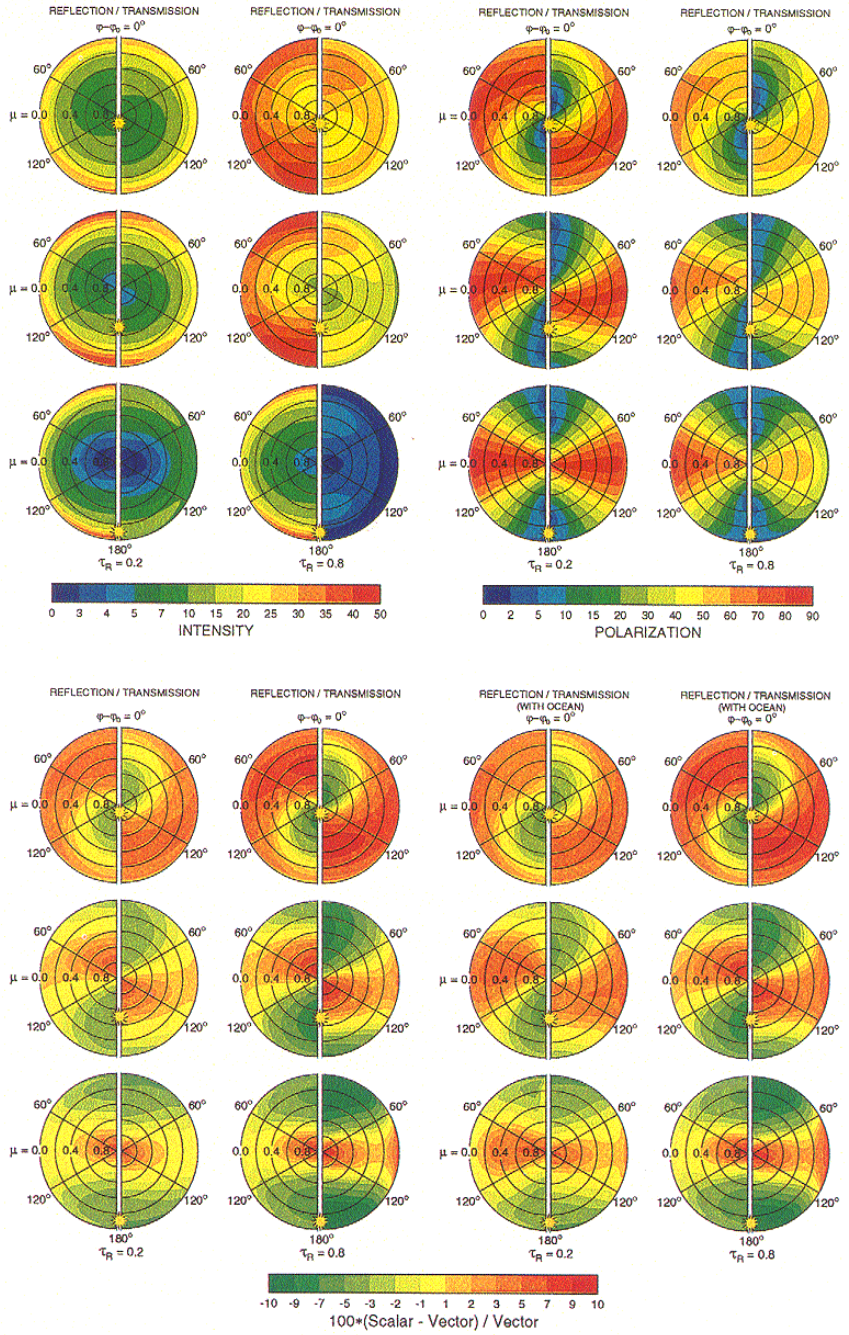
Geophysical Research Letters



JANUARY 15, 1998

Volume 25 Number 2

AMERICAN GEOPHYSICAL UNION

GEOPHYSICAL RESEARCH LETTERS, VOL. 25, NO. 2, Pages 135–138, JANUARY 15, 1998

Modeling errors in diffuse-sky radiation: Vector *vs.* scalar treatment

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Abstract. Radiative transfer calculations that utilize the scalar approximation of light produce intensity errors as large as 10% in the case of pure Rayleigh scattering. This modeling error, which arises primarily from second order scattering, is greatly reduced for flux and albedo results because of error cancellation brought about by integration over scattering angle. However, polarized light scattered from an underlying ocean surface, or from atmospheric aerosols, interacts with the pattern of Rayleigh scattered polarization to distort the error cancellation and thus incur larger flux and albedo errors. While addition of scattered radiation from clouds, aerosols or ground surface into the Rayleigh atmosphere tends to reduce the magnitude of scalar approximation intensity errors, the scalar errors in fluxes and albedos are not proportionately reduced, but are actually increased.

Introduction

Most current methods of radiative transfer treat light as a scalar, even though it is well known that the proper description of light requires explicit recognition of its electromagnetic nature. From Maxwell's equations, the radiation field possesses vector properties and consists of a large number of plane-wave packets with specific phase and polarization. A light beam is defined by its Stokes parameters, which are statistical averages that express the macroscopic properties of light in 4-vector form, $\mathbf{I} = \{I,Q,U,V\}$, where I is the beam intensity, and Q,U,V describe the beam in terms of its degree and direction of linear and elliptical polarization. Multiple scattering within a homogeneous, macroscopically isotropic, plane-parallel atmosphere is described accordingly by a vector radiative transfer equation which can be solved with high precision using numerical methods such as the vector doubling/adding method (Hansen and Travis, 1974).

Since the pioneering work by Chandrasekhar (1950), it has been recognized that for pure Rayleigh scattering, treating light as a scalar can produce errors as large as 10% in the computed intensity of the radiation field. Nevertheless, because of significantly greater modeling complexity of the 4-vector doubling/adding calculations compared to the scalar doubling/adding, there has been understandable reluctance to employ the more rigorous method when simpler treatment appears adequate.

