Absorption within Inhomogeneous Clouds and Its Parameterization in General Circulation Models

BRIAN CAIRNS

Department of Applied Physics, Columbia University, New York, New York, and NASA/Goddard Institute for Space Studies, New York, New York

ANDREW A. LACIS AND BARBARA E. CARLSON

NASA/Goddard Institute for Space Studies, New York, New York

(Manuscript received 30 January 1998, in final form 5 April 1999)

ABSTRACT

The effect on absorption in clouds of having an inhomogeneous distribution of droplets is shown to depend on whether one replaces a homogeneous cloud by an inhomogeneous cloud that has the same mean optical thickness, or one that has the same spherical albedo. For the purposes of general circulation models (GCMs), the more appropriate comparison is between homogeneous and inhomogeneous clouds that have the same spherical albedo, so that the radiation balance of the planet with space is maintained. In this case it is found, using Monte Carlo and independent pixel approximation calculations, that inhomogeneous clouds can absorb more than homogeneous clouds. It is also found that because of the different effects of cloud inhomogeneity on absorption and on the transmission of the direct beam the absorption efficiency of an inhomogeneous cloud may be either greater (for low and high optical depths) or lesser (for intermediate optical depths) than that for a homogeneous cloud of the same mean optical depth. This effect is relevant both to in-cloud absorption and to absorption below clouds. In order to include these effects in GCMs a simple renormalization of the single-scattering parameters of radiative transfer theory is derived that allows the effects of cloud inhomogeneities to be included in plane-parallel calculations. This renormalization method is shown to give reasonable results when compared with Monte Carlo calculations, has the appropiate limits for conservative and completely absorbing cases, and provides a simple interpretation of the effects of cloud inhomogeneities that could readily be incorporated in a GCM.

1. Introduction

There has been increasing evidence, accumulated over the course of some four decades, that shortwave absorption by clouds is greater than that predicted by model calculations. This phenomenon has been termed the cloud absorption anomaly and a detailed review of the various contributions to the study of the absorption of solar radiation in clouds has been given by Stephens and Tsay (1990). More recent investigations (Cess et al. 1995; Ramanathan et al. 1995) appear to reveal even larger absorption effects, though the spectrally unresolved nature of these studies makes it difficult to speculate on the physical mechanism(s) responsible for their findings. One aspect of these studies that is of interest is the comparison of theoretical calculations with experimental observations. A hierarchy of approximations exists that may be used in theoretical calculations of radiation transport, some more accurate than others. As Hignett (1987) noted regarding measurements related to anomalous absorption, when ''more computational effort is expended, then better agreement can be obtained in any particular case.'' Given that radiative transfer is an asymptotic approximation to Maxwell's equations (Furutsu 1963; Barabenkov and Finkel'berg 1968) any differences between theoretical calculations and experimental observations (assuming negligible experimental error) of cloud properties must be related to one, or more, of the following: the equation of radiative transfer is being inadequately modeled; the material constants and microphysical information that determine the scattering and absorbing properties of the cloud are not being correctly specified; and/or the equation of radiative transfer is not a good approximation to Maxwell's equations. The latter appears unlikely given that the correction to radiative transfer is asymptotic in the parameter λ/l (10^{-8} for solar radiation), where λ is the wavelength and l is the scattering mean free path (John 1985).

Corresponding author address: Dr. Brian Cairns, NASA/Goddard Institute for Space Studies, 2880 Broadway, New York, NY 10025.
E-mail: bcairns@giss.nasa.gov

In this paper we assume that the material constants (Hale and Querry 1973; Palmer and Williams 1974; Downing and Williams 1975) and scattering properties of water droplets are well known and devote our attention to the effect that different approximations to the transfer of radiation have on the amount of absorption within an inhomogeneous cloud. The radiative transfer models that we compare range from a Monte Carlo simulation of transport through an inhomogeneous cloud to the single Gauss point (SGP) approximation for plane-parallel transport through a homogeneous cloud (Hansen et al. 1983). We also introduce a theoretical renormalization of the properties of inhomogeneous clouds that can be used to correct plane-parallel calculations. This type of renormalization allows the effects of cloud inhomogeneity to be included in the simple two stream (King and Harshvardhan 1986) and more complex and accurate SGP approximations to radiative transfer that are used in current general circulation models (GCMs), without any modification to these algorithms.

2. Radiation and clouds

The problem of radiative transfer through inhomogeneous media has been of interest in the field of atmospheric radiation (McKee and Cox 1974; Romanova 1978) and in other fields for many years (Williams 1974, 1984; Borovoi 1984). For atmospheric radiation the principal inhomogeneous medium problem is that of radiative transfer through non-plane-parallel clouds, although radiative transfer through vegetation is also of some interest (Myneni et al. 1991). The transfer of radiation through non-plane-parallel clouds has been analyzed using Monte Carlo simulations (Barker and Davies 1992; Kobyashi 1993; O'Brien 1992; Jonas 1994), simple closure schemes for radiative transfer in binary mixtures (Boissé 1990; Malvagi and Pomraning 1990; Titov 1990), and other simplifying assumptions (Davis et al. 1990; Evans 1993; Peltoniemi 1993; Cahalan et al. 1994a). None of the various models of cloud fields used in these studies provides a definitive description of broken cloud fields, each having its own positive and negative aspects. We therefore choose to use Fourier power-law filtering of Gaussian noise to give a Kolmogorov (1941) spectrum, which is then exponentiated (Evans 1993) to define a droplet density distribution. Although a field of this type is not, strictly speaking, scale invariant (Schertzer and Lovejoy 1987) it has easily controlled lognormal statistics and a three-dimensional spatial structure that is in some respects similar to internal cloud structure. The statistics of such a cloud field are consistent with those observed in marine stratocumulus clouds (Cahalan et al. 1994b), while retaining both transverse (Cahalan et al. 1994a) and vertical structure (Stephens et al. 1991) of the droplet density distribution, both of which may be important for determining the average radiant properties of a cloud field. This model is appropriate for considering the effect of internal cloud inhomogeneities on absorption in clouds. By contrast, binary mixtures may be an appropriate model for broken cumulus cloud fields, though they appear to provide only a lower bound on the difference between broken cloud fields and corresponding plane-parallel calculations (Hobson and Scheuer 1993).

Initially we compare three different radiation calculations, Monte Carlo, independent pixel approximation (IPA), and plane-parallel, which are supposed to model the transfer of radiation through random distributions of cloud droplets generated on a $256 \times 256 \times 16$ grid. The number of grid points used in the vertical is adjusted to maintain the same physical scale of fluctuations in the distribution of cloud droplets for different mean optical depths. The Monte Carlo radiation calculations use an optimally biased code (Marchuk et al. 1980), with periodic boundary conditions in the horizontal plane, and trace the path of photons through the actual droplet density distribution. Most of the Monte Carlo results are based on an ensemble of calculations, or, as in the case of the spherical albedo, are an integral over a number of results. As expected, the largest relative variance in the Monte Carlo results was found to be for very thick clouds, $\tau = 256$, and was 2.5%, or less. The IPA (Ronnolm et al. 1980; Cahalan 1989) assumes that each vertical column is independent of the surrounding columns. This assumption is also made in the satellite retrieval of cloud properties (Rossow and Schiffer 1991). The histogram of cloud optical depths required by the IPA is therefore generated by integrating the droplet density distribution for each vertical column. In the following examples the IPA is calculated by discretizing the histogram of optical thicknesses into 300 bins and performing a radiation calculation for each thickness bin using a plane-parallel doubling–adding code (Hansen and Travis 1974). The IPA result is then given by the weighted sum of these results. The average cloud optical depth used in the plane-parallel calculation is simply the horizontally and vertically averaged optical depth, the radiation calculations being performed using the doubling/adding code. These calculations correspond to the calculation of radiative transfer through the actual medium, through a vertically averaged medium with no intercolumn coupling, and through a vertically and horizontally averaged medium, respectively.

As an example of the type of variability that is produced in the cloud model that we are using, a horizontal transect of the variability in extinction is shown in Fig. 1. Such variability in extinction, assuming a droplet effective radius of approximately 10 μm, is consistent with measurements that have been made by microphysics probes flown within clouds (Korolev and Mazin 1993) and lies within the range of values found by other microphysical measurements (Nakajima et al. 1990). In Fig. 2 we show the effect that redistributing the liquid water droplets has on the spherical albedo (Fig. 2a) and the absorption in a cloud (Fig. 2b), for various mean optical depths, assuming for all calculations a Henyey–

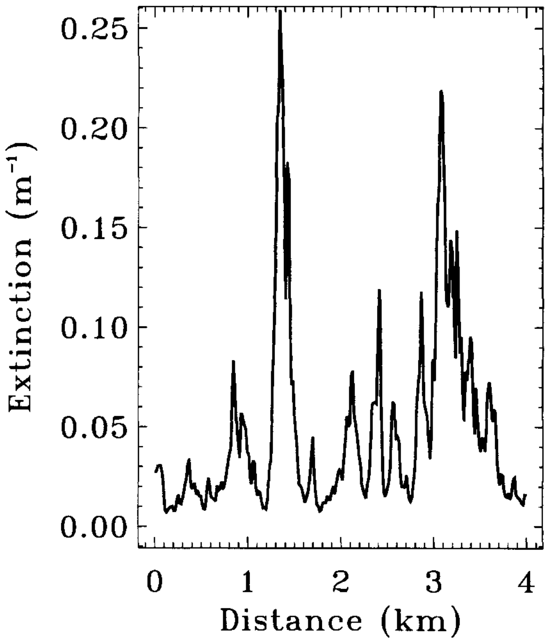

FIG. 1. Variation of extinction with distance through an inhomogeneous cloud field of the type used in this paper, with $\delta = 1.0$.

Greenstein phase function with an asymetry parameter, $g = 0.85$; a single scatter albedo, $\varpi = 0.99$; and a log standard deviation of the droplet density, $\delta = 1.0$. In Fig. 2a it is apparent that, for the same mean optical thickness, a homogeneous cloud reflects more radiation than a cloud composed of inhomogeneously distributed droplets. We also observe that the IPA underestimates the magnitude of the reduction in spherical albedo by cloud inhomogeneities, presumably because of its neglect of vertical structure and intercolumn coupling (Audic and Frisch 1993). Figure 2b shows that, for the same mean optical thickness, an inhomogeneous cloud absorbs less radiation than a cloud in which the droplets are homogeneously distributed, and that the IPA provides a good estimate of the absorption in such a cloud. These figures also indicate that reducing the optical depth of the homogeneous cloud so that its spherical albedo matches that of the inhomogeneous cloud cannot simultaneously provide a reliable estimate of absorption in the inhomogeneous cloud. Similar numerical results were obtained by Stephens and Tsay (1990) for deterministic cloud fields.

