



# Scattering, Absorption, and Emission of Light by Small Particles

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## Scattering, Absorption, and Emission of Light by Small Particles

This volume provides a thorough and up-to-date treatment of electromagnetic scattering by small particles. First, the general formalism of scattering, absorption, and emission of light and other electromagnetic radiation by arbitrarily shaped and arbitrarily oriented particles is introduced, and the relation of radiative transfer theory to single-scattering solutions of Maxwell's equations is discussed. Then exact theoretical methods and computer codes for calculating scattering, absorption, and emission properties of arbitrarily shaped particles are described in detail. Further chapters demonstrate how the scattering and absorption characteristics of small particles depend on particle size, refractive index, shape, and orientation. The work illustrates how the high efficiency and accuracy of existing theoretical and experimental techniques and the availability of fast scientific workstations result in advanced physically based applications of electromagnetic scattering to noninvasive particle characterization and remote sensing. This book will be valuable for science professionals, engineers, and graduate students in a wide range of disciplines including optics, electromagnetics, remote sensing, climate research, and biomedicine.

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M. I. MISHCHENKO, L. D. TRAVIS, and A. A. LACIS also authored a book on *Multiple Scattering of Light by Particles: Radiative Transfer and Coherent Backscattering* published in 2006 by Cambridge University Press. M. I. MISHCHENKO and L. D. TRAVIS co-edited a monograph on *Light Scattering by Nonspherical Particles: Theory, Measurements, and Applications* published in 2000 by Academic Press.



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## Preface to the electronic edition

This book was originally published by Cambridge University Press in June of 2002. The entire print run was sold out in less than 16 months, and the book has been officially out of print since October of 2003. By agreement with Cambridge University Press, this electronic edition is intended to make the book continually available via the Internet at the World Wide Web site

<http://www.giss.nasa.gov/~crmim/books.html>

No significant revision of the text has been attempted; the pagination and the numbering of equations follow those of the original hardcopy edition. However, almost all illustrations have been improved, several typos have been corrected, some minor improvements of the text have been made, and a few recent references have been added.

We express sincere gratitude to Andrew Mishchenko for excellent typesetting and copy-editing work and to Nadia Zakharova and Lilly Del Valle for help with graphics. The preparation of this electronic edition was sponsored by the NASA Radiation Sciences Program managed by Donald Anderson.

We would greatly appreciate being informed of any typos and/or factual inaccuracies that you may find either in the original hardcopy edition of the book or in this electronic release. Please communicate them to Michael Mishchenko at

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## Preface to the original hardcopy edition

The phenomena of scattering, absorption, and emission of light and other electromagnetic radiation by small particles are ubiquitous and, therefore, central to many science and engineering disciplines. Sunlight incident on the earth's atmosphere is scattered by gas molecules and suspended particles, giving rise to blue skies, white clouds, and various optical displays such as rainbows, coronae, glories, and halos. By scattering and absorbing the incident solar radiation and the radiation emitted by the underlying surface, cloud and aerosol particles affect the earth's radiation budget. The strong dependence of the scattering interaction on particle size, shape, and refractive index makes measurements of electromagnetic scattering a powerful noninvasive means of particle characterization in terrestrial and planetary remote sensing, biomedicine, engineering, and astrophysics. Meaningful interpretation of laboratory and field measurements and remote sensing observations and the widespread need for calculations of reflection, transmission, and emission properties of various particulate media require an understanding of the underlying physics and accurate quantitative knowledge of the electromagnetic interaction as a function of particle physical parameters.

This volume is intended to provide a thorough updated treatment of electromagnetic scattering, absorption, and emission by small particles. Specifically, the book

- introduces a general formalism for the scattering, absorption, and emission of light and other electromagnetic radiation by arbitrarily shaped and arbitrarily oriented particles;
- discusses the relation of radiative transfer theory to single-scattering solutions of Maxwell's equations;
- describes exact theoretical methods and computer codes for calculating the scat-

- demonstrates how the scattering and absorption characteristics of small particles depend on particle size, refractive index, shape, and orientation; and
- illustrates how the high efficiency and accuracy of existing theoretical and experimental techniques and the availability of fast scientific workstations can result in advanced physically based applications.

The book is intended for science professionals, engineers, and graduate students working or specializing in a wide range of disciplines: optics, electromagnetics, optical and electrical engineering, biomedical optics, atmospheric radiation and remote sensing, climate research, radar meteorology, planetary physics, oceanography, and astrophysics. We assume that the reader is familiar with the fundamentals of classical electromagnetics, optics, and vector calculus. Otherwise the book is sufficiently self-contained and provides explicit derivations of all important results. Although not formally a textbook, this volume can be a useful supplement to relevant graduate courses.