Errors in reflected intensity that arise due to neglect of polarization were examined by Hansen (1971) who then concluded that in most cases, the errors are less than about 1% for light reflected by spherical cloud particles with sizes of the order or larger than the incident light. This would make the scalar approximation adequate for most cloud and aerosol radiance calculations. Moreover, in climate studies, where radiative fluxes and albedos are the quantities of interest, the integration over scattering angle has the fortuitous effect of averaging out radiance errors to the point where no one has seriously worried about the adequacy of the scalar approximation. Also, since climate related applications typically rely on relative differences and radiative flux ratios, this tends to further dilute the significance of potential errors that arise from the scalar approximation.

Recent comparisons of model results and observations (e.g., Kato, *et al*., 1997; Kinne, *et al*. 1997; Charlock and Alberta, 1996; Wild, *et al*. 1995) suggest, however, that model calculations may be systematically over-estimating incident shortwave (SW) solar flux at the ground surface. King and Harshvardhan (1986) have already shown that δ 2-stream methods, which are frequently used in GCM applications, produce errors of order 10% depending on the optical depth, solar zenith angle, and scattering phase function. The extent that these apparent differences can be attributed to radiative modeling approximations, or to inadequate instrument calibration, is not yet clear at this time and thus warrants a closer examination of the scalar approximation that is used in these models.

In an earlier study, we examined the errors that are introduced by the scalar approximation for radiance calculations in simple Rayleigh-scattering atmospheres (Mishchenko *et al*., 1994). In agreement with previous studies of this problem, we found that the intensity errors (both over and under estimates) can be as large as 10%, arising in specific geometrical configurations with the maximum error occurring for optical depths near unity. Adding a Lambertian reflecting surface tends to dilute these scalar errors. Also, for optically thin atmospheres, vector/scalar intensity differences are seen to increase as the single-scattering albedo, ω_o, is reduced from unity (conservative scattering) to about 0.8, but then decrease with further increase in particle absorptivity. The source of these errors arises from low-order (except first-order) light scattering paths that involve right-angle scattering of polarized light along with right-angle rotations of the scattering plane that cannot be properly approximated when light is treated as a scalar quantity.

