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Notes

Dependence of extinction cross-section on incident polarization state and particle orientation

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ABSTRACT

This note reports on the effects of the polarization state of an incident quasi-monochromatic parallel beam of radiation and the orientation of a hexagonal ice particle with respect to the incident direction on the extinction process. When the incident beam is aligned with the six-fold rotational symmetry axis, the extinction is independent of the polarization state of the incident light. For other orientations, the extinction cross-section for linearly polarized light can be either larger or smaller than its counterpart for an unpolarized incident beam. Therefore, the attenuation of a quasi-monochromatic radiation beam by an ice cloud depends on the polarization state of the beam if ice crystals within the cloud are not randomly oriented. Furthermore, a case study of the extinction of light by a quartz particle is also presented to illustrate the dependence of the extinction cross-section on the polarization state of the incident light.

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1. Introduction

For the past several decades, the radiative effect of cirrus clouds has been a subject of active research [1–3]. The optical properties of ice crystals are fundamental for the understanding of the radiative properties of these clouds [3]; particularly, ice crystal extinction cross-section is essential for the determination of the attenuation of a radiation beam by a cirrus cloud. Unlike water droplets in warm clouds, ice crystals in the atmosphere are essentially nonspherical and their extinction cross-sections may depend on the polarization state of the incident radiation. This is an optical phenomenon analogous to the dichroism [4–6] in anisotropic materials. In addition to its sensitivity to the polarization state of radiation, the extinction of a nonspherical particle can be sensitive to the orientation of the particle with respect to the incident direction. Using spaceborne lidar observations with two beam-pointing configurations, Hu et al. [7] illustrated the presence of horizontally oriented ice plates (i.e., with their asymmetry axes oriented vertically) in the atmosphere. In the literature, the studies of the dependence of the optical properties of nonspherical ice crystals on the orientations of the particles are mainly limited to the angular distribution of the scattered electromagnetic (EM) energy [8,9]. This note is a report on the dependence of the extinction cross-section on the polarization state of radiation and the orientation of a hexagonal ice crystal based on calculations from the discrete dipole approximation (DDA) method [10]. Additionally, a similar case study for a cubic particle with the refractive index of quartz is also presented.

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2. Extinction cross-section

The extinction cross-section of a scattering particle can be calculated by the optical theorem [11]. Extensive studies of the extinction cross-section and the optical theorem have been reported in the literature. For example, Berg et al. [12] have expressed the extinction cross-section using the coefficients associated with the expansion of the scattered wave in terms of vector wave functions. Note that the extinction by a single particle and an ensemble of particles has been thoroughly investigated in conjunction with the explanation of the optical theorem by Berg et al. [13,14] and Mishchenko et al. [15]. A general form of the optical theorem that explicitly accounts for the polarization state of the incident EM radiation in terms of the Stokes parameters and the elements of the coherency extinction matrix has been reported by Mishchenko [16]. For a self-complete description, we recapture the general optical theorem using an alternative approach to the previous derivation. Specifically, we generalize a method suggested by van de Hulst [17] who derived the optical theorem without considering polarization, and explicitly accounts for the polarization state of the incident EM radiation in terms of the Stokes parameters associated with the incident EM radiation.

Assume that the incident EM wave propagates in the z-direction of the coordinate system of concern. In the far-field region, the approximation $R=(x^2+y^2+z^2)^{1/2}\approx z+(x^2+y^2)/(2z)$ is valid in the forward direction, leading to the simplification of the scattered field at scattering angle $\vartheta=0^\circ$ as follows:

$$\begin{pmatrix} E_{//sca} \\ E_{\perp sca} \end{pmatrix} = \frac{\exp[ik(R-z)]}{-ikR}\begin{pmatrix} S_2 & S_3 \\ S_4 & S_1 \end{pmatrix}\begin{pmatrix} E_{//inc} \\ E_{\perp inc} \end{pmatrix} \approx \frac{\exp[ik(x^2+y^2)/(2z)]}{-ikz}\begin{pmatrix} S_2(0^\circ)E_{//inc}+S_3(0^\circ)E_{\perp inc} \\ S_4(0^\circ)E_{//inc}+S_1(0^\circ)E_{\perp inc} \end{pmatrix}, \tag{1}$$

where we follow the notations of Bohren and Huffman [18] for the amplitude scattering matrix elements S_j, $j=1$–4.