At this point we might conclude that the presence of inhomogeneities in the distribution of water within a cloud reduces the amount of absorption in the cloud. However, the question remains as to whether we should compare homogeneous and inhomogeneous clouds with the same mean liquid water content or compare the absorption in homogeneous and inhomogeneous clouds that have the same spherical albedo. This is an important point given that the cloud albedo is probably more accurately measured than the liquid water content (Nakajima et al. 1990). Also relevant is the observation (Harshvardhan and Randall 1985) that in order to have models that are in radiative balance with space, the water content in GCM clouds is generally low compared to observed liquid water contents. We should, however, note that in GCMs with prognostic liquid water parameterizations the radiative heating for a synoptic-scale region over a period of several days is effectively calculated using an IPA over a distribution of cloud optical properties (Del Genio et al. 1996). Thus, although a plane-parallel bias will exist in GCMs at smaller space scales and timescales the magnitude of the problem is probably not as great as has been previously suggested (Cahalan et al. 1994a). Nonetheless, if GCMs had inhomogeneous clouds at a grid box level, then any residual bias in the water content of these clouds could be removed while maintaining the same planetary al-

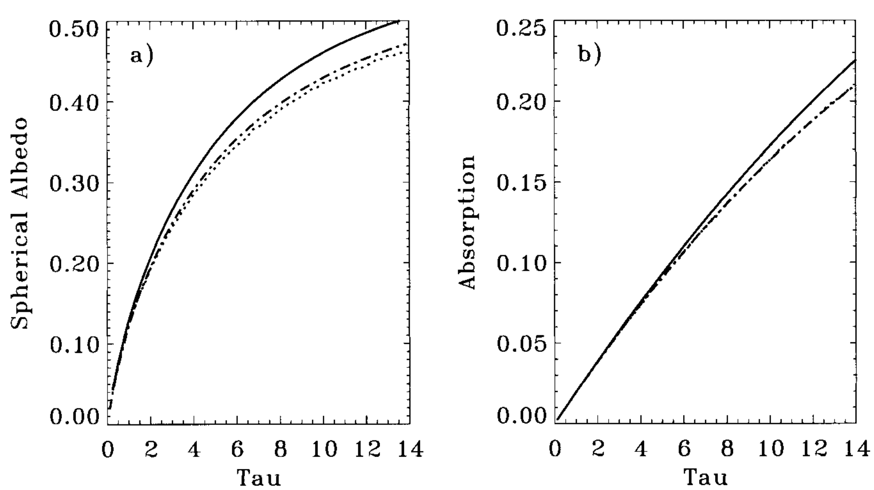

FIG. 2. Variation of (a) spherical albedo and (b) absorption with mean optical thickness for a homogeneous cloud (solid line), and for an inhomogeneous cloud as calculated using the Monte Carlo method (dotted line) and the IPA (dotted–dashed line).

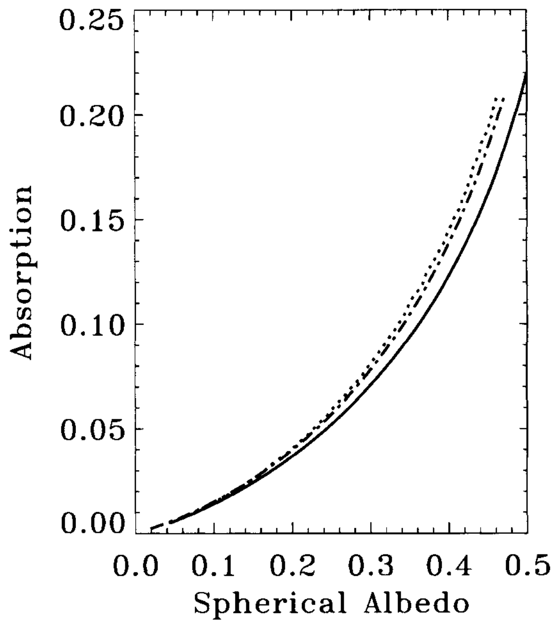

FIG. 3. Variation of absorption with spherical albedo for a homogeneous cloud (solid line), and for an inhomogeneous cloud as calculated using the Monte Carlo method (dotted line) and the IPA (dotted–dashed line).

bedo and radiation balance with space (cf. Fig. 2a). The comparison of the amount of absorption in homogeneous and inhomogeneous clouds with the same spherical albedo is therefore of particular relevance to GCMs.

In Fig. 3 we compare the absorption in clouds with an inhomogeneous distribution of water with the absorption in homogeneous clouds having the same spherical albedo. For the same spherical albedo, we see that there is more absorption in inhomogeneous clouds than in homogeneous clouds. A simple explanation for this effect is that absorption is less affected by cloud inhomogeneities than scattering. This means that an inhomogeneous cloud that has had its mean optical thickness increased, so that it has the same spherical albedo as a homogeneous cloud, will absorb more than the homogeneous cloud. For the same spherical albedoes we see that the IPA underestimates the increase in absorption in inhomogeneous clouds, principally because it overestimates the spherical albedo (cf. Fig. 2a). Given recent results concerning the cloud absorption anomaly (Cess et al. 1995; Ramanathan et al. 1995), the increase in absorption that can be obtained from inhomogeneous cloud fields is interesting in its own right (Kliorin et al. 1990; Li et al. 1995). Nonetheless one might question how generally applicable such results are, as well as question their relevance to either remote sensing or climate model parameterizations, given that the results may depend sensitively on the model cloud fields chosen. In the next section we introduce a renormalization procedure that allows the effects of cloud inhomogeneities to be included in plane-parallel calculations and allows us to address the issue of the generality of the effects caused by cloud inhomogeneities.

3. Renormalization theory

There have been many attempts to obtain simple approximations to the propagation of light through randomly inhomogeneous media, some of which make statistical assumptions without providing any analysis of the errors implicit in such assumptions (Anisimov and Fukshansky 1992; Kliorin et al. 1990), while others have been developed for isotropic scattering phase functions (Malvagi and Pomraning 1990; Hobson and Scheuer 1993) and only have simple forms for binary mixtures. Using standard methods from statistical physics (Abrikisov et al. 1963; Feynman and Hibbs 1965; Arya and Zeyher 1983) it is possible to obtain a closed form solution for the effect of fluctuations in the droplet density on the transport of radiation through a randomly inhomogeneous medium. Although this solution is in the form of a nonlocal integro-differential equation that is not readily soluble, an expansion can be obtained that allows us to calculate the lowest-order consistent correction to plane-parallel transport theory and demonstrates that such a correction is a universal feature of radiative transfer in randomly inhomogeneous media.

We will now briefly define the problem of radiative transfer in a general inhomogeneous random medium and present the solution of the problem for the mean specific intensity. It should be noted that similar results can be obtained for the higher-order moments of the specific intensity, which are relevant to satellite remote sensing (Cairns 1992). Our starting point is the equation of radiative transfer for the specific intensity $I(\mathbf{r}, \mathbf{s})$ in an inhomogeneous medium,

$$\mathbf{s} \cdot \nabla I(\mathbf{r}, \mathbf{s}) + \sigma_{\text{ext}} N(\mathbf{r}) \int_{4\pi} B(\mathbf{s} \cdot \mathbf{s}') I(\mathbf{r}, \mathbf{s}')\, d\mathbf{s}' = 0, \quad (1)$$

where σ_{ext} is the extinction cross section; $N(\mathbf{r})$ is the number concentration of scatterers; $B(\mathbf{s} \cdot \mathbf{s}')$ is the Boltzmann collision operator; and $\mathbf{r}$ and $\mathbf{s}$ are, respectively, a three-dimensional position vector and a two-dimensional direction vector. The Boltzmann collision operator is defined to be

$$B(\mathbf{s} \cdot \mathbf{s}') = \delta(\mathbf{s} \cdot \mathbf{s}') - \varpi p(\mathbf{s} \cdot \mathbf{s}'), \quad (2)$$

where $\delta(\mathbf{s} \cdot \mathbf{s}')$ is the solid angle delta function, ϖ is the single scatter albedo, and $p(\mathbf{s} \cdot \mathbf{s}')$ is the phase function. Although this is not standard notation for atmospheric radiative transfer it provides a compact and natural notation for the following analysis and it is, in fact, an eigenanalysis of a discretized version of the Boltzmann collision operator on which the commonly used discrete ordinates method is based.

The solution of (1) with appropriate boundary conditions, for the unaveraged specific intensity $I(\mathbf{r}, \mathbf{s})$, describes the distribution of incident and emitted photons that will be transported through the particular distribution of droplets $N(\mathbf{r})$, to the point $\mathbf{r}$ traveling in the direction $\mathbf{s}$. However, we are generally not interested in how a photon is transported through a particular cloud, but rather in the mean transport properties for a statistically representative ensemble of clouds. We therefore need to calculate the mean specific intensity, based on the statistical properties of the cloud fields that we are interested in. Since we are interested in the effects of inhomogenities in the scatterer concentration on the transport of radiation, it is useful to separate the number concentration of scatterers into its mean N, which is spatially invariant, and a spatially fluctuating random component of zero mean $\eta(\mathbf{r})$, namely,

$$N(\mathbf{r}) = N + \eta(\mathbf{r}). \tag{3}$$

We will assume for the purposes of this analysis that the scatterers are bounded by a slab geometry within which N is constant, so that the geometry admits the usual well-known solutions for plane-parallel problems when there are no fluctuations. Smooth variations of N that are slow compared with the mean diffusion length do not affect the following results and are more naturally the subject of the independent pixel approximation than the method presented here. We can now average (1) to obtain an expression for the mean specific intensity:

$$\mathbf{s}\cdot\boldsymbol{\nabla}\langle I(\mathbf{r},\mathbf{s})\rangle + \sigma_{\mathrm{ext}} N \int_{4\pi} B(\mathbf{s}\cdot\mathbf{s}')\langle I(\mathbf{r},\mathbf{s}')\rangle\, d\mathbf{s}'_1$$
$$= -\sigma_{\mathrm{ext}} \int_{4\pi} B(\mathbf{s}\cdot\mathbf{s}')\langle \eta(\mathbf{r}) I(\mathbf{r},\mathbf{s}')\rangle\, d\mathbf{s}', \tag{4}$$

where the angle brackets denote an ensemble average over random fluctuations in $\eta(\mathbf{r})$. This is not a closed equation for the mean specific intensity because of the presence of the term $\langle \eta(\mathbf{r}) I(\mathbf{r},\mathbf{s}')\rangle$ on the right-hand side of (3). This coupling of the mean specific intensity to a higher-order moment is a classic problem in fluid mechanics, statistical physics, and quantum mechanics (Abrikisov et al. 1963; Feynman and Hibbs 1965; Frisch 1968; Arya and Zeyher 1983) and can be dealt with using a number of different techniques. Conceptually, the simplest method is to rewrite (1) as an integral equation,

$$I(\mathbf{r},\mathbf{s}) = I_0(\mathbf{r},\mathbf{s}) - \sigma_{\mathrm{ext}} \int_V \int_{4\pi} G_0(\mathbf{r},\mathbf{s};\mathbf{r}',\mathbf{s}') \int_{4\pi} B(\mathbf{s}',\mathbf{s}'')\eta(\mathbf{r}') I(\mathbf{r}',\mathbf{s}'')\, d\mathbf{s}''\, d\mathbf{s}'\, d\mathbf{r}', \tag{5}$$

where $I_0(\mathbf{r},\mathbf{s})$ is the solution of (1), with appropriate boundary conditions, when there are no fluctuations in the scatterer concentration and the mean concentration of scatterers is N, that is, the solution of the usual plane-parallel problem. The function $G_0(\mathbf{r},\mathbf{s};\mathbf{r}',\mathbf{s}')$ is the associated Green's function for this problem (Rybicki 1971; van der Mark et al. 1988). If we perform a perturbation expansion of (5) and then average this series term by term, we obtain a solution for the mean specific intensity in terms of an infinite sum over statistical moments of $\eta(\mathbf{r})$ of all orders. The terms in this expansion can be implicitly resummed to give an expression for the mean specific intensity that has the functional form

$$\mathbf{s}\cdot\boldsymbol{\nabla}\langle I(\mathbf{r},\mathbf{s})\rangle + \sigma_{\mathrm{ext}} N \int_{4\pi} B(\mathbf{s}\cdot\mathbf{s}')\langle I(\mathbf{r},\mathbf{s}')\rangle\, d\mathbf{s}'$$
$$= \int_V \int_{4\pi} Q(\mathbf{r},\mathbf{s};\mathbf{r}',\mathbf{s}')\langle I(\mathbf{r}',\mathbf{s}')\rangle\, d\mathbf{s}'\, d\mathbf{r}'. \tag{6}$$

This perturbative expansion and the approach to the implicit summation is described in the appendix. It should be emphasized that although (6) is an exact formal solution to the problem of obtaining an equation for the mean specific intensity, in general only approximations to the function $Q(\mathbf{r},\mathbf{s};\mathbf{r}',\mathbf{s}')$ can be determined. Nonetheless, the functional form of (6) is useful in that it demonstrates that the effect of fluctuations in the scatterer concentration is to turn the equation of radiative transfer into one where the collision process is effectively nonlocal in space. This means that if (6) were solved using a Monte Carlo method, then the model would have to allow for the probability of scattering being conditional on the location of the previous scattering event. The necessity of including such conditional processes in modeling radiative transfer in inhomogeneous media has previously been noted by Evans (1993).

The perturbative method noted above and discussed in the appendix is essentially a diagrammatic resummation (Feynmann and Hibbs 1965) and can be used to derive and to analyze the conditions of validity of approximate expressions for the mean specific intensity that have been justified elsewhere by ''dishonest'' (Keller 1962) closure assumptions (Anisimov and Fukshansky 1992; Kliorin et al. 1990). Furthermore, aspects of scattering that are particular to randomly inhomogeneous media, such as the hot spot effect, are naturally included in such an iterative expansion and may readily be identified using a diagrammatic analysis (Frisch 1968). The backscattering direction is a special direction in the case of inhomogeneous distributions of infinitesimal scatterers because of the complete correlation between fluctuations along the forward- and backward-scattering paths. This contrasts with the hot spot in vegetation, which is due to the effects of shadowing

(Myneni et al. 1991), or coherent backscattering, which is a result of the effects of phase coherence (MacKintosh and John 1989; van der Mark et al. 1988).

To demonstrate that the formal solution (6) has simple, readily calculable consequences, we will use one particular infinite subseries of terms in an estimate of the function $Q(\mathbf{r}, \mathbf{s}; \mathbf{r}', \mathbf{s}')$. This approximation, sometimes known as the nonlinear approximation (Rosenbaum 1971; Frisch 1968), yields a transport equation for the mean specific intensity:

$$\mathbf{s}\cdot\boldsymbol{\nabla}\langle I(\mathbf{r}, \mathbf{s})\rangle + \sigma_{\text{ext}} N \int_{4\pi} B(\mathbf{s}\cdot\mathbf{s}')\langle I(\mathbf{r}, \mathbf{s}')\rangle\, d\mathbf{s}'$$
$$= \sigma_{\text{ext}}^2 \int_V \int_{4\pi} \int_{4\pi} \int_{4\pi} B(\mathbf{s}\cdot\mathbf{s}')\langle G(\mathbf{r}, \mathbf{s}'; \mathbf{r}', \mathbf{s}'')\rangle\langle \eta(\mathbf{r})\eta(\mathbf{r}')\rangle B(\mathbf{s}''\cdot\mathbf{s}''')\langle I(\mathbf{r}', \mathbf{s}''')\rangle\, d\mathbf{s}'\, d\mathbf{s}''\, d\mathbf{s}'''\, d\mathbf{r}'. \tag{7}$$

Evidently, for a homogeneous medium the covariance of the droplet density distribution $\langle \eta(\mathbf{r})\eta(\mathbf{r}')\rangle$ is zero and we recover the usual plane-parallel radiative transfer equation. The reason that this approximation is known as the nonlinear approximation is because the Green's function $\langle G(\mathbf{r}, \mathbf{s}'; \mathbf{r}', \mathbf{s}'')\rangle$ is the Green's function associated with the solution of (7). As we noted for (6), the effect of fluctuations in scatterer concentration is to make (7) a nonlocal transport equation, which can also be written as a pair of coupled transport equations with structure similar to those for a two-state binary mixture (Malvagi et al. 1993). However, if we make the following approximation,

$$\int_V \langle G(\mathbf{r}, \mathbf{s}'; \mathbf{r}', \mathbf{s}'')\rangle\langle \eta(\mathbf{r})\eta(\mathbf{r}')\rangle\langle I(\mathbf{r}', \mathbf{s}''')\rangle\, d\mathbf{r}'$$
$$\approx \int_V \langle G(\mathbf{r}, \mathbf{s}'; \mathbf{r}', \mathbf{s}'')\rangle\langle \eta(\mathbf{r})\eta(\mathbf{r}')\rangle\, d\mathbf{r}'\langle I(\mathbf{r}, \mathbf{s}''')\rangle, \tag{8}$$

we can reduce (7) to the usual type of transport equation with a spatially local collision integral and renormalized single scattering parameters. This approximation may be justified by observing that the contribution of the diffuse (long range) part of the mean Green's function $\langle G(\mathbf{r}, \mathbf{s}'; \mathbf{r}', \mathbf{s}'')\rangle$, in the integrals in (8), is of the order $\varpi^2(1 - \varpi)$ compared with its coherent, or direct beam, contribution (Frisch 1968). A more detailed analysis of the behavior of the mean Green's function that includes ballistic transport (MacKintosh and John 1989) does not significantly alter this result. Thus, although the transport of radiation in the solar spectrum tends to be dominated by multiple scattering interactions that are long ranged, the dominant part of the correction term given by (8) is short ranged and may therefore be considered to be local in nature. In essence, the multiply scattered diffuse radiation self-averages over the cloud variability. This observation is born out by the work of Gabriel and Evans (1996) who found that evaluating the source term exactly for an inhomogeneous cloud and then performing multiple scattering calculations for the average cloud properties provides a reasonable approximation to exact calculations of radiative transfer through inhomogeneous cloud fields.

When the approximation given by (8) is substituted into (7) the following renormalized single scattering parameters, denoted by a prime, are obtained:

$$\sigma'_{\text{ext}} = \sigma_{\text{ext}}(1 - \varepsilon), \tag{9a}$$

$$\varpi' = \varpi\left[1 - \frac{\varepsilon}{1 - \varepsilon}(1 - \varpi)\right], \quad \text{and} \tag{9b}$$

$$p'(\mathbf{s}\cdot\mathbf{s}') = \left[p(\mathbf{s}\cdot\mathbf{s}') - \frac{\varepsilon}{1-\varepsilon} \times \left(p(\mathbf{s}\cdot\mathbf{s}') - \varpi\int_{4\pi} p(\mathbf{s}\cdot\mathbf{s}'')p(\mathbf{s}''\cdot\mathbf{s})\, d\Omega''\right)\right] \div \left[1 - \frac{\varepsilon}{1-\varepsilon}(1 - \varpi)\right]. \tag{9c}$$

The self-consistent expression for the correction factor, ε, that is obtained by substituting (8) into (7) and solving the nonlinear equation for the coherent part of the mean Green's function is

$$\varepsilon = \frac{\alpha - \sqrt{\alpha^2 - 4V}}{2}, \tag{10a}$$

where V is the relative variance

$$V = \frac{\langle \eta(\mathbf{r})\eta(\mathbf{r})\rangle}{N^2} \quad \text{and} \tag{10b}$$

$$\alpha = \frac{1 + \sigma_{\text{ext}} l_c}{\sigma_{\text{ext}} l_c}. \tag{10c}$$

For thin clouds the integral on the right-hand side of (8) is defined by the optical depth of the cloud. This modifies the renormalization of single scattering parameters given above such that as the cloud becomes thinner the effects of renormalization become smaller (Borde and Isaka 1996). One expects this behavior on the grounds that when single scattering dominates the radiative interaction there should be no renormalization. This effect is given by multiplying the correction term ε by the factor $[1 - \exp(\tau/\cos\theta_0)]$ where τ is the mean optical depth of the cloud and θ_0 is the solar zenith angle.