The literature on electromagnetic scattering is notorious for discrepancies and inconsistencies in the definition and usage of terms. Among the commonly encountered differences are the use of right-handed as opposed to left-handed coordinate systems, the use of the time-harmonic factor  $\exp(-i\omega t)$  versus  $\exp(i\omega t)$ , and the way an angle of rotation is defined. Because we extensively employ mathematical techniques of the quantum theory of angular momentum and because we wanted to make the book self-consistent, we use throughout only right-handed (spherical) coordinate systems and always consider an angle of rotation positive if the rotation is performed in the *clockwise* direction when one is looking in the positive direction of the rotation axis (or in the direction of light propagation). Also, we adopt the time-harmonic factor  $\exp(-i\omega t)$ , which seems to be the preferred choice in the majority of publications and implies a non-negative imaginary part of the relative refractive index.

Because the subject of electromagnetic scattering crosses the boundaries between many disciplines, it was very difficult to develop a clear and unambiguous notation system. In many cases we found that the conventional symbol for a quantity in one discipline was the same as the conventional symbol for a different quantity in another discipline. Although we have made an effort to reconcile tradition and simplicity with the desire of having a unique symbol for every variable, some symbols ultimately adopted for the book still represent more than one variable. We hope, however, that the meaning of all symbols is clear from the context. We denote vectors using the Times bold font and matrices using the Arial bold or bold italic font. Unit vectors are denoted by a caret, whereas tensors, dyads, and dyadics are denoted by the dyadic symbol  $\leftrightarrow$ . The Times italic font is usually reserved for scalar variables. However, the square root of minus one, the base of natural logarithms, and the differential sign are denoted by Times roman (upright) characters  $i$ ,  $e$ , and  $d$ , respectively.

A table containing the symbols used, their meaning and dimension, and the section where they first appear is provided at the end of the book, to assist the reader.

We have not attempted to compile a comprehensive list of relevant publications and often cite a book or a review article where further references can be found. In this regard, two books deserve to be mentioned specifically. The monograph by Kerker (1969) provides a list of nearly a thousand papers on light scattering published prior to 1970, while the recent book edited by Mishchenko *et al.* (2000a) lists nearly 1400 publications on all aspects of electromagnetic scattering by nonspherical and heterogeneous particles.

We provide references to many relevant computer programs developed by various research groups and individuals, including ourselves, and made publicly available through the Internet. Easy accessibility of these programs can be beneficial both to individuals who are mostly interested in applications and to those looking for sources of benchmark results for testing their own codes. Although the majority of these programs have been extensively tested and are expected to generate reliable results in most cases provided that they are used as instructed, it is not inconceivable that some of them contain errors or idiosyncrasies. Furthermore, input parameters can be used that are outside the range of values for which results can be computed accurately. For these reasons the authors of this book and the publisher disclaim all liability for any damage that may result from the use of the programs. In addition, although the publisher and the authors have used their best endeavors to ensure that the URLs for the external websites referred to in this book are correct and active at the time of going to press, the publisher and the authors have no responsibility for the websites and can make no guarantee that a site will remain live or that the content is or will remain appropriate.

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We have greatly benefited from extensive discussions with Oleg Bugaenko, Brian Cairns, Barbara Carlson, Helmut Domke, Kirk Fuller, James Hansen, Joop Hovenier, Vsevolod Ivanov, Kuo-Nan Liou, Kari Lumme, Andreas Macke, Daniel Mackowski, Alexander Morozhenko, William Rossow, Kenneth Sassen, Cornelis van der Mee, Bart van Tiggelen, Gorden Videen, Tõnu Viik, Hester Volten, Ping Yang, Edgard Yanovitskij, and many other colleagues.

We thank Cornelis van der Mee and Joop Hovenier for numerous comments which resulted in a much improved manuscript. Our computer codes have benefited from comments and suggestions made by Michael Wolff, Raphael Ruppin, and many other individuals using the codes in their research. We thank Lilly Del Valle for contributing excellent drawings and Zoe Wai and Josefina Mora for helping to find papers and books that were not readily accessible.

We acknowledge with many thanks the fine cooperation that we received from the staff of Cambridge University Press. We are grateful to Matt Lloyd, Jacqueline Garget, and Jane Aldhouse for their patience, encouragement, and help and to Susan Parkinson for careful copy-editing work.

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