Vector Doubling/Adding Results

Our aim here is to examine the magnitude and angular distribution of the (Scalar – Vector)/Vector relative error caused by the scalar approximation for diffusely reflected and diffusely transmitted radiation, and to determine its dependence on solar zenith angle, atmospheric optical depth, and surface properties. For this purpose, and also to relate this to potential problems that may impact the calibration of instruments used to measure whole-sky radiation, we first examine the distribution of clear-sky intensity and degree of linear polarization calculated with the vector doubling/adding method.

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Paper Number 97GL03613.

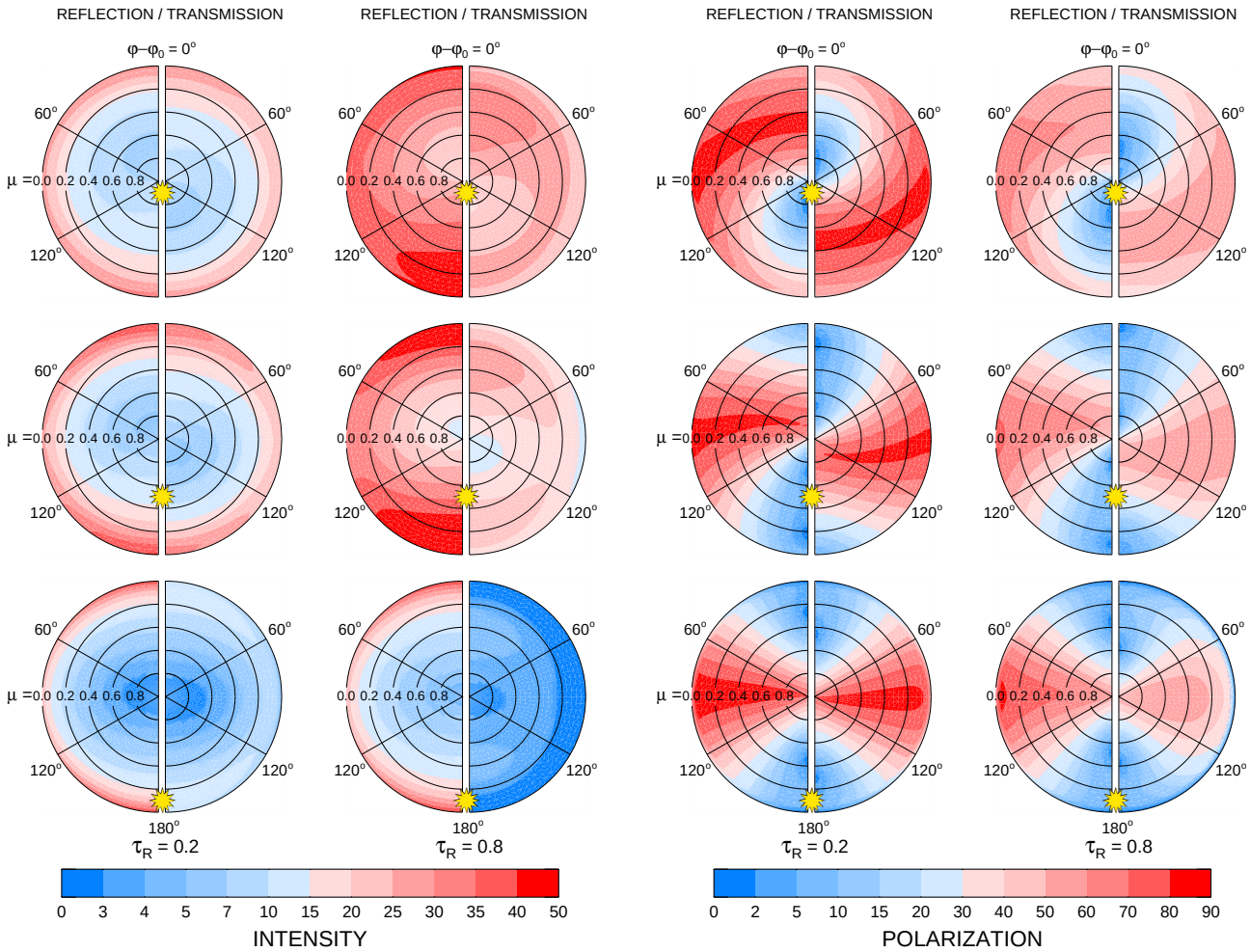

Figure 1. Vector doubling/adding intensity and degree of linear polarization for Rayleigh scattering atmospheres. The left two columns show polar plots of scattered intensity with the left hemispheres displaying diffuse reflection and the right hemispheres showing diffuse transmission for τ_R=0.2 and 0.8, respectively. Solar zenith angles are modeled for μ_o=0.9, 0.5, and 0.1. The right two columns show per cent linear polarization for the respective cases.

The examples in Fig. 1, while simple in concept are rigorous in treatment. The atmosphere is assumed to be plane-parallel and horizontally homogeneous, but it does include depolarization due to anisotropy of air molecules. In the figure, the nadir/zenith is centrally located in each polar plot. The sun is placed at typical high-medium-low solar zenith angles, and is identified by its μ_o position at μ_o=0.9, 0.8, 0.1, respectively. The horizon is at μ=0.

Since there is left/right hemispherical symmetry with respect to the φ-φ_o=0° principal plane for both reflected and transmitted radiances, we use split-hemispheres to display the intensity of the scattered radiation. Thus, the bi-directional reflectance $\mathbf{R}(\mu,\mu_o,\varphi-\varphi_o)$, as would be seen from space or high flying aircraft, is shown in the left hemisphere, and the diffuse transmission $\mathbf{T}(\mu,\mu_o,\varphi-\varphi_o)$, corresponding