According to the definition of the Stokes vector, (I, Q, U and V) [4], the first element of the Stokes vector or the irradiance associated with the total (incident plus scattered) wave in the forward direction is

$$I_{total} = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\Big[(E_{//inc}+E_{//sca})(E^*_{//inc}+E^*_{//sca}) + (E_{\perp inc}+E_{\perp sca})(E^*_{\perp inc}+E^*_{\perp sca})\Big], \tag{2}$$

where ε_0 and μ_0 are the permittivity and permeability in vacuum, respectively. Note, in many texts, the constant factor $I_{total}=(1/2)\sqrt{\varepsilon_0/\mu_0}$ is usually omitted in the definition of the Stokes vector because only the relative magnitudes of the Stokes parameters usually matter in many practical applications [18,19]. The irradiance in Eq. (2) can be decomposed into two components as follows:

$$I_{total} = I_{inc} - I_{ext} + O(1/z^2), \tag{3}$$

where I_{inc} is the incident irradiance and I_{ext} is associated with the extinction process, which are given by

$$I_{inc} = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}(E_{//inc}E^*_{//inc}+E_{\perp inc}E^*_{\perp inc}), \tag{4}$$

$$\begin{aligned} I_{ext} &= \sqrt{\frac{\varepsilon_0}{\mu_0}}\mathrm{Re}\Big\{\frac{\exp[ik(x^2+y^2)/(2z)}{-ikz}\Big[S_2(0^\circ)E_{//inc}E^*_{//inc} \\ &\quad + S_3(0^\circ)E_{\perp inc}E^*_{//inc}+S_4(0^\circ)E_{//inc}E^*_{\perp inc}+S_1(0^\circ)E_{\perp inc}E^*_{\perp inc}\Big]\Big\} \\ &= \mathrm{Re}\Big\{\frac{\exp[ik(x^2+y^2)/(2z)}{-ikz}(I_{inc}[S_2(0^\circ)+S_1(0^\circ)] \\ &\quad + Q_{inc}[S_2(0^\circ)-S_1(0^\circ)])+U_{inc}[S_3(0^\circ)+S_4(0^\circ)] \\ &\quad + iV_{inc}[S_3(0^\circ)-S_4(0^\circ)]\Big\}. \end{aligned} \tag{5}$$

The extinction cross-section of the particle can be calculated via

$$C_{ext} = \frac{1}{I_{inc}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I_{ext}\, dx\, dy. \tag{6}$$

From Eq. (6) and the double Fresnel integral given by

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i(x^2+y^2)}\, dx\, dy = i\pi, \tag{7}$$

it is straightforward to formulate the extinction cross-section as follows:

$$\begin{aligned} C_{ext} &= \frac{2\pi}{k^2}\mathrm{Re}\{[S_2(0^\circ)+S_1(0^\circ)]+[S_2(0^\circ)-S_1(0^\circ)](Q_{inc}/I_{inc}) \\ &\quad + [S_3(0^\circ)+S_4(0^\circ)](U_{inc}/I_{inc})+i[S_3(0^\circ)-S_4(0^\circ)](V_{inc}/I_{inc})\}. \end{aligned} \tag{8}$$

