On the Semiannual Variation of Geomagnetic Activity and its Relation to the Solar Corpuscular Radiation

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ABSTRACT

The semiannual variation of geomagnetic activity has been in the past tentatively explained by two different theories, the so-called "equinoctial hypothesis" and the "axial hypothesis." The latter relates the effect to the inclination of the sun's axis with respect to the sun-earth line, i.e., to the annual variation of the heliographic latitude of the earth.

From a reëxamination of the data for geomagnetic activity calculated by Bartels (1932, 1940) we have found a strong modulation of the amplitude of the semiannual variation during the eleven year period of the solar cycle, which is apparently related to the heliographic latitudes of the sunspot-zones and provides indirect support for the axial hypothesis. The amplitude of the modulation is 13 per cent during the increasing phase of solar activity, when the mean sunspot latitude is 20°; it reaches 27 per cent during the late decreasing phase when the mean sunspot latitude is 10°. Three axial-symmetric models are derived, which give the statistical distribution of the solar particle flux for three different 3-yr periods during the solar cycle according to mean sunspot-latitudes of 20°, 15° and 10°. The distribution in heliographic latitude is given by two Gaussian functions centered at the heliographic latitudes of the active zones, as defined by the mean latitudes of the sunspots. These models explain the observed semiannual geomagnetic variations, except for a phase-lag of 20 to 30 days. We suggest that a semiannual density variation of charged solar particles which are trapped in the earth's magnetosphere can provide the required phase-lag mechanism. This implies a lifetime for these particles of the order of one month. The models can also provide a quantitative explanation of the observed delay of geomagnetic activity behind solar activity during the 11-yr cycle. Furthermore, they are in reasonable agreement with preliminary results on the statistical latitude distribution of comets with ionized tails.

A space probe in a heliocentric orbit is proposed in order to obtain further evidence on the latitude distribution of solar particle streams.

1. Introduction

In the course of deriving densities of the upper atmosphere from the observed changes of satellite periods, H. K. Paetzold and H. Zschoerner (1960) have found a semiannual period in the density fluctuations with minima in June-July and December-January and maxima in March-April and September-October. These variations are superposed upon the well-known variations with local time and solar activity (as measured by the solar flux in the wavelength range of 3 to 30 cm). [See Priester and Martin (1960); Jacchia (1960); Priester (1961).] It has been noticed by Martin and Priester (1961) [see also Paetzold and Zschoerner (1961) and Martin *et al.* (1961)] that these density variations show a striking similarity to the semiannual variations of geomagnetic activity, which have been discussed in detail by Bartels (1932) and more recently by McIntosh (1959). The same behavior is also well known for auroral frequency.

These developments drew our attention to the possible interpretations of the semiannual variation. There are two rival theories for the explanation of the geomagnetic semiannual effect (Bartels, 1932):

(1) The "equinoctial hypothesis," which relates the cause of the effect to the annual variation of the direction of the earth's axis or more specifically to the annual variation of the geomagnetic axis.

(2) The "axial hypothesis," first pointed out by Cortie (1912). He explained the effect as caused by the

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varying inclination of the sun's axis with respect to the sun-earth line or in other words by the annual variation of the heliographic latitude of the earth, which covers the range from $+7°.2$ to $-7°.2$. Strong support for the axial hypothesis has been given by Bell and Glazer (1957) in an investigation of the relations between geomagnetism and coronal emission lines.

Bartels (1932) and McIntosh (1959) preferred the equinoctial hypothesis. Their main reason for this choice is the fact that the observed maxima of the semiannual variation occur late in March and September, or even in April and October. They are therefore closer to the equinoxes than to the maximum heliographic latitudes of the earth (7 September: $B=+7°.2$, 6 March: $B=-7°.2$) which would be emphasized in the axial hypothesis. Furthermore, McIntosh supports his explanation by pointing out the existence of a very small geomagnetic variation which is a function of universal time, and in which the maximum disturbance occurs when the geomagnetic axis is normal to the sun-earth line.

The axial hypothesis of Cortie (1912) is based on the assumption that the solar corpuscular streams are emitted radially from the active zones (defined by the sunspots) and further that the heliographic latitude of the particles has not decreased appreciably by the time that they reach the distance of the earth. As the active zones move from higher latitudes toward the equator during the solar cycle, we should expect to find a related variation in the relative amplitude of the semiannual geomagnetic effect, if the "axial hypothesis" is the right explanation. We have found this amplitude variation from an analysis of geomagnetic activity data, and then have used the observed maxima and minima to calculate models for the statistical structure of the solar particle streams.