to ground-based observations, is displayed in the right side hemisphere of each polar plot. Since the sunlight incident on the atmosphere is unpolarized, it is described by the flux vector $\pi\mathbf{F} = \pi\{1,0,0,0\}$. Brightness of the scattered radiation is expressed in per cent relative to that of a perfectly reflecting Lambert surface.

The left two columns of Fig.1 show the intensity of scattered light for Rayleigh optical depths of τ_R=0.2 and τ_R=0.8, which correspond approximately to wavelengths of 455 nm (blue) and 320 nm (UVA), respectively, while the two right-side columns show corresponding maps of degree of linear polarization, $(Q^2+U^2)^{1/2}/I$. For simplicity, the surface albedo in these examples is set to zero.

As can be seen, intensity variations in azimuth tend to be smaller than a factor of 2 for reflected and transmitted radiances. On the other hand, brightness variation with μ can be quite large, with far greater brightness along the horizon compared to the zenith direction, particularly in those cases for small τ_R, where also the intensity patterns for reflected and transmitted light are very similar. Bright horizons are characteristic of diffuse reflection for low solar zenith angles. Also notable is the large change in μ dependence for both reflection and transmission as τ_R is increased. This might be an important consideration in evaluating the calibration of flux measurements where a substantial cosine correction is required to account for the angle dependence of detector response. It can be seen that for small values of τ_R, both reflected and transmitted fluxes receive proportionately more radiative energy from small μ values than is the case for large optical depths.

Degree of polarization ranges from near-zero at neutral points to over 90% near 90° scattering angle. Maximum polarization occurs at small optical depths, and decreases with the addition of multiply scattered light. The sky polarization patterns are clearly different from those for intensity. The patterns change with μ_o and persist as τ_R increases, albeit with decreasing amplitude.