Note that the approximation $(x^2+y^2+z^2)^{1/2}\approx z+(x^2+y^2)/(2z)$ assumed in Eq. (1) can be avoided by using an alternate approach based on the asymptotic behavior [20] of the integral $\iint \exp[ikzf(x,y)]g(x,y)\, dx\, dy$ (see Bohren and Huffman [18] for details). It is evident from Eq. (8) that the extinction cross-section is determined by the amplitude scattering matrix in the forward direction and by the polarization state of the incident EM radiation, although scattering occurs in all directions and the extinction cross-section is the sum of the scattering and absorption cross-sections. If the incident radiation is unpolarized, the extinction cross-section given by Eq. (8) reduces to

$$C_{ext} = \frac{2\pi}{k^2}\mathrm{Re}[S_2(0^\circ)+S_1(0^\circ)]. \tag{9}$$

If $S_2(0^\circ)$ and $S_1(0^\circ)$ are the same [hereafter, indicated by $S(0^\circ)$] such as in the case of the scattering of light by a sphere, Eq. (9) reduces to the form of the optical theorem given in most texts as

$$C_{ext} = \frac{4\pi}{k^2}\mathrm{Re}[S(0^\circ)]. \tag{10}$$

3. Numerical results and discussion

Let the direction of the incident radiation be along the z-axis in the laboratory coordinate system and the scattering plane makes an angle of ϕ with the x–z plane. As the optical theorem (or Eq. (8)) is valid with the reference to any scattering planes, we compute the amplitude

scattering matrix in the forward scattering direction with respect to the x–z plane (i.e., $\phi=0$). The orientation of an ice particle is described through Euler angles (α, β, γ) in the laboratory coordinate system according to the common z–y–z convention (Fig. 1). Note that the rotation of the scatterer with respect to the direction of the incident plane wave is specified in terms of the angle α. Therefore, the amplitude scattering matrix with respect to the x–z plane ($\phi=0$) for the orientation (α, β, γ) is equal to that with respect to the scattering plane ($\phi=\pi-\alpha$) for the orientation (0, β, γ). In the DDA simulations, we let angle α be zero and varied the other two Euler angles, i.e., β and γ. The amplitude matrix in the forward scattering direction with respect to the x–z plane for a nonzero value of α can be obtained through a coordinate transformation. If the extinction cross-section is averaged over α, Eq. (8) reduces to the following form

$$C_{ext}=\frac{2\pi}{k^2}\mathrm{Re}\{[S_2(0^\circ)+S_1(0^\circ)]+i[S_3(0^\circ)-S_4(0^\circ)](V_{inc}/I_{inc})\}. \tag{11}$$

As the scattering by a hexagonal ice crystal has no analytical solution, it is beneficial to analyze the properties of the forward amplitude scattering matrix through symmetry relations. Hu et al. [21] systematically investigated these symmetry relations. However, when considering the symmetry relations to understand the properties of the amplitude scattering matrix simulated from the DDA method, one should keep in mind the shape errors introduced in the dipole representation of the hexagonal geometry. For example, the six-fold rotational symmetry and some planes of mirror symmetry no longer exist or are only approximately guaranteed (the accuracy depends on the number of dipoles). However, the symmetry of the mirror inversion with respect to the x–y plane and the mirror symmetry with respect to two planes shown in Fig. 2 rigorously remain, because the current DDA model employs dipoles in constructing the particle geometry (note that all the geometric symmetry relationships of a cube or box remain).

For orientations specified by (0, β and 30°n) where n is an integer, the particle has its own mirror image with respect to the x–z plane. This implies that (see van de Hulst [17]) $S_3=-S_3$ and $S_4=-S_4$, thus $S_3=S_4=0$, and Eq. (8) can be written as

$$C_{ext}=\frac{2\pi}{k^2}\mathrm{Re}\{[S_2(0^\circ)+S_1(0^\circ)]+[S_2(0^\circ)-S_1(0^\circ)](Q_{inc}/I_{inc})\}. \tag{12}$$