2. Relation between the geomagnetic semiannual variation and the heliographic latitude of the sunspot-zones

In order to check the statistical relation between the relative amplitude of the semiannual variation and the mean latitude of the sunspot-zones, we used the monthly geomagnetic activity data u and u_1 given by Bartels (1940) for the time interval 1872 through 1938. The quantity u was called by Bartels the "interdiurnal variability of the horizontal component at the magnetic equator." He derived it from the measurements of nine stations with world-wide distribution in longitude; two of the stations were in the southern hemisphere. The quantity u_1 is a transformation of u which has more suitable statistical properties. It reduces the contributions from the outstanding large storms. [For details see Bartels (1932).] In Bartel's analysis the data were classified according to high, medium and low solar activity, defined by the sunspot numbers. He found a statistically significant semiannual variation with nearly the same relative amplitude for the three cases of high, medium and low activity, despite the fact that the annual averages for high and low activity differ by more than a factor of two. Using this classification, any dependence on the heliographic latitude of the active zones would tend to be smoothed out.

We therefore repeated his statistics, but grouped the data according to the mean latitude of the sunspots. We selected two groups with spots in high (H) and low (L) latitudes. Generally, 3-yr intervals were chosen. In case (H) they belong to the increasing phase of the solar cycle, in case (L) to the decreasing phase, close to solar minimum. An interval of 2 yr around the minimum was excluded since at these times spots at low latitudes often appear together with spots at very high latitudes, indicating some overlapping of the cycles. The mean latitudes are 18 to 20° for case (H) and 9 to 10° for case (L). The years included in case (H) are: 1880–82, 1891–93, 1902–04, 1914–16, 1924–26, 1935–37; and in case (L): 1875–77, 1886–88, 1897–99, 1908–10, 1920–22, 1930–32.

The calculated relative averages $u_1/\bar{u}_1$ are given in Table 1. The effect of the increase or decrease of activity during our 3-yr intervals has been removed by deriving individual $\bar{u}_1$-values for each month of the 3-yr intervals using linear interpolation of the yearly averages.

TABLE 1. Geomagnetic semiannual variation $u_1/\bar{u}_1$ for three different heliographic latitudes B of the active zones: High: $|B|=18°–20°$; Med: $|B|=14°–15°$; Low: $|B|=9°–10°$. The bottom line gives the annual averages $\bar{u}_1$.

$u_1/\bar{u}_1$	High	Med.	Low
Jan.	0.90	0.87	0.86
Feb.	0.99	0.97	0.88
Mar.	0.96	1.12	1.17
Apr.	1.13	1.03	0.98
May	0.97	0.93	1.20
June	1.00	0.86	0.83
July	0.96	0.94	0.77
Aug.	1.00	1.04	0.94
Sep.	1.04	1.08	1.27
Oct.	1.15	1.17	1.14
Nov.	1.12	1.04	0.98
Dec.	0.87	0.90	0.95
$\bar{u}_1$	54.1	68.7	47.2

For the representation in Fig. 1 we used slightly smoothed data, following Bartel's procedure, by calculating overlapping averages according to the formula $\frac{1}{4}(a+2b+c)$. The mean standard deviations of the points are ± 6 per cent. As the data given by Bartels for high solar activity refer to sunspots in medium latitudes of about 15°, we also included these data in Table 1 and Fig. 1, marked by M for medium latitudes. The yearly averages of u_1 are given at the right side of the diagram. The significant increase of the relative amplitude of the semiannual variation with decreasing heliographic latitude of the sunspots can easily be seen.

Comparing case (*H*) and (*L*) the amplitude increases from 13 per cent (*H*) to 27 per cent (*L*), while the yearly average remains nearly the same i.e., 54.1 (*H*) and 47.2 (*L*). The amplitude for case (*M*) is 19 per cent, which is satisfactorily placed between the two extreme cases.