If $\alpha^2 > 4V$, (10a) can be expanded in a Taylor series. The validity of (10a) as a resummation of a subset of terms in approximating the function $Q(\mathbf{r}, \mathbf{s}; \mathbf{r}', \mathbf{s}')$ can then be checked by direct calculation. In calculating the correction term in (10) we have assumed that the random medium is statistically isotropic and homogeneous with an effective correlation length l_c (Rytov et al. 1989), though these restrictions can easily be removed. Statistical anisotropy of the random medium can be included in this formalism and its effect is to give a renormalized extinction length and single scatter albedo that depend on the direction of propagation and a renormalized phase function that is anisotropic. Spatial variation in the statistical properties of the droplet concentration can also be included in the effectively plane-parallel renormalization method by using a quasi-homogeneous model (Rytov et al. 1989) of statistical variability. Similar expressions for the renormalized coefficients were given by Kliorin et al. (1990), although their expression for the correction factor is not self-consistent and only reduces to that given here when $V \ll 1$ and $\alpha \gg 1$. It should be noted that (9) and (10) make no assumption regarding the statistics of the droplet density distribution. They are generic corrections to plane-parallel radiative transfer, caused by introducing weak variability into the distribution of droplets in a cloud, and only depend on the second-order statistics of the droplet density distribution.

There are a number of aspects of the renormalization equations (9) and (10) that should be noted. The atomic mix result, that very rapid fluctuations in optical properties should have no effect on the radiative transfer equation, is correctly retrieved since as the correlation length l_c tends to zero the renormalization correction ε also tends to zero. There is no renormalization of the single scatter albedo for purely scattering or purely absorbing media, and when the initial phase function is isotropic there is no renormalization of the phase function. This means that no unrealistic modification of the single scattering properties is caused by this renormalization.

A limit of this perturbative approach is demonstrated by the fact that if the relative variance becomes too large, the correction term given by (10) becomes imaginary. This is simply an indication that the infinite sum used to approximate the function $Q(\mathbf{r}, \mathbf{s}; \mathbf{r}', \mathbf{s}')$ is inadequate and that higher-order statistics than the second need to be included in the evaluation of $Q(\mathbf{r}, \mathbf{s}; \mathbf{r}', \mathbf{s}')$, which can readily be done. Since no closure assumptions were made in deriving (7), we can always check whether any given order of approximation is adequate and estimate the corrections to (9) and (10) required to include higher-order terms. In the random medium we are considering in this paper, the correlation length is of the same order as the mean free path and the droplet density fluctuations have lognormal statistics. For this case a better estimate of the renormalization equations than (10) that fits a simple functional form to the correction terms obtained from higher-order moments is

$$\sigma'_{\text{ext}} = \sigma_{\text{ext}}(1 + V)^{-1} \tag{11a}$$

$$\varpi' = \varpi/[1 + V(1 - \varpi)] \quad \text{and} \tag{11b}$$

$$g' = g[1 + V(1 - \varpi)]/[1 + V(1 - \varpi g)]. \tag{11c}$$

In these equations, V is the relative variance (10b), which is related to δ, the log standard deviation of the droplet density distribution, by the formula $V = \exp(\delta^2) - 1$. Rather than renormalizing the phase function, in (11c) we give a renormalization equation for the asymmetry parameter g. This is done because higher-order corrections to the phase function require successively higher-order convolutions [cf. (10c)] that are not readily calculable. If we assume that the phase function is well approximated by a Henyey–Greenstein phase function with asymmetry parameter g, the expression (14c) provides a renormalized asymmetry parameter.

The renormalization equations (11) show the usual feature of radiative transfer though a randomly inhomogeneous medium. The medium appears more transparent than a homogeneous medium with the same mean droplet density, since the effective extinction length is greater (Dolin 1984; Borovoi 1984). The effective single scatter albedo is reduced, which has been noted for binary mixtures (Boisse 1990; Hobson and Scheuer 1993) and in fractal cloud calculations (Borde and Isaka 1996), and the effective phase function is more isotropic (Kliorin et al. 1990; Cairns 1992), which is the manifestation of limb brightening as a result of inhomogeneities in this method. The poor results that have been observed when only the extinction length is renormalized (Barker 1992; Evans 1993; Audic and Frisch 1993; Cahalan et al. 1994a) are therefore not unexpected, since such an ad hoc method for including the effect of cloud inhomogeneities is inconsistent with the equations of radiative transfer. The renormalization of single scattering parameters that we have derived here provides a theoretical basis for the type of ''closures'' (Stephens 1988) and renormalizations (Borde and Isaka 1996) that have been presented previously. In the following section we will compare the simple, effectively plane-parallel renormalization equations for radiative transfer through an inhomogeneous cloud, with the more numerically intensive IPA and Monte Carlo methods.

4. Using the renormalization method

In this section we apply the renormalization method to calculate scattering from an inhomogeneous medium that has lognormal statistics, as discussed in section 2. We use (11) to obtain renormalized single scattering pa-

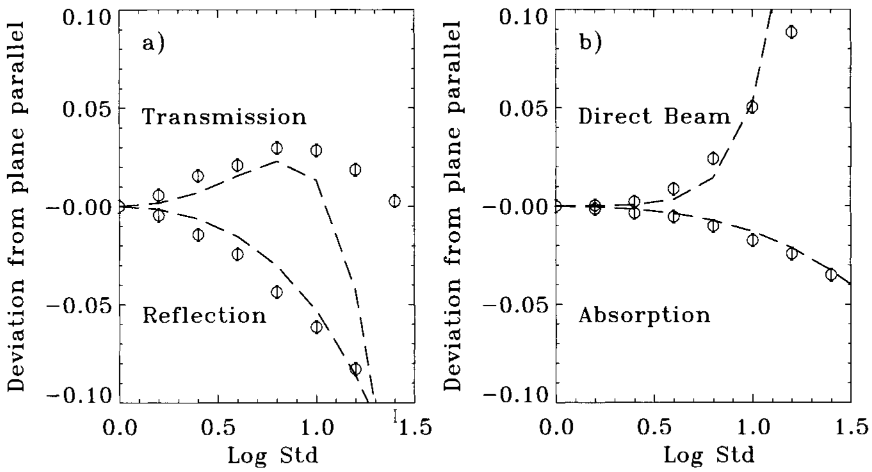

FIG. 4. Deviation of (a) diffuse reflection and diffuse transmission coefficients and (b) direct transmission and absorption coefficients for an inhomogeneous cloud from those for a homogeneous cloud with the same mean optical thickness, as a function of the log standard deviation of the droplet density distribution for $\tau = 8$, $\varpi = 0.99$, $\mu_0 = 1.00$, and $g = 0.85$. The renormalization method is shown with a dashed line and the Monte Carlo method by circles with error bars.

rameters (single scatter albedo and asymmetry parameter) and optical depth, that are then used in a doubling–adding plane-parallel radiative transfer code (Hansen and Travis 1974) to calculate the mean reflection and transmission from the inhomogeneous random medium.

The range of log standard deviations over which the renormalization method is valid is shown in Fig. 4, for a cloud of mean optical thickness $\tau = 8$ (two grid boxes used in vertical for Monte Carlo calculations), asymmetry parameter $g = 0.85$, and single scatter albedo $= 0.99$, with a solar zenith angle of $\mu_0 = 1.0$. In this figure the deviation from their plane-parallel values of the fractional flux reflected and the fractional flux diffusely (Fig. 4a) and the fractional flux directly transmitted and the fractional flux absorbed (Fig. 4b) are plotted as a function of the log standard deviation of the droplet density variations. The usual reduction in reflection and increase in transmission for an inhomogeneous cloud compared with a plane-parallel cloud with the same mean optical depth can be seen in this figure. The dashed line represents the renormalization method and the circles represent Monte Carlo calculations, with one-sigma-error bars estimated from five realizations of the inhomogeneous cloud field. It is apparent that at a log standard deviation of $\delta > 1.0$ the renormalization method starts to break down. In particular, the partitioning between diffuse and direct transmission is incorrectly modeled, although the estimates of absorption and reflection continue to be reasonable. However, up to a log standard deviation of $\delta \approx 1.0$ the renormalization method gives fairly good results, which is encouraging given its extreme simplicity. To put these values in context, we note that Cahalan et al. (1994a) estimated that the log standard deviation of the liquid water path in stratocumulus was $\delta = 0.4$ and that it would be necessary to have a log standard deviation $\delta = 0.7$ in order to have albedos in GCMs that are consistent with observed liquid water paths. If such a range of variability of liquid water paths can be considered representative, then the renormalization method would appear to provide a useful model for the modification of radiative transfer by inhomogeneities in the distribution of droplets in clouds. It should, however, be noted that the liquid water is distributed inhomogeneously in three dimensions in the calculations presented here, whereas Cahalan et al. (1994a) use a one-dimensional distribution of variability. The effects of inhomogeneity that we find here are therefore more directly comparable to the work of Evans (1993) and Borde and Isaka (1996).