Considering the shape errors, the above formula has higher accuracy when $n=0$, 3, 6 and 9 than with the other integers. For the specific case of (0, 0, 0), the particle has six-fold rotational symmetry with respect to the z-axis and mirror inversion symmetry with respect to the x–y plane. The former geometric symmetry requires that the forward amplitude scattering matrix does not depend on the scattering plane and $S_1=S_2$. When the incident wave is perpendicular to the basal face of the hexagonal particle, the extinction cross-section is independent of polarization state. In the DDA simulations, the symmetry of the mirror inversion is observed perfectly, however, the 6-fold rotational symmetry is only approximately

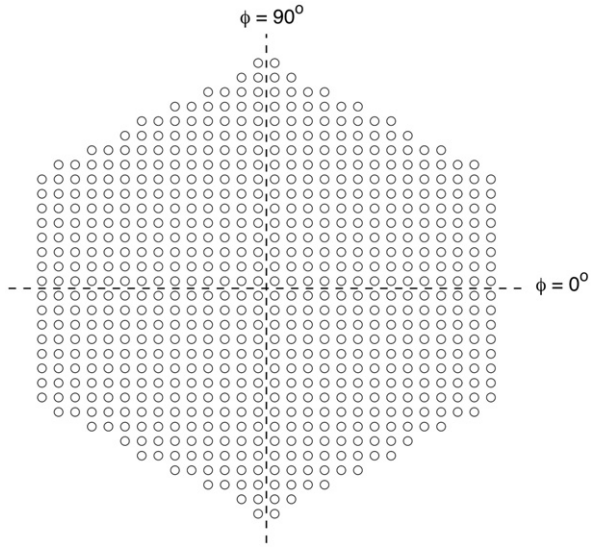

Fig. 2. Dipole representation of the geometry of a hexagonal ice particle (top view).

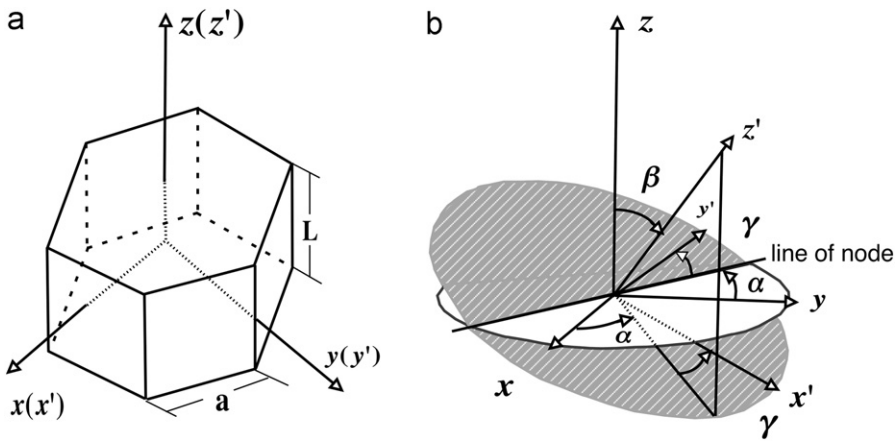

Fig. 1. Geometry of a hexagonal ice crystal (a) and Euler angles transforming the laboratory coordinate system (x, y, z) to the particle coordinate system (x', y', z') (b).

observed. Small differences between S_1 and S_2 and their scattering-plane-dependent behavior are noticed.

With the knowledge of the accuracy of the DDA method and symmetry relations in mind, we simulate the dependence of the extinction cross-section on particle orientation and the polarization state of the incident radiation beam. The ADDA code developed by Yurkin et al. [22] is employed for the simulation. To demonstrate the effect of polarization on the extinction of the radiation beam, a factor f_{ext} is defined as

$$f_{ext} = \frac{C_{ext}(1,Q/I,U/I,V/I)}{C_{ext}(1,0,0,0)}. \tag{13}$$

Fig. 3 shows f_{ext} as a function of the orientation of a hexagonal plate for the incident light with a Stokes vector of (1, 1, 0, 0). In the simulation, the aspect ratio of the hexagonal particle is unity, the refractive index is $1.3+i0.0$, and the size parameter defined in terms of the thickness of the hexagonal plate is 20. As shown in Fig. 3, f_{ext} can be either larger or less than unity, depending on the particle orientation. Note that Shefer and Popov [23] investigated the elements of the extinction matrix based on physical optics and reported that the off-diagonal elements differ from the first element by an order of magnitude.

