1993ApJ...409..495G

THE ASTROPHYSICAL JOURNAL, 409:495–503, 1993 June 1
© 1993. The American Astronomical Society. All rights reserved. Printed in U.S.A.

COSMIC TURBULENCE REVISITED

I. GOLDMAN[1,2] AND V. M. CANUTO[2]

Received 1992 July 22; accepted 1992 November 19

ABSTRACT

Primordial cosmic turbulence has been suggested in the past as a mechanism for the formation of large-scale structure in the universe, since it is more efficient than the growth of density perturbations. However, difficulties arose when it was pointed out that (1) the amplitude of the turbulence required to explain the large-scale structure would be in conflict with the observed high degree of isotropy of the microwave background radiation. (2) In addition, following recombination, the turbulence would have become supersonic and produced too large density contrasts on scales of galaxies and clusters of galaxies. (3) Finally, a difficulty of fundamental rather than observational nature: the lack of a specific physical process that could have generated the turbulence in the first place.

In this paper, we argue that inflation naturally provides mechanisms for the direct generation of turbulence on the same scales on which density perturbations are formed, thus solving problem 3 above. This calls for a reexamination of the issue of cosmic turbulence since the observational constraints on turbulence (problems 1 and 2, above) could translate into constraints on inflation itself. We find that by the end of inflation, the amplitude of the generated turbulent velocity has been suppressed by a factor $\gtrsim 10^{100}$ (this result also applies to any turbulence assumed to be part of the initial conditions). Thus, inflation guarantees the absence of turbulence on scales of galaxies and clusters of galaxies and avoids difficulties 1 and 2 above. Alternatively, they constitute an additional support for inflationary cosmology.

We show that the density fluctuations generated by inflation can excite longitudinal turbulence after they reenter the Hubble radius at later cosmic epochs (at $z \gtrsim 27 z_{eq}$). The scales on which this happens are *much smaller* than those of galaxies. The largest scale corresponds to a present-epoch size of $\lesssim 6.3$ kpc and contains a mass of $\lesssim 3.6 \times 10^4\ M_\odot$. The smallness of these scales renders the turbulence immune from the observational difficulties 1 and 2 above. Since the generation of this "secondary" and "small-scale" turbulence seems unavoidable, it should be regarded as yet another ingredient of cosmology.

In spite of its small scale, this turbulence can have an important impact on the formation of structure on scales of galaxies and clusters of galaxies. This is so because any part of the turbulence that survived dissipation by the radiative viscosity will become supersonic following the decoupling time. Shocks collisions would lead to large density contrasts on the above small scales. Such an early population of objects of mass $\lesssim 3.6 \times 10^4\ M_\odot$ can serve as a seed that could help the growth of density on the scales of galaxies and clusters of galaxies.

Subject headings: cosmology: theory — turbulence

1. INTRODUCTION

Cosmic turbulence has long been suggested as a mechanism for the formation of large-scale structure in the universe (von Weizsäcker 1951; Gamow 1952; Nariai 1956a, b; Ozernoi & Chernin 1968a, b). This idea was further developed by Tomita et al. (1970), by Kurshov & Ozernoi (1974a, b, 1975, 1976), by Tanabe et al. (1975) and by Nariai & Tanabe (1978). The hope was that velocity fluctuations could excite density fluctuations at a level sufficient for gravitational growth of galaxies to overcome the cosmic expansion. Moreover, Ozernoi & Chibisov (1971a, b) suggested that the angular momentum of galaxies originated from the vortical eddies from which they formed. Harrison (1970a, 1973) showed that cosmic turbulence in the radiation-dominated era could have generated a magnetic field which could serve as a seed for galactic magnetic fields (for a review of this and other mechanisms for generating cosmic magnetic fields, see Rees 1987a). This extensive work on cosmic turbulence came, to a halt however, because of difficulties of an observational and fundamental nature.