Since the renormalization method appears to work adequately for a log standard deviation of $\delta = 1.0$, Fig. 5 shows the results of applying this method to the problem discussed in section 2. The curves in Fig. 5a use the same set of parameters as in Fig. 3, namely, $g = 0.85$, $\varpi = 0.99$ with $\delta = 1.0$, and show the difference in absorption caused by replacing a plane-parallel cloud with an inhomogeneous cloud of the same spherical albedo. In this case the renormalization method, although underestimating the effect of cloud inhomogeneities on absorption, is comparable in accuracy to the IPA. Figure 5b shows the difference in absorption caused by replacing a plane-parallel cloud with an inhomogeneous cloud of the same spherical albedo, for the same phase function, $g = 0.85$, and degree of inhomogeneity, $\delta = 1.0$, as Fig. 5a but with a reduced single scatter albedo, $\varpi = 0.98$. Comparing Figs. 5a and 5b we can see that, for a fixed degree of inhomogeneity, the increase in absorption caused by replacing a plane-parallel cloud with an inhomogeneous cloud of the same spherical albedo is roughly proportional to the coalbedo $(1 - \varpi)$. Based on these two examples the renormalization method appears to offer a simple, ef-

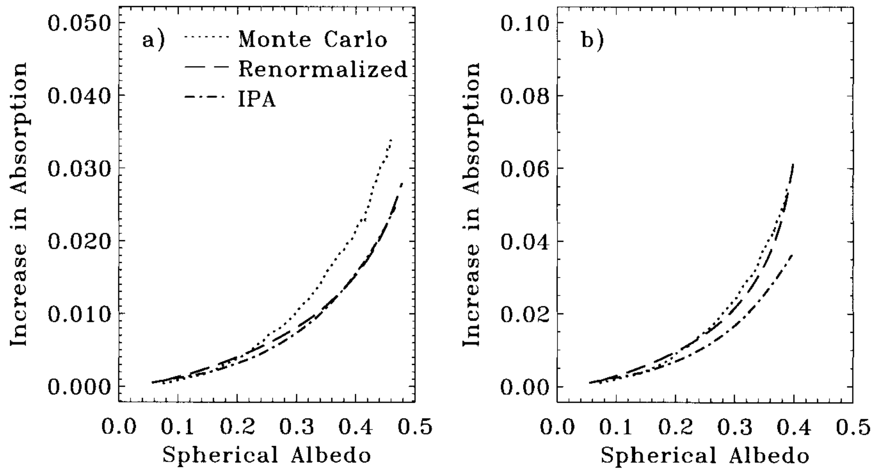

FIG. 5. Difference in absorption between an inhomogeneous cloud and a homogeneous cloud with the same spherical albedo, calculated using the Monte Carlo method (dotted line), the IPA (dotted–dashed line), and the renormalization method (dashed line) with $\delta = 1.0$ and $g = 0.85$: (a) $\varpi = 0.99$ and (b) $\varpi = 0.98$.

ficient, and relatively accurate model of the effect of cloud inhomogeneities on absorption in clouds.

We will now present some more results comparing the effect of cloud inhomogeneities on cloud absorption with both plane-parallel doubling–adding calculations and a simplified SGP scheme for plane-parallel calculations, which is used in the Goddard Institute for Space Studies (GISS) GCM. A useful measure of cloud absorption is the absorption efficiency, which is defined such that it is unity for very thin clouds and is given by the expression

$$A_{\mathrm{eff}} = \frac{A}{(1 - \varpi) \times [1 - \langle \exp(-\tau/\mu_0) \rangle]}, \qquad (12)$$

where τ is the optical thickness of the clouds, μ_0 is the solar zenith angle, and A is the fraction of the incident flux that is absorbed. The angle brackets indicate that an ensemble average is performed, numerically in the case of the Monte Carlo calculations and analytically in the case of the renormalization method. It is this measure that we will use to evaluate the effect that including cloud inhomogeneities might have on absorption in a GCM. In Fig. 6 the separate contributions to the absorption efficiency are shown as they vary with the mean optical depth for $\varpi = 0.99$ and $g = 0.85$. These provide the basis for understanding the following set of absorption efficiency plots. It is apparent from Fig. 6 that the term in the denominator, $[1 - \langle \exp(-\tau/\mu_0) \rangle]$, is always less for an inhomogeneous cloud than for a homogeneous cloud of the same mean optical depth (Jensen 1906). By contrast, the absorption, which is initially less for inhomogeneous clouds than homogeneous clouds of the same mean optical thickness, becomes greater for inhomogeneous clouds than for homogeneous clouds when the mean optical depth is very large ($\tau \approx 100$). This behavior can be understood on the basis of photons in thin inhomogeneous clouds flying farther between scattering events than photons in homogeneous clouds and therefore making fewer (absorbing) collisions before being transmitted, or reflected. However, when the optical depth becomes very large there is negligible transmission and the photons in inhomogeneous clouds penetrate farther than photons in homogeneous clouds of the same mean optical depth and must subsequently undergo more

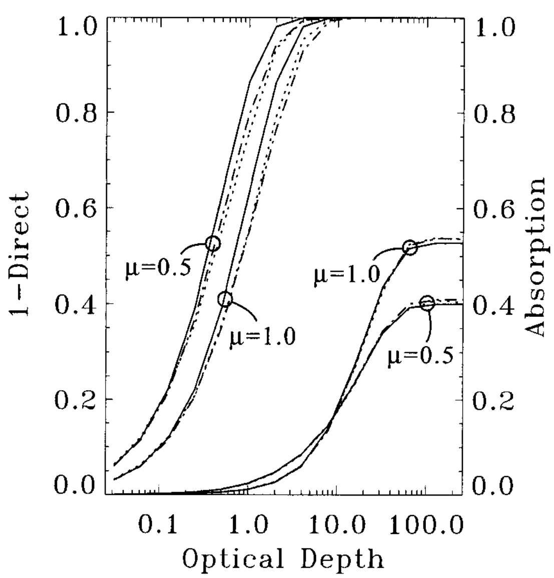

FIG. 6. The two components of the absorption efficiency that vary with the mean optical depth are shown for the single scattering parameters $g = 0.85$ and $\varpi = 0.99$. The left two families of curves compare the variation of $[1 - \langle \exp(-\tau/\mu_0) \rangle]$ for an inhomogeneous cloud with $\delta = 0.7$, using the Monte Carlo method (dotted line) and the renormalization method (dashed line), with the variation for a homogeneous cloud of the same mean optical depth, using the doubling/adding method (solid line), for the solar zenith angles $\mu_0 = 0.5$ and $\mu_0 = 1.0$. The right two families of curves show a similar comparison for absorption.

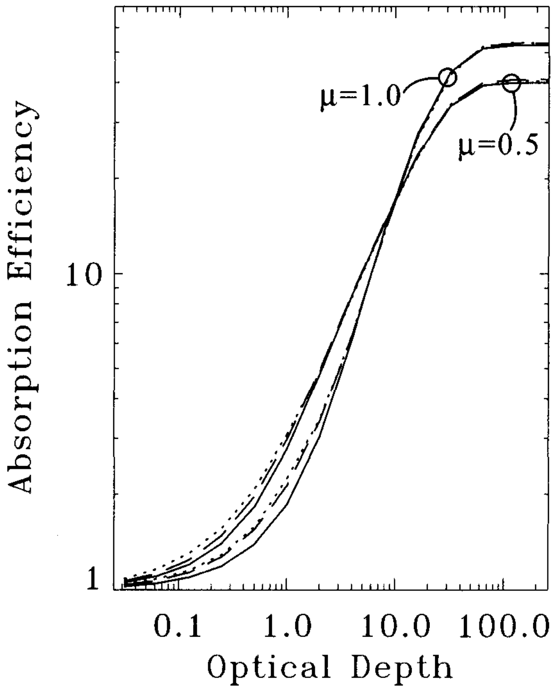

FIG. 7. Variation of absorption efficiency as a function of mean optical depth for a homogeneous cloud using the doubling/adding method (solid line) and for an inhomogeneous cloud using the Monte Carlo method (dotted line) and the renormalization method (dashed line). The two families of curves in this plot are for the solar zenith angles $\mu_0 = 0.5$ and $\mu_0 = 1.0$ and use the parameters $\delta = 0.7$, $\varpi = 0.99$, and $g = 0.85$.

(absorbing) collisions in returning to the surface. This type of behavior has been noted and studied extensively for binary mixtures (Boissé 1990; Hobson and Scheuer 1993) and for some types of fractal clouds (Borde and Isaka 1996). This feature of inhomogeneous clouds cannot be obtained using the IPA since it depends on the penetration of radiation through voids in the clouds and its subsequent trapping as a result of scattering deep within the cloud. The competing effects of the denominator and the numerator of the absorption efficiency explain why the absorption efficiency at small mean optical depths is initially larger for inhomogeneous clouds than for homogeneous clouds (the effect of $[1 - \langle \exp(-\tau/\mu_0)\rangle]$ in the denominator) and why one expects the difference to decrease as the optical depth increases. As the mean optical depth increases, the absorption efficiency of a homogeneous cloud may become larger than that of an inhomogeneous cloud of the same mean optical depth (see the numerator A as shown in Figs. 2b and 6) before reaching its asymptotic value where the absorption efficiency (absorption) will be larger for an inhomogeneous cloud than for a homogeneous cloud of the same mean optical depth. This behavior can be seen in Fig. 7, for solar zenith angles of $\mu_0 = 1.00$ and $\mu_0 = 0.5$ with $\delta = 0.7$, $\varpi = 0.99$, and $g = 0.85$. Similar functional behavior was found for a wide range of single scattering properties. Although for a fixed mean optical depth the renormalization method was found to be adequate for values of δ up to 1.0, it was found when comparing the Monte Carlo and renormalization results for absorption efficiency, over a range of optical depths, that a more reasonable value at which to stop using the renormalization method is $\delta = 0.7$. In Fig. 7 we compare the absorption efficiency of clouds with the same mean optical depth. Evidently if the comparison was for clouds with the same spherical albedo the absorption efficiency of inhomogeneous clouds would tend to be greater than that for homogeneous clouds with the same spherical albedo (see Figs. 3 and 6).