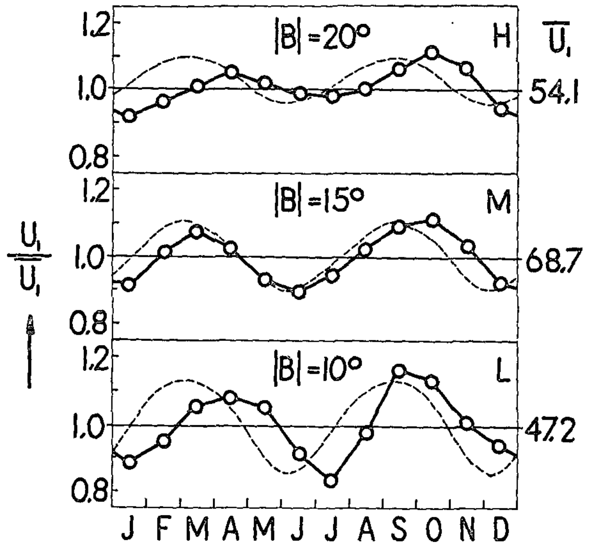

FIG. 1. The geomagnetic semiannual variation $u_1/\bar{u}_1$ for three different heliographic latitudes of the active zones on the sun: (*H*) High latitudes ($|B|=20°$), (*M*) Medium latitudes ($|B|=15°$), (*L*) Low latitudes ($|B|=10°$). The theoretical curves, derived from the model (formula 2, Table 2), are given by the dotted lines. The phase shift between the observed and the theoretical curves is discussed in section three. The annual averages $\bar{u}_1$ are given on the right.

The two maxima of the semiannual variations are not identical. This is also true for the minima. This might be due to additional effects, for example the northern and southern solar hemispheres can differ considerably from each other in terms of activity (Bell, 1961). Further, we have to account for a remaining annual effect, which has its maximum in summer and minimum in winter. Its amplitude increases with the absolute value of geomagnetic latitude (McIntosh, 1959). In an ideal planetary index this effect would be removed. Bartels' data, however, are derived mostly from geomagnetic measurements at northern hemisphere observatories. Therefore, we should expect a less intense summer minimum. As we intend to use only average values for the semiannual maxima and minima, we can, however, omit these secondary effects from our discussion.

We consider the increase of the amplitude of the semiannual variation with decreasing latitude of the active zones as a strong evidence for the suggestion that the semiannual variation is in fact due to the varying heliographic latitude of the earth ("axial hypothesis").

3. Model of the solar corpuscular radiation

In order to explain the properties of the semiannual variation, we have derived a model of the average energy flux distribution of the solar corpuscular radiation, as a function of the heliographic latitude, at the distance of the earth from the sun. The model gives the right values for the maxima and minima of u_1 and also the increase of their ratio during the 11-yr cycle. The geomagnetic disturbances are probably not only a function of the energy flux of the solar corpuscular radiation at the position of the earth, but may also depend on the physical properties of the earth's magnetosphere. Thus, our model really describes the efficiency of the mechanism for producing geomagnetic disturbances as a function of heliographic latitude, rather than describing the actual energy flux of the solar corpuscular radiation. But it seems reasonable at the present state of our knowledge to consider it as an approximate statistical model for the solar particle streams. As there are two active zones on the sun, we used two Gaussian distributions centered at heliographic latitudes corresponding to the mean latitudes of the active zones, which were assumed to be nearly identical with the mean latitudes of the sunspots. As we deal with statistical averages over a long period of time, the effects of the northern and southern streams of the corpuscular radiation can be assumed to be simply additive. Therefore, we give our "efficiency-distribution" by the following formula:

$$U = a_N e^{-[(B-B_N)/\alpha]^2} + a_S e^{-[(B-B_S)/\alpha]^2}, \qquad (1)$$

where B represents the heliographic latitude, B_N and B_S the mean heliographic latitudes of the stream centers at the earth's distance from the sun for the northern and the southern solar hemisphere, respectively. The symbol a gives the central intensity of the stream and α the width. For reasons mentioned in the previous paragraph we used for each of the three curves one average value for the maximum related to $|B| = 7\overset{\circ}{.}2$, the maximum heliographic latitude of the earth at 7 September ($B = +7\overset{\circ}{.}2$) and 6 March ($B = -7\overset{\circ}{.}2$), and one minimum value related to $B = 0$ occurring at 7 June and 8 December. The data are given in Table 2. The phase lag of about 20 to 30 days which can be seen from Fig. 1 will be considered later. With this simplification mentioned, formula (1) changes to:

$$U = a\left[e^{-[(B-B_0)/\alpha]^2} + e^{-[(B+B_0)/\alpha]^2}\right], \qquad (2)$$

where $B_0 = B_N = |B_S|$. Since only two free parameters can be derived from the given u_1 (MAX) and u_1 (MIN), we chose the mean heliographic latitude of the stream center at the earth's distance (B_0) to be equal to the mean heliographic latitude of the sunspots. This assumes that the corpuscular streams leave the sun almost radially and that therefore the heliographic latitude of the stream center has not decreased appreciably when the stream reaches the earth. This assumption is supported by a relation between the number of magnetic storms and the heliographic latitude of the spots (Gnevishev and Ol, 1946). (Actually, a decrease in latitude of 30 per cent would not affect our results qualitatively.)

