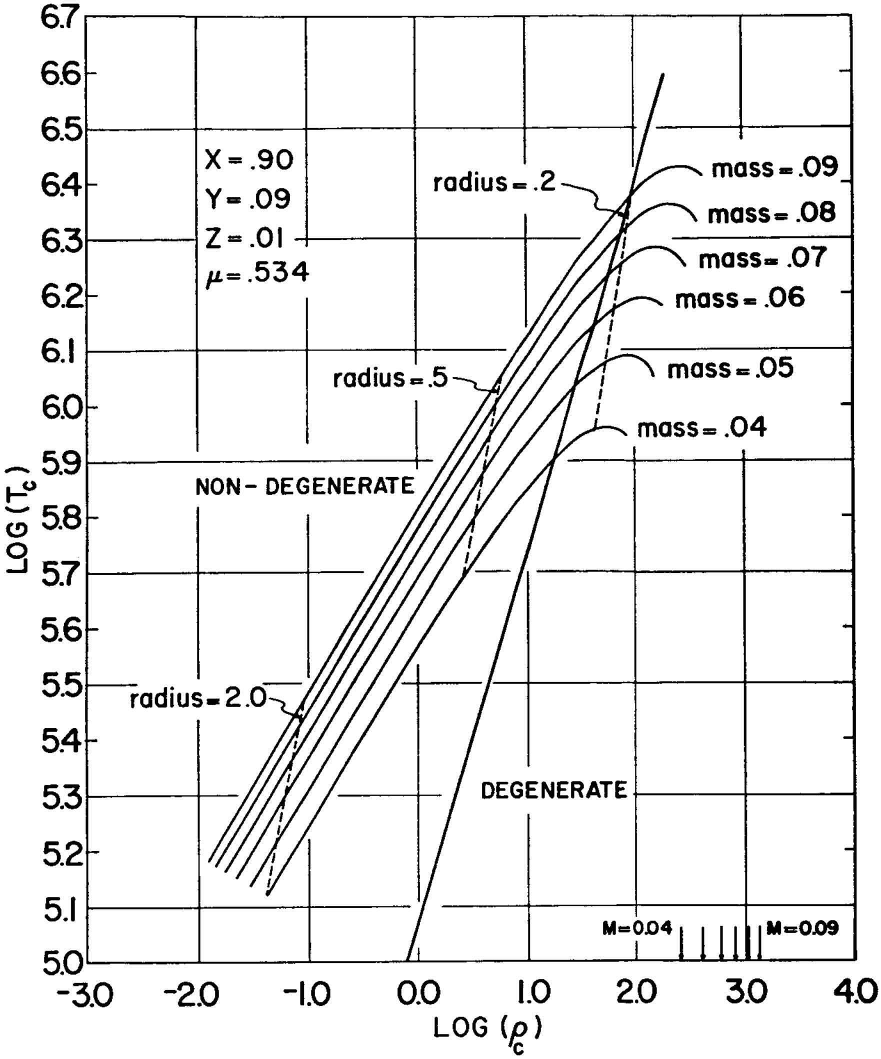

FIG. 4.—The temperature-density diagram for completely convective models. Population II

It is a pleasure to express my gratitude to Dr. A. G. W. Cameron and Dr. Edward Upton for their helpful discussions. Mr. A. Liebman is thanked for his assistance with numerical computations. I am grateful to Dr. R. Jastrow for his hospitality at the Institute for Space Studies.

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