In addition to a hexagonal ice crystal, we also conducted a case study for a cubic particle with a refractive index of $1.54+i0.0$, which represents a quartz particle. Similar to Fig. 3, Fig. 4 shows f_{ext} as a function of the orientation of the cubic particle. For the orientation with Euler angles (0, 0, 0), the x, y and z axes are perpendicular to the faces of the cube. When $\alpha=0^\circ$ and $\gamma=0^\circ$, the orientation with β is the same as the orientation with $\beta-45^\circ$. The behavior of f_{ext} with respect to β for $\gamma=0^\circ$ illustrates the aforementioned symmetry of geometry. As shown in Figs. 3 and 4, the deviation of f_{ext} from unity is larger for the quartz particle than that for the hexagonal ice crystal shown in Fig. 3 due to the larger refractive index of the quartz particle.

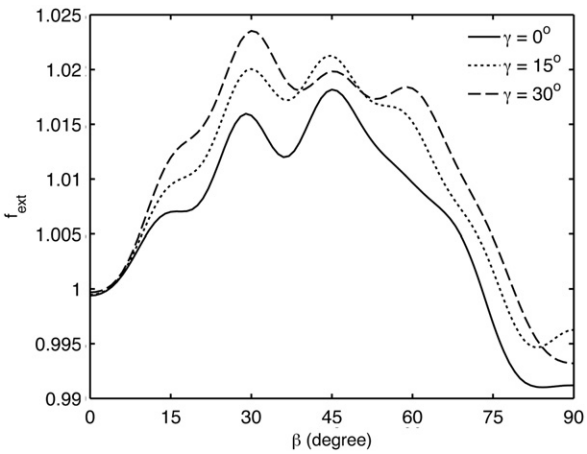

Fig. 3. Effect of the polarization state and the orientation of a hexagonal ice crystal with a refractive index of $m=1.3+i0.0$ on the extinction of light.

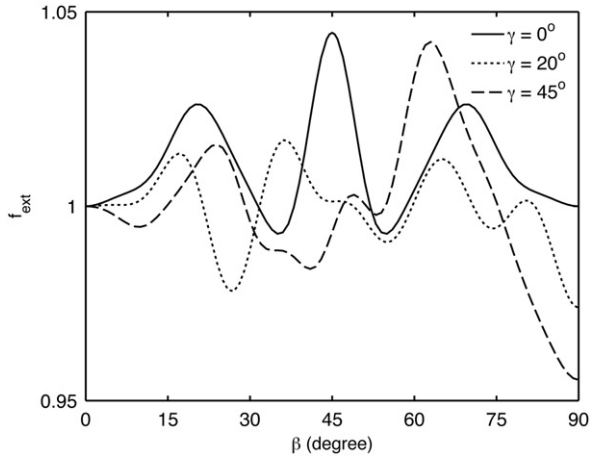

Fig. 4. Effect of the polarization state and the orientation of a cubic particle with a refractive index of $m=1.54+i0.00$ on the extinction of light.

4. Summary

In this study, the effect of the particle orientation on the amplitude scattering matrix in the forward direction is discussed in conjunction with some symmetry relationships. Based on the DDA method and a formula for the extinction cross-section, which explicitly accounts for the polarization state of the incident light in terms of the Stokes parameters, it is shown that the extinction cross-section depends on the particle orientation and the polarization state of the incident light. Specifically, the extinction cross-section for a linearly polarized radiation beam can be larger or smaller than that for an unpolarized radiation beam, depending on the orientation of the scattering particle. It should be pointed out that the complete extinction matrix [24] rather than the scalar extinction cross-section is required to rigorously characterize the attenuation of the intensity and the variation of the polarization state of the incident radiation beam in the extinction process.

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