We calculated two sets of Gaussian distributions, one for the u_1 data in order to compare with the curves (Fig. 2) using $B_0=20°$, 15° and 10° in formula 2 for the three cases, respectively. The results are given in the upper part of Table 2. As the quantities u_1 do not have a good relation to the energies involved, we transformed the maximum and minimum values back again into u data. Their unit is 10^{-4} Gauss (10γ). We recall that in these data the contributions of the outstanding storms are somewhat reduced. The relation between u and the energy which is transformed into magnetic energy can be estimated to be of the order of $3\times10^{21}\,u$ erg. As u is a measure of geomagnetic activity in a daily scale we cannot deduce easily the rate of energy per second which causes the change of the field. It might be in the order of $10^{-2}\,u$ erg cm^{-2} sec^{-1}. This gives a lower limit for the flux of the corpuscular radiation. The real flux could be higher by orders of magnitude.

The parameters of the Gaussian distributions derived from u_{MAX} and u_{MIN} are given in the lower part of Table 2; and in Fig. 2 in polar coordinates. For the stream center we chose the heliographic latitudes $B_0=18°$, 14° and 9°. This takes account of the expected slight decrease of the stream latitudes. The parameter α is approximately proportional to the heliographic latitudes of the stream centers. This can easily be understood because the latitude dispersion of the sunspots is also approximately proportional to their mean latitude.

In Table 2 the integral $E=\int_{-\pi/2}^{+\pi/2} UdB$ is given in arbitrary units. In our approximation it should be proportional to the total power of the corpuscular radiation. For comparison the average sunspot numbers R are also given in Table 2. The maximum value of the integral is found for case (H), almost 2 yr before sunspot maximum. This may perhaps change a little in a modified model which takes into account a greater decrease of the heliographic latitude of the stream centers as mentioned above.

From our model (Table 2, formula 2) and from the heliographic latitudes of the earth given in the American Ephemeris, we calculated the theoretical semiannual variation and plotted it in Fig. 1 (dotted curve). These curves give a satisfactory representation of the observational data except for a significant phase shift of about 20 to 30 days. In case (M) the phase shift seems to be somewhat smaller than in the other cases.

Bartels (1932) and McIntosh (1959) have tried to explain the semiannual variation by the "equinoctial" theory. Since this theory fails to explain the dependence of the amplitude on the heliographic latitude of the active zones, and furthermore is not able to explain the delay in the 11-yr cycle mentioned below, we can not consider the "equinoctial" theory as the primary explanation. In order to explain the observed phase-shift, however, we could think of a secondary effect, provided by the "equinoctial" theory, which could be superposed on the primary cause. But this should lead to maxima between 7 and 23 September and 6 and 21 March. Actually, however, the maxima occur merely one week or more after the equinoxes. Therefore, such a possibility does not seem very likely.

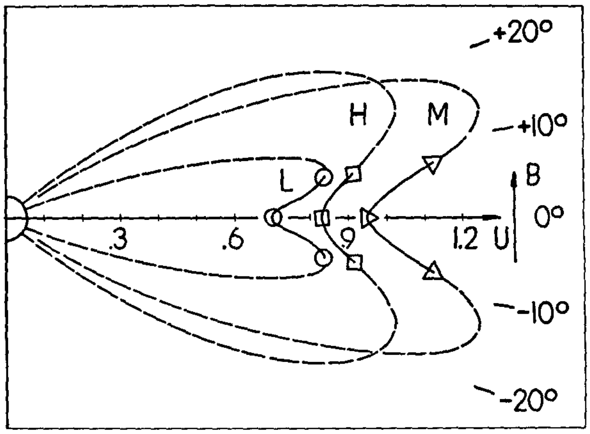

FIG. 2. Models of the statistical structure of the solar corpuscular radiation at the distance of the earth in polar coordinates for three different heliographic latitudes of the active zones (see Fig. 1). B is the heliographic latitude. The radius vector is given in u-units of geomagnetic disturbances. (For transformation in energy-flux see section three.) The circles, squares and triangles give the observed maxima and minima of the semiannual variation for the cases (L), (H) and (M), respectively.