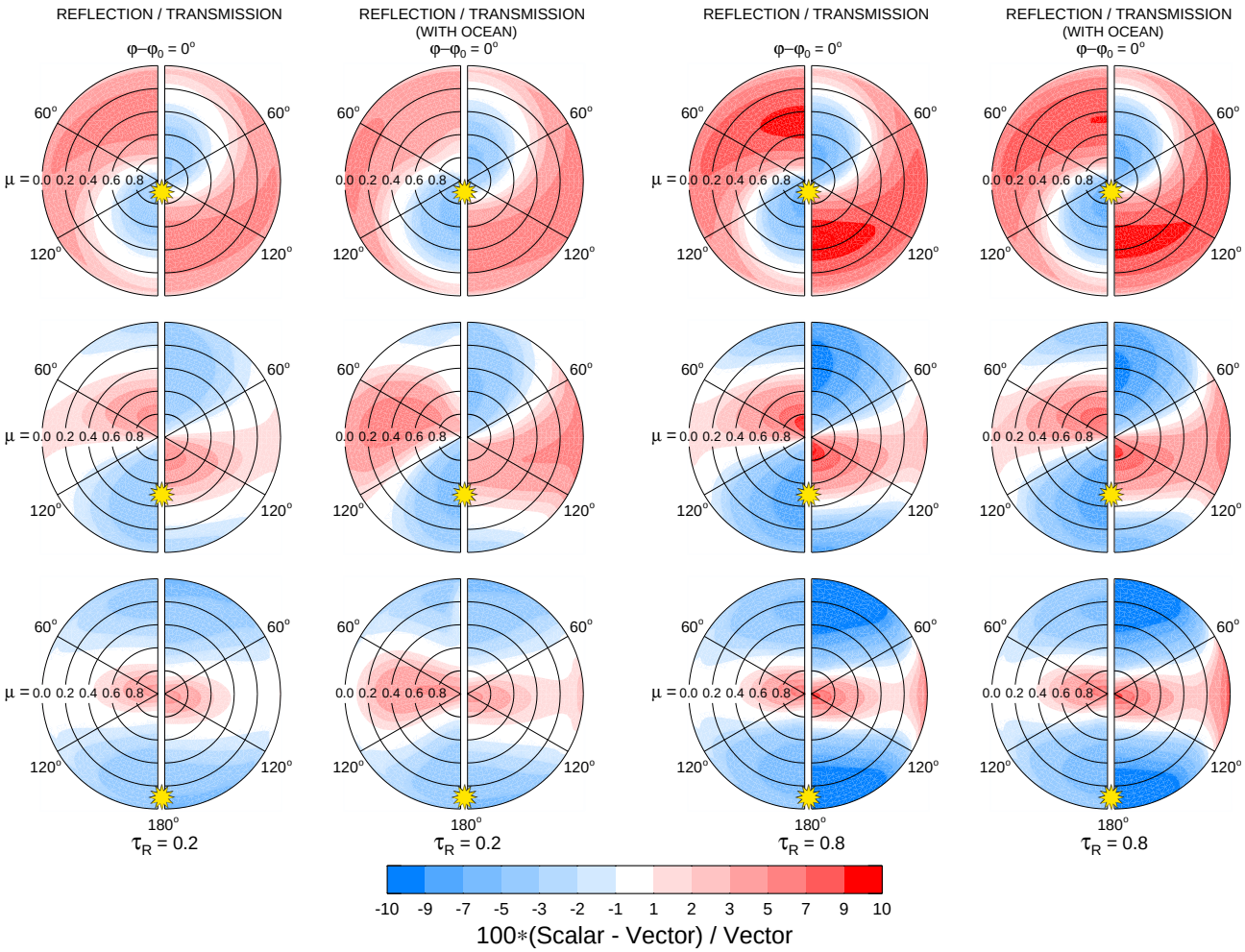

Figure 2. Scalar approximation intensity errors in Rayleigh scattering atmospheres. The split-hemisphere polar plots display relative intensity errors for reflection in the left-side hemispheres and for diffuse transmission in the right-hand hemispheres. The left two columns show results for τ_R=0.2, the right hand columns for τ_R=0.8, respectively. An ocean surface is included for the results shown in columns two and four. Sun angles are for μ_o=0.9, 0.5, and 0.1, respectively.