We will now examine how the absorption efficiency calculated using the doubling–adding method compares with the SGP method in order to compare the different effects of approximations in the radiative transfer calculations and the inclusion of cloud inhomogeneity. In GCMs, radiation calculations are usually made using two-stream, or equivalent, methods. The fact that the optical path of radiation through a plane-parallel cloud is inadequately modeled by having only two fluxes means that GCMs will not consistently get the correct absorption in clouds for all optical depths and solar zenith angles. The SGP method uses a doubling–adding calculation, but with only one Gauss point to calculate the layer-to-layer scattering of solar radiation by clouds and one ‘‘extra’’ angle (Lacis and Hansen 1974) to keep track of the solar zenith angle dependence. The SGP method is tuned to give the correct albedo for all optical depths and solar zenith angles when there is no absorption. This is accomplished by means of a lookup table, which for any given optical depth and asymmetry parameter selects the required backscatter ratio in the SGP formulation to give the correct albedo at all solar zenith angles. The SGP formalism also gives the correct result in the totally absorbing case when $\omega = 0$. It is the more common case of intermediate absorption where the SGP parameterization shows deviations from reference doubling–adding results. In Fig. 8 we compare the absorption eficiency calculated using the SGP method with the absorption efficiency calculated using the full doubling–adding method (31 Gauss points) for the same pairs of single scatter albedo and asymmetry parameter as used in Fig. 7. We see that, in all cases, for a solar zenith angle of 60° ($\mu_0 = 0.5$) the SGP method underestimates the absorption efficiency for smaller values of mean optical depth and marginally overestimates the absorption efficiency for large values of the mean optical depth. At a solar zenith angle of 0° ($\mu_0 = 1.00$) the converse is true, with the SGP method initially overestimating the absorption efficiency at smaller mean optical depths and slightly underestimating the absorption efficiency at large mean optical depths. The absorption efficiency of the SGP method does not, therefore, change as a function of solar zenith angle in the same way as the full doubling–adding method at intermediate optical depths and intermediate levels of absorption. This emphasizes the difficulty in obtaining accurate estimates of absorption effi-

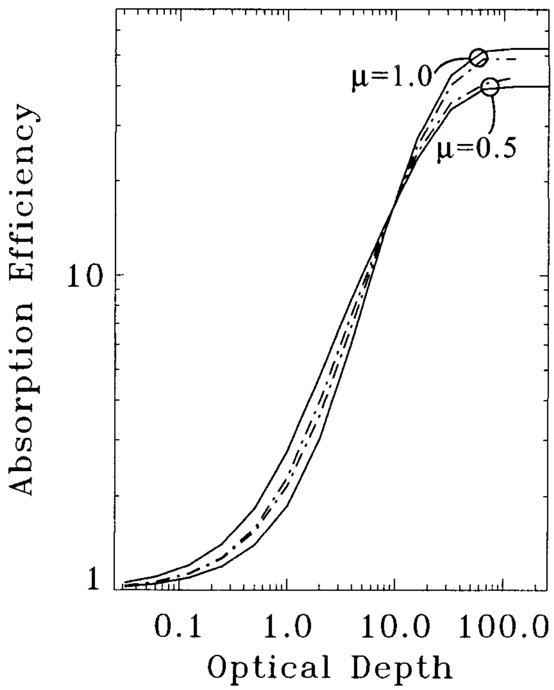

FIG. 8. Variation of absorption efficiency as a function of mean optical depth for a homogeneous cloud using the doubling/adding method (solid line) and the SGP method. The two families of curves in this plot are for the solar zenith angles $\mu_0 = 0.5$ and $\mu_0 = 1.0$ and use the parameters $\varpi = 0.99$ and $g = 0.85$.

ciency using simple effective two-stream methods, even though the SGP method is exact for conservative and completely absorbing media. The effect on absorption efficiency of the SGP approximation contrasts with the effect of including inhomogeneous clouds (see Fig. 7) where the inhomogeneous cloud biases depend on both optical depth and level of variability but have a similar sign for different solar zenith angles.

The more common alternative to the SGP approximation that is used in GCM radiation calculations is the two-stream approximation. An advantage of this approximation that has been noted by Barker (1996) is that the effects of inhomogeneity can be included by performing an IPA average over a gamma distribution of optical depths analytically. This yields a parameterization of the effects of cloud inhomogeneity of great simplicity and speed. Since this approach is not readily applicable to other more accurate radiative transfer approximations (such as the SGP approximation), one limitation of this method is the lack of accuracy of the two stream approximation (King and Harshvardhan 1986). Thus, even for current GCM radiation codes, the neglect of cloud inhomogeneities may introduce a bias in the efficiency with which GCM clouds absorb radiation, compared with inhomogeneous clouds.

5. Conclusions

In this paper we showed, using both Monte Carlo and independent pixel approximation (IPA) calculations, that for a particular model of cloud inhomogeneity, the absorption in inhomogeneous clouds is greater than that in homogeneous clouds with the same spherical albedo. We also showed that for a range of mean optical depths the absorption efficiency of inhomogeneous clouds is greater than that for homogeneous clouds of the same mean optical depth. Although the differences in absorption and absorption efficiency between homogeneous and inhomogeneous clouds depends on the degree of inhomogeneity, these results may be pertinent to recent suggestions regarding anomalous absorption in clouds (Cess et al. 1995; Ramanathan et al. 1995). It should be noted that this change in the absorbing properties of clouds does not depend on unknown physics or on making arbitrary changes to the single scattering parameters of radiative transfer theory. The physics has been known about at least since 1906, when Schwarzchild (1906) commented that it is ''customary, as a first approximation to substitute mean steady state conditions for spatial and temporal variations''; the inclusion of a statistical model of cloud inhomogeneities is merely a second approximation.

If the effects of such cloud inhomogeneities are to be included in GCMs, then a simple, physically based method for parameterizing their effect is required. To this end we presented a set of coupled equations that provide an exact representation of the mean radiative properties of an inhomogeneous medium. Such a theoretical understanding of how the mean properties of radiative transfer are affected by cloud inhomogeneities provides a rigorous means for determining which characteristics of a cloud field are relevant to the transfer of radiation and how they affect it. We showed that, under certain circumstances, the mean radiative properties of an inhomogeneous medium can be evaluated using plane-parallel radiative transfer theory, but with a simple renormalization of the single scattering parameters to include the effects of an inhomogeneous distribution of droplets within the cloud. Since this simple renormalization method is derived from a set of exact equations, we can easily evaluate the domain of validity and corrections to this and other (Kliorin et al. 1990; Anisimov and Fukshansky 1992) methods. Although the renormalization method developed here is no more of a universal panacea than any other approximate method for modeling cloud inhomogeneities (Evans 1993; Malvagi et al. 1993; Peltoniemi 1993; Zuev and Titov 1995), there are a number of advantages to using such a simple method compared with more numerically intensive and complicated methods, which may be more accurate for particular cloud models. The first is its simplicity, which allows us to identify in a relatively simple way how the transfer of radiation will be modified by inhomogeneities, for clouds of different mean optical depths, single scatter albedos, and phase functions. The second is its speed, because the effects of inhomogeneities can be included in plane-parallel codes that have already been heavily optimized. The effects of inhomogeneity can therefore be included in any GCM that includes a prognostic cloud water parameterization capable of predicting the variability of liquid water in a cloud. The third is its parametric dependence on measurable properties of cloud variability, which means that it is both experimentally testable and also of use in estimating the sensitivity of the retrieval of microphysical quantities to the spatial structure of cloud inhomogeneities. Finally, the lack of assumptions, in this theory, about the statistical properties of the random medium means that features such as a hot spot effect, that depend on the non-Markovian nature of backscattering can be identified and evaluated. A number of other studies (Stephens 1988; Cahalan et al. 1994a,b; Borde and Isaka 1996) have also indicated that much of the effect of cloud inhomogeneity can be included in plane-parallel calculations by some rescaling, or renormalization of the single scattering parameters. This study merely emphasizes the theoretical justification for such a simple approach.

The type of renormalization developed here, which is based on a continuous droplet density distribution, is particularly appropriate for treating internal cloud inhomogeneities. An alternative approach (Titov 1990; Malvagi et al. 1993; Zuev and Titov 1995) makes simplifying assumptions about the droplet density distribution in order to obtain a relatively simple, but accurate, set of equations for radiative transfer through a model of broken cumulus clouds. This model approximates the density distribution of droplets by a discrete set of droplet densities. In its simplest incarnation the sky is treated as a binary mixture of clear and cloudy random elements, which, though it may be a good approximation for large-scale cloud variability, tends to underestimate the effect of droplet density fluctuations on the transfer of radiation (Hobson and Scheuer 1993). The combination of the renormalization method, developed here, for a general statistical model of smaller-scale variability and the binary mixture approach for larger-scale variability (Malvagi et al. 1993) might therefore provide a realistic model of the transfer of radiation by inhomogeneous cumulus clouds.

Acknowledgments. This work was supported in part by the Department of Energy Interagency Agreement under the Atmospheric Radiation Measurements Program, and by NASA through Grants NAGW-5221 and NAGW-4693. Suggestions for improving the readability of the manuscript by one of the referees and by W. B. Rossow are gratefully acknowledged.

APPENDIX

Perturbative Expansion

Here we describe how (5) can be expanded in a perturbation series and then resummed. We first rewrite (5) as

$$I(\mathbf{r}, \mathbf{s}) = I_0(\mathbf{r}, \mathbf{s}) - \int_V \int_{4\pi} F(\mathbf{r}, \mathbf{s}; \mathbf{r}', \mathbf{s}')\eta(\mathbf{r}')I(\mathbf{r}', \mathbf{s}')\, d\mathbf{s}'\, d\mathbf{r}'. \tag{A1}$$

The function $F(\mathbf{r}, \mathbf{s}; \mathbf{r}', \mathbf{s}')$ is defined to be

$$F(\mathbf{r}, \mathbf{s}; \mathbf{r}', \mathbf{s}') = \sigma_{\mathrm{ext}} \int_{4\pi} G_0(\mathbf{r}, \mathbf{s}; \mathbf{r}', \mathbf{s}'')B(\mathbf{s}'' \cdot \mathbf{s}')\, d\mathbf{s}' \tag{A2}$$

and is introduced simply to compress the length of the equations that we will obtain when we expand (A1), so that a representative set of terms in the expansion can be written down. If we now expand (A1) and average over fluctutations in $\eta(\mathbf{r})$, we obtain the expression