Perhaps the most serious observational difficulty follows from the high degree of isotropy of the 3 K radiation observed on angular scales larger than few arcminutes. This severely constrains the assumed turbulent spectrum (Anile et al. 1976; Kurskov & Ozernoi 1978), implying that turbulence could not have been strong enough to generate density fluctuations at the levels required for the formation of galaxies and clusters of galaxies. Another difficulty stems from the fact that, after decoupling, the turbulence would have become supersonic forming shocks whose collisions would have produced too large density contrasts on scales of galaxies and clusters of galaxies (Peebles 1980).

There were also difficulties of a more fundamental nature related to the fact that no specific physical mechanism for the generation of turbulence, or a cosmic epoch at which it was generated, was suggested. Thus, it was assumed that turbulence originated at some cosmic time earlier than t_{eq}, which is the time when the energy densities of radiation and matter were equal. In the context of big bang cosmology this brings in a causality problem. Scales that contain a mass equal to or larger than that of a galaxy and that were at the late epochs ($t > t_{eq}$)

[1] School of Physics and Astronomy, Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv 69978, Israel (postal address).

[2] NASA Goddard Institute for Space Studies, 2880 Broadway, New York, NY 10025.

© American Astronomical Society • Provided by the NASA Astrophysics Data System

1993ApJ...409..495G

within the Hubble radius, $l_H = cH^{-1}$, must have had a size larger than the Hubble radius at some early enough cosmic time. If a spectrum of turbulence is assumed to be part of the initial conditions of the big bang cosmology, then no generation mechanism is needed and all scales are in principle possible. However, one may wish to avoid such an ad hoc assumption and instead regard turbulence as having been generated by some physical mechanism in the early universe. In this case, only scales smaller than the Hubble radius at the generation time are possible. This is so because no causal physical mechanism can operate on scales exceeding the Hubble radius (Bardeen, Steinhardt, & Turner 1983). At later times, these scales will have sizes so small as to be *irrelevant* for the formation of galaxies.

Inflationary cosmology (for reviews, see Blau & Guth 1987; Narlikar & Padmanabhan 1991) calls for a reexamination of the issue since it naturally solves the causality problem and provides mechanisms for turbulence generation—the GUT phase transition that initiated inflation, and the quantum fluctuations that generated the density fluctuations. This raises the question of whether the observational constraints on turbulence imply constraints on inflation itself. In the present work we find that these turbulent velocities (or relic from the preinflation era) were suppressed by a factor $\gtrsim 10^{100}$ by the end of the inflationary phase. Therefore, while inflation can provide a mechanism for the generation of cosmic turbulence and also solve the causality problem, it also implies that by the end of inflation turbulence would have been reduced to an *unobservable* amplitude, thus avoiding the above observational conflicts. This is an additional confirmation of the consistency of inflationary cosmology with observations.

Inflation naturally generates density fluctuations (Blau & Guth 1987; Narlikar & Padmanabhan 1991). The scale invariant spectrum (Harrison 1970b; Zeldovich 1972) is consistent with the fluctuations in the temperature of the microwave background radiation detected recently by *COBE* (Silk 1992). This opens an interesting possibility since density fluctuations can excite turbulence, as shown recently by Batchelor, Canuto, & Chasnov (1992). Even though the cosmological case is different than that considered by Batchelor et al. (1992), the latter work points to the possibility that density fluctuations could excite turbulence after they reenter the Hubble radius at later cosmic times. To investigate this possibility, we extend the formalism for incompressible cosmic turbulence and derive coupled equations for density and velocity fluctuations evolving on a cosmic background. Density fluctuations can excite longitudinal velocity fluctuations, which in turn, through the nonlinear velocity interactions, can generate a longitudinal turbulence.

We study analytically the feasibility of longitudinal turbulence generation by the density fluctuations on the basis of the time scales of the competing physical processes. We obtain the necessary conditions which translate into constraints on the generation epoch and on the maximal possible scale. We find that the process could have occured at two different epochs. In the earlier one, defined by $z \gtrsim 27 z_{\rm eq}$, turbulence is generated on scales that correspond to a present-epoch size smaller than ~ 6.3 kpc, and contain a mass of $\lesssim 3.6 \times 10^4\ M_\odot$. The amplitude of the generated turbulent velocity is $\sim 0.01c$ and it is subsonic at the time of generation. The second epoch is the postdecoupling era, and the generated scales are smaller: 0.2 kpc, containing a mass of $\sim 1\ M_\odot$. The turbulent velocity is $\sim 2 \times 10^{-6}\ c$, and it is subsonic.