Figure 2 shows maps of (Scalar –Vector)/Vector relative intensity error that arises from the scalar approximation. The results are for the same solar geometry and idealized Rayleigh atmosphere as depicted in Fig. 1. Reflectance errors are shown in the left-side hemisphere of each polar plot, and with diffuse transmission error in the right-side hemisphere. The two left-side columns depict results for Rayleigh optical depth τ_R=0.2, with zero surface albedo for the first column maps, and with an underlying ocean roughened by 7.2 m/sec winds for the second column. The two right-side columns depict results for τ_R=0.8 for the same respective surface conditions.

Regions of the sky that are colored blue indicate those areas where scalar approximation errors underestimate the intensity of diffuse radiation. The red regions depict areas where the intensity is overestimated. The regions in white indicate basic agreement (to within 1%) between the vector and scalar calculations. Maximum intensity errors tend to be of order 5-10% for both over- and under-estimates for the cases considered. The maps of intensity error are clearly different from those of either intensity or the degree of polarization. There is a 'flip-symmetric' similarity between the reflection and the transmission errors, along with systematic change in the error pattern with increasing μ_o. The scalar approximation tends to under-estimate near-zenith radiances at high μ_o, and to over-estimate the zenith sky brightness (also the nadir brightness for reflection) in the case of low μ_o. Notable also is the quasi-polar error pattern that occurs at low μ_o, where the scalar approximation underestimates diffuse radiances along the φ-φ_o=0° principal plane direction, but over-estimates intensity in the orthogonal direction. The error patterns for both reflection and transmission tend to be preserved as the optical depth changes. Also, there is a tendency for regions of minimum polarization to coincide with regions of maximum under-estimates arising from the scalar approximation, while the scalar over-estimates tend to coincide with regions of maximum polarization. Adding a Lambertian surface reduces the intensity errors, while retaining the basic error pattern.

Based on our earlier study (Mishchenko, *et al*., 1994), that maximum scalar errors occur near τ_R=1, and since no scalar approximation errors occur in the optically thin limit where single scattering dominates, it follows then that the error pattern amplitude increases with increasing optical depth until about τ_R=1. As optical depth increases beyond τ_R=1, the magnitude of the scalar error decreases, approaching an asymptotic limit that depends on ω_o. The degree of scalar intensity error cancellation is remarkable for plane albedo and plane transmission for a Rayleigh-scattering atmosphere (upper panels in Fig. 3). Similar error cancellation occurs for Lambertian surfaces.

Over ocean surfaces, the effective surface reflectivity depends on the polarization of the incident light. Here, the albedo and plane transmission are under-estimated for moderate τ_R and high μ_o, as shown in bottom panels of Fig. 3, with opposite signs at low μ_o. The errors tend to decrease with increasing τ_R, to their asymptotic Rayleigh values. The ocean introduces subtle shifts in the intensity error pattern (see Fig. 2), particularly for small τ_R and for moderate μ_o, that act to compromise the error cancellation (relative to the pure Rayleigh case), and thus produce unexpectedly large flux and albedo errors when the intensity is integrated over angle. The plane transmission over ocean is underestimated by as much as 0.5% for high μ_o, while the albedo errors also produce under-estimates by up to 1.3% at μ_o=1. Cancellation of the scalar intensity error is most effective for intermediate values μ_o.

The center panels in Fig. 3 show plane albedo and flux errors for a Rayleigh atmosphere layer above an aerosol layer of optical depth unity (0.5 μm radius). Aerosols larger than the wavelength of incident light act to reduce the scalar approximation intensity errors to order 1%, even though they may have strong polarization signatures at rainbow angles. For aerosols, the plane transmission is typically underestimated for high μ_o by about 0.2%, and overestimated for low μ_o by as much as 1%. Scalar error for albedo is opposite in sign, with overestimates ($\lesssim$0.5) for high μ_o, and underestimates ($\lesssim$0.3%) at low μ_o.