$$\begin{aligned}\langle I(\mathbf{r}, \mathbf{s})\rangle = I_0(\mathbf{r}, \mathbf{s}) &+ \int_V \int_{4\pi} \int_V \int_{4\pi} F(\mathbf{r}, \mathbf{s}; \mathbf{r}_2, \mathbf{s}_2)F(\mathbf{r}_2, \mathbf{s}_2; \mathbf{r}_1, \mathbf{s}_1)\langle \eta(\mathbf{r}_2)\eta(\mathbf{r}_1)\rangle I_0(\mathbf{r}_1, \mathbf{s}_1)\, d\mathbf{s}_1\, d\mathbf{r}_1\, d\mathbf{s}_2\, d\mathbf{r}_2 \\
&- \int_V \int_{4\pi} \int_V \int_{4\pi} \int_V \int_{4\pi} F(\mathbf{r}_4, \mathbf{s}_4; \mathbf{r}_3, \mathbf{s}_3)F(\mathbf{r}_3, \mathbf{s}_3; \mathbf{r}_2, \mathbf{s}_2)F(\mathbf{r}_2, \mathbf{s}_2; \mathbf{r}_1, \mathbf{s}_1) \\
&\qquad \times \langle \eta(\mathbf{r}_3)\eta(\mathbf{r}_2)\eta(\mathbf{r}_1)\rangle I_0(\mathbf{r}_1, \mathbf{s}_1)\, d\mathbf{s}_1\, d\mathbf{r}_1\, d\mathbf{s}_2\, d\mathbf{r}_2\, d\mathbf{s}_3\, d\mathbf{r}_3 \\
&+ \int_V \int_{4\pi} \int_V \int_{4\pi} \int_V \int_{4\pi} \int_V \int_{4\pi} F(\mathbf{r}, \mathbf{s}; \mathbf{r}_4, \mathbf{s}_4)F(\mathbf{r}_4, \mathbf{s}_4; \mathbf{r}_3, \mathbf{s}_3)F(\mathbf{r}_3, \mathbf{s}_3; \mathbf{r}_2, \mathbf{s}_2)F(\mathbf{r}_2, \mathbf{s}_2; \mathbf{r}_1, \mathbf{s}_1) \\
&\qquad \times \langle \eta(\mathbf{r}_4)\eta(\mathbf{r}_3)\eta(\mathbf{r}_2)\eta(\mathbf{r}_1)\rangle I_0(\mathbf{r}_1, \mathbf{s}_1)\, d\mathbf{s}_1\, d\mathbf{r}_1\, d\mathbf{s}_2\, d\mathbf{r}_2\, d\mathbf{s}_3\, d\mathbf{r}_3\, d\mathbf{s}_4\, d\mathbf{r}_4,\end{aligned} \tag{A3}$$

contribution from fifth-order moment of $\eta(\mathbf{r})$, etc. This expansion can obviously not be resummed since all the terms depend on different moments that, depending on the distributional properties of $\eta(\mathbf{r})$, will have different relations with respect to one another. However, if we recall that moments can be written as the sum of permutations of products of lower-order moments and the cummulant of that order (Gnedenko 1963; Fradkin 1966), then we can at least resum an infinite subseries of the terms generated in the expansion (A3). For example, the fourth moment of $\eta(\mathbf{r})$ can be written as

$$\begin{aligned}\langle\eta(\mathbf{r}_4)\eta(\mathbf{r}_3)\eta(\mathbf{r}_2)\eta(\mathbf{r}_1)\rangle &= \langle\eta(\mathbf{r}_4)\eta(\mathbf{r}_3)\rangle\langle\eta(\mathbf{r}_2)\eta(\mathbf{r}_1)\rangle + \langle\eta(\mathbf{r}_4)\eta(\mathbf{r}_1)\rangle\langle\eta(\mathbf{r}_3)\eta(\mathbf{r}_2)\rangle \\ &\quad + \langle\eta(\mathbf{r}_4)\eta(\mathbf{r}_2)\rangle\langle\eta(\mathbf{r}_3)\eta(\mathbf{r}_1)\rangle + K_4(\mathbf{r}_4, \mathbf{r}_3, \mathbf{r}_2, \mathbf{r}_1), \end{aligned} \tag{A4}$$

where $K_4(\mathbf{r}_4, \mathbf{r}_3, \mathbf{r}_2, \mathbf{r}_1)$ is the fourth-order cummulant of the random process $\eta(\mathbf{r})$. Similar relations apply for higher-order moments. The cummulant can therefore be regarded as that part of the moment that cannot be factorized into products of lower-order moments and in that sense can be regarded as the only irreducible contribution from a given moment. The last term in (A3) is therefore determined by products of second-order moments and the fourth-order cummulant. The contribution of the first term in (A4) and a contribution to all of the even higher-order moments in the expansion (A3) can be included in an implicit resummation that is given by the expression

$$\langle I(\mathbf{r}, \mathbf{s})\rangle = I_0(\mathbf{r}, \mathbf{s}) + \int_V\int_{4\pi}\int_V\int_{4\pi} F(\mathbf{r}, \mathbf{s}; \mathbf{r}_2, \mathbf{s}_2)F(\mathbf{r}_2, \mathbf{s}_2; \mathbf{r}_1, \mathbf{s}_1)\langle\eta(\mathbf{r}_2)\eta(\mathbf{r}_1)\rangle\langle I(\mathbf{r}_1, \mathbf{s}_1)\rangle\, d\mathbf{s}_1\, d\mathbf{r}_1\, d\mathbf{s}_2\, d\mathbf{r}_2 \tag{A5}$$

for the mean specific intensity. Expanding this expression in a perturbation series indicates which terms in (A3) are included in such an implicit summation. Equation (A5) is simply an integral form of the equation of radiative transfer, which, when written in its differential form, becomes

$$\begin{aligned}&\mathbf{s}\cdot\nabla\langle I(\mathbf{r}, \mathbf{s})\rangle + \sigma_{\text{ext}}N\int_{4\pi} B(\mathbf{s}\cdot\mathbf{s}')\langle I(\mathbf{r}, \mathbf{s}')\rangle\, d\mathbf{s}' \\ &\quad = \sigma_{\text{ext}}^2\int_V\int_{4\pi}\int_{4\pi}\int_{4\pi} B(\mathbf{s}\cdot\mathbf{s}')G_0(\mathbf{r}, \mathbf{s}'; \mathbf{r}', \mathbf{s}'')\langle\eta(\mathbf{r})\eta(\mathbf{r}')\rangle B(\mathbf{s}''\cdot\mathbf{s}''')\langle I(\mathbf{r}', \mathbf{s}''')\rangle\, d\mathbf{s}'\, d\mathbf{s}''\, d\mathbf{s}'''\, d\mathbf{r}'.\end{aligned} \tag{A6}$$

It should be noted that (A6) is identical to the results obtained using the method of smoothing (Frisch 1968; Pomraning 1991). The expression given in the body of the text is

$$\begin{aligned}&\mathbf{s}\cdot\nabla\langle I(\mathbf{r}, \mathbf{s})\rangle + \sigma_{\text{ext}}N\int_{4\pi} B(\mathbf{s}\cdot\mathbf{s}')\langle I(\mathbf{r}, \mathbf{s}')\rangle\, d\mathbf{s}' \\ &\quad = \sigma_{\text{ext}}^2\int_V\int_{4\pi}\int_{4\pi}\int_{4\pi} B(\mathbf{s}\cdot\mathbf{s}')\langle G(\mathbf{r}, \mathbf{s}'; \mathbf{r}', \mathbf{s}'')\rangle\langle\eta(\mathbf{r})\eta(\mathbf{r}')\rangle B(\mathbf{s}''\cdot\mathbf{s}''')\langle I(\mathbf{r}', \mathbf{s}''')\rangle\, d\mathbf{s}'\, d\mathbf{s}''\, d\mathbf{s}'''\, d\mathbf{r}',\end{aligned} \tag{A7}$$

which is a better approximation to (A3), in that it includes the first two terms of (A4) and a substantially larger number of higher-order terms than are provided by using (A6). Evidently, if the distribution of $\eta(\mathbf{r})$ has significant skew, or kurtosis, associated with it, then (7) must be modified to include the higher-order cummulants. The local approximation to (7) that is used in this paper to provide renormalized single scatter parameters that model the effect of cloud inhomogeneities on radiative transfer is only one method for solving (7). A set of coupled transport equations that are structurally similar to those used for binary and ternary mixtures (Malvagi et al. 1993; Hobson and Scheuer 1993) can be derived that is equivalent to (7). This coupled set of equations reduces to those given by Anisimov and Fukshansky (1992) when the kurtosis of the distribution of $\eta(\mathbf{r})$ is negigible and when the correlation lengths associated with the second and third moments of $\eta(\mathbf{r})$ are long compared with the transport mean free path.

REFERENCES

Abrikosov, A. A., L. P. Gorkov, and I. E. Dzyaloshinski, 1963: *Methods of Quantum Field Theory in Statistical Physics.* Prentice Hall, 352 pp.

Anisimov, O., and L. Fukshansky, 1992: Stochastic radiation in macro-heterogeneous random optical media. *J. Quant. Spectrosc. Radiat. Transfer,* **2,** 169–186.

Arya, K., and R. Zeyher, 1983: Optical response from a randomly rough surface: Theory and numerical results. *Phys. Rev.,* **28B,** 4090–4105.

Audic, S., and H. Frisch, 1993: Monte Carlo simulation of a radiative transfer problem in a random medium: application to a binary mixture. *J. Quant. Spectrosc. Radiat. Transfer,* **50,** 127–147.

Barabenkov, Yu. N., and V. M. Finkel'berg, 1968: Radiation transport equation for correlated scatterers. *Sov. Phys. JETP,* **26,** 587–591.

Barker, H. W., 1992: Solar radiative transfer through clouds posessing isotropic variable extinction coefficient. *Quart. J. Roy. Meteor. Soc.,* **118,** 1145–1162.

——, 1996: A parameterization for computing grid-averaged solar fluxes for inhomogeneous marine boundary layer clouds. Part I: Methodology and homogeneous biases. *J. Atmos. Sci.,* **53,** 2289–2303.

——, and J. A. Davies, 1992: Solar radiative fluxes for stochastic, scale-invariant broken cloud fields. *J. Atmos. Sci.,* **49,** 1115–1125.

Boissé, P., 1990: Radiative transfer inside clumpy media: The penetration of UV photons inside clumpy clouds. *Astron. Astrophys.,* **228,** 483–501.

Borde, R., and H. Isaka, 1996: Radiative transfer in multifractal clouds. *J. Geophys. Res.,* **101,** 29 461–29 478.

Borovoi, A. G., 1984: Radiative transfer in inhomogeneous media. *Sov. Phys. Dokl.,* **29,** 490–492.

Cahalan, R. F., 1989: Overview of fractal clouds. *Advances in Remote Sensing,* A. Deepak, Ed., A. Deepak, 317–389.

——, W. Ridgway, W. J. Wiscombe, and T. L. Bell, 1994a: The albedo of fractal stratocumulus clouds. *J. Atmos. Sci.,* **51,** 2434–2455.

——, ——, ——, S. Gollmer, and Harshvardhan, 1994b: Independent pixel and Monte Carlo estimates of stratocumulus albedo. *J. Atmos. Sci.,* **51,** 3776–3790.

Cairns, B., 1992: An investigation of radiative transfer and multiple scattering. Ph.D. thesis, University of Rochester. [Available from UMI Dissertation Information Service, 300 N. Zeeb Rd., Ann Arbor, MI 48106.]