TABLE 2. Observed maxima and minima of the semiannual variation, which are used to derive the statistical flux-distribution of the solar corpuscular radiation, as represented by the parameters B_0, α, and a of the Gaussian functions (formula 2). In the upper part the data are given in u_1-units, in the lower part in u-units. The total flux E is given in the next to last line; for comparison the Zürich sunspot numbers R are given in the last line. The Gaussian functions and the observed maxima and minima are plotted in Fig. 2.

	High	Med.	Low
u_1 (Max)	59	75	53
u_1 (Min)	52	62	40
B_0	20°	15°	10°
α	19°.3	14°.7	10°.0
a	76	88	54
u (Max)	0.92	1.13	0.84
u (Min)	0.83	0.95	0.70
B_0	18°	14°	9°
α	18°.8	14°.3	9°.8
a	1.035	1.24	0.815
E	69.3	63.2	28.4
R	48.9	67.7	22.6

To explain the observed phase-shift of 20 to 30 days on the basis of our model, we are led to require a phase-lag mechanism which is physically associated with the earth. In this connection we may recall that the density of trapped particles in the earth's magnetosphere is likely to show a semiannual variation in the statistical averages, if our models describe correctly the solar particle flux arriving at the earth. This density variation of the trapped particles would have a phase-shift with respect to the times of *extrema* expected from our model, due to their lifetime. If the lifetime is of the order of one month, it could provide a possible explanation of the observed phase-lag in the geomagnetic semiannual variation. Independent evidence that daily geomagnetic disturbances arise from drifting particles in the magnetosphere is given by Vestine (1961).

Our models of the solar corpuscular radiation provide further an immediate explanation for two other findings: the delay of geomagnetic activity in the solar cycle and the statistical distribution of comets with ionized tails. The delay of the geomagnetic activity behind the general variation of the sunspot numbers, discussed in detail by Bartels (1932), can be understood quantitatively if we assume a solar cycle variation of the total corpuscular radiation characterized by the three E-values and a distribution represented by the widths α given in Table 2. The delay would be explained by the combined effect of the 11-yr variation of the total power and the decrease of the latitude dispersion of the streams.

The statistical distribution of comets with ionized tails ("type I") was investigated by Stumpff (1961) with respect to its dependence on the ecliptical latitudes β of the perihelia of the comets. Type I tails are believed to be caused by the interaction of solar corpuscular radiation with the gaseous component of comets according to Biermann's theory (1951). The latitude distribution is in satisfactory agreement with a distribution which we would expect from our model, when we take into account the additional scatter that the use of the ecliptical latitudes of the perigees in Stumpff's histogram introduces. His result is still preliminary; a more detailed paper has been announced.

Our model does not contain an isotropic component, as it is not possible to get information on it from geomagnetic data. The existence of such a component in addition to our model is, however, rather likely, as comets which have once developed a type I tail seem to show this tail without interruption as long as they are sufficiently close to the sun, even at higher heliographic latitudes (Biermann, 1961).

4. A proposal for a space flight experiment on solar corpuscular radiation

As the solar corpuscular radiation is believed to provide the energy for a great variety of effects on earth and in interplanetary space (e.g., geomagnetic activity, auroral activity, hydromagnetic waves in the upper atmosphere, partial heating of the upper atmosphere, outer Van Allen belts, ionized comet tails, modulation of galactic cosmic ray intensity), it is highly desirable to obtain further evidence about the solar corpuscular streams (densities, energy flux, composition) and their spatial distribution. A space probe in an orbit around the sun with instrumentation for energetic particle and magnetic field measurements will furnish the relevant data, provided that its orbit has a sufficient inclination relative to the sun's equatorial plane. Our models can be used as a guide for determining lower limits to the desired inclination. They depend on the phase of the solar cycle. We obtain in this way 20° for the interval 1962–64, 35° for 1965–67 and 30° for 1968–70. The required inclinations with respect to the ecliptic plane are 13°, 28° and 23°, respectively, if the launchings are made close in time to the optimum dates (7 June and 8 December). This implies velocities of 12.9, 18.1 and 16.2 km/s, respectively, at the burnout of the last stage, assuming an altitude of 200 km at this time. An orbit with small eccentricity and a period of about 1 yr would be optimum with respect to minimum requirements for launching energy; during the flight the space probe would also remain sufficiently close to the earth for convenient telemetry. A space probe with one of the proposed orbits would be the logical continuation of the experiments made with Explorer X (Bridge *et al.*, 1961). The space probe would also yield valuable information on the phase-lag mechanism discussed above if simultaneous measurements of particle densities and energies in the outer magnetosphere by earth satellites would be available.

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