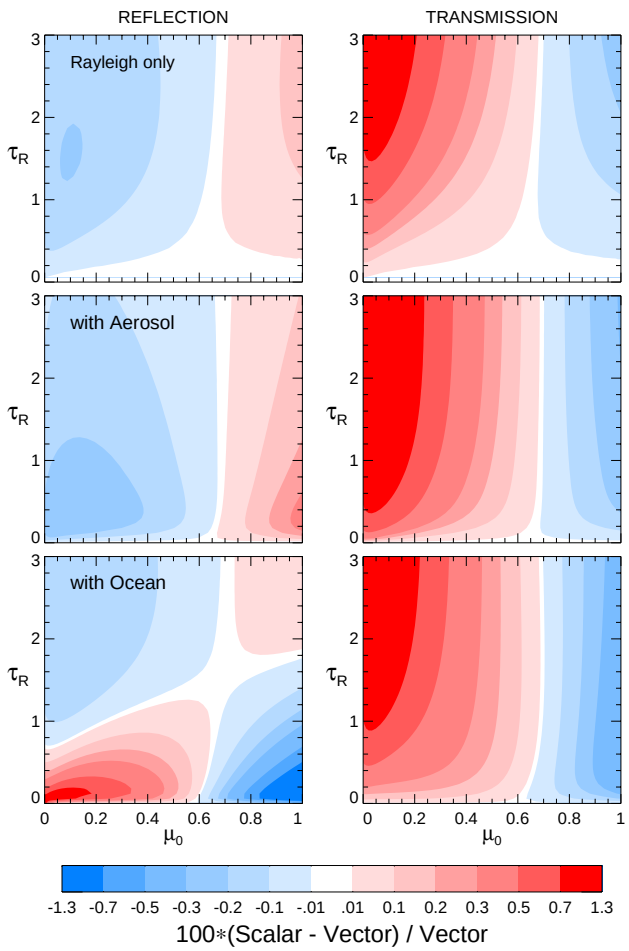

Figure 3. (Scalar –Vector)/Vector errors for plane albedo (left panels) and plane transmission (right panels) for a Rayleigh atmosphere (upper panels), and over an ocean surface (bottom) as functions of τ_R and μ_o. The center panels show Rayleigh results above a τ_A = 1 aerosol layer.

Conclusions

The scalar approximation for light propagation, though widely used, is intrinsically deficient, and consequently not adequate for those situations where high precision is required for measurement analysis. The magnitude of the error is 5-10% for typical clear-sky intensities. The plane transmission is typically under-estimated by $\lesssim$0.2% for high μ_o, (by $\lesssim$0.5% over the ocean). Consequently, scalar approximation errors are not the source of the reported over-estimates of modeled solar irradiance.

There is further cancellation of scalar errors when the plane albedos and fluxes are integrated over μ_o to obtain spherical values of albedo and transmission relevant for global energy balance. Hence, the scalar approximation continues to be more than adequate for a wide range of climate related modeling applications.

It is also clear from the complex dependence of scalar radiance errors on scattering geometry, etc., that simple scaling corrections are not likely to correct these errors. In situations where high-precision modeling is required, there really is no substitute for vector radiative transfer.

The results presented also have relevance to instrument calibration issues, particularly instruments that are used to measure clear-sky atmospheric flux. Because of strong azimuth and sun angle variation in degree of polarization and sky brightness of Rayleigh scattered light, moderated by aerosols and surface reflectance, it may be difficult to maintain the calibration of wide-field instruments that require substantial cosine corrections, particularly if their detectors have a significant sensitivity to angle, azimuth, and polarization of the incident light.

Acknowledgments. This research was supported by the Environmental Sciences Division of U.S. Department of Energy Atmospheric Radiation Measurement Program. We thank N. Zakharova for assistance with graphics.

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(Received April 24, 1997; accepted November 13, 1997.)