Cess, R. D., and Coauthors, 1995: Absorption of solar radiation by clouds: Observation versus models. *Science,* **267,** 496–499.

Davis, A., P. Gabriel, S. Lovejoy, D. Schertzer, and G. L. Austin, 1990: Discrete angle radiative transfer: 3. Numerical results and meteorological applications. *J. Geophys. Res.,* **95** 11 729–11 742.

Del Genio, A. D., M.-S. Yao, W. Kovari, and K.-W. Lo, 1996: A prognostic cloud water parameterization for global climate models. *J. Climate,* **9,** 270–303.

Dolin, L. S., 1984: Randomizing of a radiation field in a medium with a fluctuating extinction coefficient. *Sov. Phys. Dokl.,* **29,** 544–546.

Downing, H. D., and D. Williams, 1975: Optical constants of water in the infrared. *J. Geophys. Res.,* **80,** 1656–1661.

Evans, K. F., 1993: A general solution for stochastic radiative transfer. *Geophys. Res. Lett.,* **20,** 2075–2078.

Feynman, R. P., and H. Hibbs, 1965: *Quantum Mechanics and Path Integrals.* McGraw-Hill, 365 pp.

Fradkin, E. S., 1966: Application of functional methods in quantum field theory and quantum statistics. *Nucl. Phys.,* **76,** 588–593.

Frisch, U., 1968: Wave propagation in random media. *Probabilistic Methods in Applied Mathematics,* A. T. Barucha-Reid, Ed., Vol. 1, Academic Press, 75–199.

Furutsu, K., 1963: On the statistical theory of electromagnetic waves in a fluctuating medium. *J. Res. Nat. Bur. Stand.,* **67D,** 303–323.

Gabriel, P. M., and K. F. Evans, 1996: Simple radiative transfer methods for calculating domain-averaged solar fluxes in inhomogeneous clouds. *J. Atmos. Sci.,* **53,** 858–877.

Gnedenko, B. V., 1963: *Theory of Probability.* Chelsea, 529 pp.

Hale, G. M., and M. R. Query, 1973: Optical constants of water in the 200-nm to 200-μm wavelength region. *Appl. Opt.,* **12,** 555–563.

Hansen, J. E., and L. D. Travis, 1974: Light scattering in planetary atmospheres. *Space Sci. Rev.* **16,** 527–610.

——, G. Russell, D. Rind, P. Stone, A. Lacis, S. Lebedeff, R. Ruedy, and L. Travis 1983: Efficient three-dimensional global models for climate studies: Models I and II. *Mon. Wea. Rev.,* **111,** 609–662.

Harshvardhan, and D. A. Randall 1985: Comments on ''The parameterization of radiation for numerical weather prediction and climate models.'' *Mon. Wea. Rev.,* **113** 1832–1833.

Hignett, P., 1987: A study of the short-wave radiative properties of marine stratus: Aircraft measurements and model comparisons. *Quart. J. Roy. Meteor. Soc.,* **113,** 1011–1024.

Hobson, M. P., and P. A. G. Scheuer, 1993: Radiative transfer in a clumpy medium-I. An analytical Markov-process solution for an N-phase slab. *Mon. Not. Roy. Astron. Soc.,* **264,** 145–160.

Jensen, J. L. W. V., 1906: Sur les fonctions convexes et les inégalitiés entre les valeur moyenne. *Acta Math.,* **30,** 175–193.

John, S., 1985: Localization and absorption of waves in weakly dissipative disordered media. *Phys. Rev.,* **B31,** 304–309.

Jonas, P. J., 1994: On the reflectance of cellular cloud layers. *Quart. J. Roy. Meteor. Soc.,* **120,** 221–229.

Keller, J. B., 1962: Wave propagation in random media. *Proceedings of Symposia in Applied Mathematics XIII,* G. Birkhoff, R. Bellman and C. C. Lin, Eds., American Mathematical Society, 227–246.

King, M. D., and Harshvardhan, 1986: Comparative accuracy of selected multiple scattering approximations. *J. Atmos. Sci.,* **43,** 784–801.

Kliorin, N. I., Yu. A. Kravtsov, A. E. Mereminskii, and V. G. Mirovskii, 1990: Clearing effects and radiative transfer in a medium with large scale fluctuations in the density of scatterers. *Radiofizika,* **32,** 1072–1078.

Kobyashi, T., 1993: Effects due to cloud geometry on biases in the albedo derived from radiance measurements. *J. Climate,* **6,** 120–128.

Kolmogorov, A. N., 1941: The local structure of trubulence in incompressible turbulent viscous fluid for very large Reynolds number. *Dokl. Akad. Nauk SSSR,* **30,** 301–305.

Korolev, A. V., and I. P. Mazin, 1993: Zones of increased and decreased droplet concentration in stratiform clouds. *J. Appl. Meteor.,* **32,** 760–773.

Lacis, A. A., and J. E. Hansen, 1974: A parameterization for the absorption of solar radiation in the earth's atmosphere. *J. Atmos. Sci.,* **31,** 118–133.

Li, Z.-Q., H. W. Barker, and L. Moreau, 1995: The variable effect of clouds on atmospheric absorption of solar radiation. *Nature,* **376,** 486–490.

Malvagi, F., and G. C. Pomraning 1990: Renormalized equations for linear transport in stochastic media. *J. Math. Phys.,* **31,** 892–900.

——, R. N. Byrne, G. C. Pomraning, and R. C. J. Somerville, 1993: Stochastic radiative transfer in a partially cloudy atmosphere. *J. Atmos. Sci.,* **50,** 2146–2158.

MacKintosh, F. C., and S. John, 1989: Diffusing-wave spectroscopy and multiple scattering of light in correlated random media. *Phys. Rev.,* **40B,** 2383–2406.

Marchuk, G. I., G. A. Mikhailov, M. A. Nazoraliev, R. A. Dorbinjan, B. A. Kargin, and B. S. Elepov, 1980: *Monte Carlo Methods in Atmospheric Optics.* Series in Optical Science, Vol. 12, Springer-Verlag, 208 pp.

McKee, T. B., and S. K. Cox, 1974: Scattering of visible radiation by finite clouds. *J. Atmos. Sci.,* **31,** 1885–1892.

Myneni, R. B., A. L. Marshak, and Y. V. Knyazikhin, 1991: Transport theory for a leaf canopy of finite dimensional scattering centers. *J. Quant. Spectrosc. Radiat. Transfer,* **46,** 259–280.

Nakajima, T., MD. King, J. D. Spinhirne, and L. F. Radke 1990: Determination of the optical thickness and effective particle radius of clouds from reflected solar measurements. Part II: Marine stratocumulus observations. *J. Atmos. Sci.,* **47,** 1878–1893.

O'Brien, D. M., 1992: Accelerated quasi-Monte Carlo integration of the radiative transfer equation. *J. Quant. Spectrosc. Radiat. Transfer,* **48,** 41–59.

Palmer, K. F., and D. Williams. 1974: Optical properties of water in the near infrared. *J. Opt. Soc. Amer.,* **64,** 1107–1110.

Peltoniemi, J. I., 1993: Radiative transfer in stochastically inhomogeneous media. *J. Quant. Spectrosc. Radiat. Transfer,* **50,** 655–671.

Pomraning, G. C., 1991: *Linear Kinetic Theory and Particle Transport in Stochastic Mixtures.* World Scientific, 255 pp.

Ramanathan, V., B. Subasilar, G. J. Zhang, W. Conant, R. D. Cess, J. T. Kiehl, H. Grassl, and L. Shi, 1995: Warm pool heat budget and short wave cloud forcing: A missing physics? *Science,* **267,** 499–503.

Romanova, L. M., 1978: Principles of invariance for horizontally inhomogeneous media. *Izv. AN SSSR FAO,* **12,** 820–833.

Ronnolm, K., M. B. Baker, and H. Harrison, 1980: Radiative transfer through media with uncertain or variable parameters. *J. Atmos. Sci.,* **37,** 1279–1290.

Rosenbaum, S., 1971: The mean Green's function: A non-linear approximation. *Radio Sci.,* **6,** 379–386.

Rossow, W. B., and R. A. Schiffer, 1991: ISCCP cloud data products. *Bull. Amer. Meteor. Soc.,* **72,** 1–20.

Rybicki, G. B., 1971: The searchlight problem with isotropic scattering. *J. Quant. Spectrosc. Radiat. Transfer,* **11,** 827–849.

Rytov, S. M., Yu. A. Kravtsov, and V. I. Tatarskii, 1989: *Statistical Radiophysics. Vols.* 1–4, Springer Verlag.

Schertzer, D., and S. Lovejoy, 1987: Physical modelling and analysis of rain and clouds by anisotropic scaling multiplicative processes. *J. Geophys. Res.,* **92,** 9693–9714.

Schwarzchild, K., 1906: On the equilibrium of the sun's disc. *Nachr. Königlichen Ges. Göttingen Math.-Phys. Kl.,* **195,** 41–53. [Reprinted (1966) in *Selected Papers on the Transfer of Radiation,* D. H. Menzel, Ed., Dover, 25–34.]

Stephens, G. L., 1988: Radiative transfer through arbitrarily shaped media. Part II: Group theory and closures. *J. Atmos. Sci.,* **45,** 1836–1848.

——, and S. Tsay, 1990: On the cloud absorption anomaly. *Quart. J. Roy. Meteor. Soc.,* **116,** 671–704.

——, P. M. Gabriel, and S.-C. Tsay, 1991: Statistical radiative transport in one-dimensional media and its application to the terrestrial atmosphere. *Transp. Theory Stat. Phys.,* **20,** 139–175.

Titov, G. A., 1990: Statistical description of radiative transfer in clouds. *J. Atmos. Sci.,* **47,** 24–38.

van der Mark, M. B., M. P. van Albada, and A. Lagendijk, 1988: Light scattering in strongly scattering media: Multiple scattering and weak localization. *Phys. Rev.,* **37B,** 3575–3592.

Williams, M. M. R., 1974: The effect of random material density on reactor criticality. *Atomkernenergie,* **22,** 248.

——, 1984: *Random Processes in Nuclear Reactors.* Pergamon Press, 243 pp.

Zuev, V. E., and G. A. Titov, 1995: Radiative transfer in cloud fields with random geometry. *J. Atmos. Sci.,* **52,** 176–190.