The small size of these scales renders the turbulence immune to the observational constraints imposed by the isotropy of the background radiation, and by the absence of overdense structures on scales of galaxies and clusters of galaxies. In spite of its small scale, this turbulence can have interesting implications regarding the formation of structure on the larger scales. After matter and radiation decouple, there is a sharp decrease in the sound velocity. As a result, if part of the turbulence that was generated in the earlier epoch survived viscous dissipation, it will become supersonic. The resulting shock collisions would generate large density enhancements which could serve as seeds helping the growth of density on the larger scales of galaxies and clusters of galaxies.

2. INCOMPRESSIBLE COSMIC TURBULENCE

We briefly review the formalism for incompressible, nonrelativistic cosmic turbulence by Tomita et al. (1970) and Kurshov & Ozernoi (1974a) based on the work by Nariai (1956a, b). The energy-momentum densities associated with turbulence are negligible compared to those of the unperturbed cosmic fluid, implying a cosmic evolution unaffected by turbulence. The situation is that of a "passive" turbulence evolving in an expanding cosmic background. In the radiation-dominated era, the sound velocity is $3^{-1/2}c \gg v$, the turbulent velocity, so that the assumptions of quasi-incompressibility and a purely vortical turbulent velocity are justified. As a result of the sharp drop in the sound speed after decoupling time, compressibility can become important.

These authors used a Robertson-Walker metric describing a spatially flat, homogeneous isotropic universe:

$$ds^2 = c^2 dt^2 - R^2(t)(dx^{1^2} + dx^{2^2} + dx^{3^2}) . \quad (1)$$

Indeed, for $z \gtrsim z_d$ the spatial curvature term in the dynamical equation for $R(t)$ is small compared to the matter term. Using the energy-momentum tensor appropriate to a viscous fluid, Tomita et al. (1970) obtained a general relativistic generalization of the Navier-Stokes equations

$$\frac{\partial u^i}{\partial t} + u^j \frac{\partial u^i}{\partial x^j} = -c^2 R^{-2}(\rho + p)^{-1} \frac{\partial p_1}{\partial x^i} - \frac{\partial \ln[(\rho + p)R^5]}{\partial t} u^i + R^{-2} \nu \frac{\partial^2 u^i}{\partial x^j \partial x^j} , \quad (2)$$

where ρ, p are the energy density and pressure, respectively, of the unperturbed cosmic fluid and are functions of time only, while p_1 denotes the fluctuating pressure. The unperturbed fluid four-velocity is $U^\mu = \delta^{\mu 0}$. The turbulent four-velocity u^i is related to the physical velocity v^i, measured by an observer at rest in the coordinate system, by $v^i = R(t)u^i$. Because of incompressibility, the turbulence is vortical

$$\frac{\partial v^i}{\partial x^i} = \frac{\partial u^i}{\partial x^i} = 0 . \quad (3)$$

The kinematic viscosity ν is the radiative viscosity to be discussed in § 5. In terms of the physical velocity, v^i, equation (2) takes the form

$$\frac{\partial v^i}{\partial t} + R^{-1} v^j \frac{\partial v^i}{\partial x^j} = -c^2 R^{-1}(\rho + p)^{-1} \frac{\partial p_1}{\partial x^i} + R^{-2} \nu \frac{\partial^2 v^i}{\partial x^j \partial x^j} - \frac{\partial \ln[(\rho + p)R^4]}{\partial t} v^i . \quad (4)$$

© American Astronomical Society • Provided by the NASA Astrophysics Data System

1993ApJ...409..495G

Using the standard method of correlation functions and spatial Fourier transforms, it is easy to obtain the equations describing the time evolution of the turbulent kinetic energy spectrum $E(k)$ (taken to be isotropic and homogeneous, in accordance with the homegeneity and isotropy of the background)

$$-\frac{\partial E(k)}{\partial t} = T(k) + 2\left\{R^{-2}\nu k^2 + \frac{\partial \ln[(\rho + p)R^4]}{\partial t}\right\}E(k) , \quad (5)$$

$$\tfrac{1}{2}v^2(k) = \int_k^\infty E(k')dk' , \quad (6)$$

where k is a wavenumber defined with respect to the *comoving* coordinates x^i, $v(k)$ is the turbulent velocity corresponding to k, and $T(k)$ is the transfer term representing the nonlinear interactions among eddies of different wavenumbers.

As pointed out by Nariai & Tanabe (1978), the transfer term is defined in terms of the *physical* wavenumber $\hat{k} = k/R(t)$. The spectral function $\hat{E}(\hat{k})$ and the transfer $\hat{T}(\hat{k})$ in terms of $\hat{k}$ are related to those in terms of k by the relations

$$\hat{E}(\hat{k}) = R(t)E(k) , \quad (7)$$

$$\hat{T}(\hat{k}) = R(t)T(k) . \quad (8)$$

One may use equation (5) provided that $T(k)$ is determined from equation (8) where the physical transfer is $\hat{T}(\hat{k})$. As discussed by Tanabe (1981), for the specific form of the transfer term suggested by Heisenberg (Batchelor 1973), namely $T(k) = [E(k)k]^{3/2}$, it turns out that

$$\hat{T}(\hat{k}) = \hat{T}(k) \quad (9)$$

so that equation (8) yields

$$T(k) = R^{-1}\hat{T}(k) . \quad (10)$$

We note that equation (9) is satisfied for a class of transfer functions defined as functionals of the spectral function and the wavenumber and satisfying (for any λ)

$$\hat{T}(\lambda\hat{E}, \lambda^{-1}\hat{k}) = \hat{T}(\hat{E}, \hat{k}) . \quad (11)$$

Therefore, equation (10) will hold also for this class of transfer terms.

In order for any physical process to operate in an expanding universe, its characteristic time scale must be shorter than $H^{-1}(t)$. In the case of turbulence, the relevant time scale is that of the nonlinear interaction among eddies $l/v(l)$, where $l = R(t)\pi/k$ is the physical size corresponding to the wavenumber k, and $v(l)$ is given by equation (6), so that the above condition becomes

$$l/v(l) \lesssim H^{-1} . \quad (12)$$

For spectra $E(k)$ where $l/v(l)$ is an increasing function of l (a condition satisfied by most spectra), equation (12) defines the largest scale for which the eddy interaction is effective. Scales exceeding the above limit will be frozen, that is, the velocity and scale will change only due to the cosmic expansion. In particular, since $v(l) < c$, this implies that all scales larger than the Hubble radius are frozen.

3. TURBULENCE IN THE INFLATIONARY ERA

Let us turn now to the inflationary era (Blau & Guth 1987; Narlikar & Padmanabhan 1991). The preinflation (classic, post-Planckian) universe is radiation dominated with the radiation energy density contributed by all particles that are relativistic. Inflation started at some initial time t_i in a region where the temperature fell below a critical value $T_i \sim T_{\rm GUT} \sim 10^{14}$ GeV. Subsequently, the universe underwent an exponential expansion

$$R(t) = R_i e^{(t-t_i)/\tau} , \quad (13)$$

where $\tau \sim \hbar(kT_{\rm GUT})^{-1} \sim 3 \times 10^{-35}$ s. In the course of the expansion, the temperature decreased exponentially until $t = t_f$ when a phase transition caused a sudden reheating to a temperature $T_f \sim T_i \sim 10^{14}$ GeV. From this time on, the universe continued to evolve as a radiation-dominated Friedmann universe. During the inflationary phase, every scale expanded by a factor Z given by

$$Z = e^{(t_f - t_i)/\tau} \quad (14)$$

where $Z \sim 10^{25}$ is the minimal value required for the entire presently observed universe to be within the Hubble radius at $t = t_i$.

Inflation provides a natural mechanism for the generation of turbulence. Possible sources are the thermal fluctuations following the GUT phase transition that started inflation, and the quantum fluctuations in the scalar field that generated the density fluctuations. When generated, any scale in the turbulence spectrum is within the Hubble radius and thus causally connected. As the universe expands exponentially, the above scale crosses the Hubble radius and goes out of causal contact. These same scales will reenter the Hubble radius and resume causal contact at some late epoch in the Friedmann phase. This solves the causality problem.

Thus, inflation naturally solves the two long-standing obstacles faced by cosmic turbulence. However, in what follows we show that any turbulence generated at that epoch will be suppressed by a factor $\gtrsim 10^{100}$ when inflation is over, at $t = t_f$.

At any given cosmic time t, the generated largest scale of turbulence cannot exceed the Hubble radius $l_H(t)$ at that time. From equation (13) follows

$$l_H(t) = x_H(t)R(t) = c\tau , \quad (15)$$

where x_H denotes the corresponding *comoving* scale. Therefore, only comoving scales satisfying $x \leq x_H(t)$ can be generated at the time t. Equations (13) and (15) imply that $x_H(t)$ decreases exponentially, so that at some later time, a given x that was originally smaller than $x_H(t)$, exceeds it and is no longer in causal contact. In contrast, in a Friedmann cosmology, x_H increases monotonically with time so that any x contained within it at a given time, will never cross it.

Let t_* be the time when some scale crosses the Hubble radius. For $t \geq t_*$ the above scale is frozen out and cannot interact with other scales or be dissipated by viscosity. Thus, equations (4) and (5) imply that it evolves according to

$$\frac{\partial}{\partial t}[(\rho + p)R^4 v] = 0 , \quad (16)$$

so that

$$(\rho + p)(t)R^4(t)v(t) = B , \quad (17)$$

with B constant. In the inflationary era

$$\rho + p = \rho_v + p_v + \tfrac{4}{3}\rho_r , \quad (18)$$

© American Astronomical Society • Provided by the NASA Astrophysics Data System

1993ApJ...409..495G

where ρ_v and p_v are the vacuum energy density and pressure responsible for inflation, and ρ_r is the energy density of radiation, respectively. During the inflationary era $\rho_v \gg \rho_r$, except at the beginning and at the end. Moreover, during inflation ρ_v and p_v effectively stay constant and effectively $p_v = -\rho_v$. Therefore, $\rho_v + p_v$ equals the sum of the deviations of ρ_v and p_v from their constant parts. For scalar inflation these deviations are equal to each other (Narlikar & Padmanabhan 1991). Thus, the covariant conservation equation for the vacuum energy-momentum tensor implies that the deviation of $\rho_v + p_v$ from zero satisfies

$$\rho_v + p_v \propto R^{-6} \tag{19}$$

so it is decreasing with R faster than does $\rho_r \propto R^{-4}$. Since at the beginning of inflation, at t_i, $\rho_r \sim \rho_v \gg (\rho_v + p_v)$ then at t_* one has $\rho_r \gg (\rho_v + p_v)$. With this result, evaluating equation (17) at t_* and using $\rho_r \propto R^{-4}$ yields

$$B = \tfrac{4}{3}\rho_r(t_*)R^4(t_*)v(t_*) = \tfrac{4}{3}\rho_r(t_i)R^4(t_i)v(t_*) \,. \tag{20}$$

After reheating, the universe enters a radiation-dominated Friedmann phase. Equation (17) still holds with the same value of B since the velocity field is frozen. Thus,

$$B = \tfrac{4}{3}\rho_r(t_f)R^4(t_f)v(t_f) \,, \tag{21}$$

but $T_f \sim T_i$, implying that $\rho_r(t_f) \sim (t_i)$ so that equations (20) and (21) yield

$$\begin{aligned} v(t_f) &= v(t_*)[R(t_i)/R(t_f)]^4 \\ &= v(t_*)Z^{-4} \lesssim v(t_*)10^{-100} \,, \end{aligned} \tag{22}$$

where we used equations (13) and (14).

Therefore, any turbulence generated during the inflationary era will be essentially *erased*. This conclusion will also apply to preinflation turbulence, either postulated as part of the initial conditions or generated in the preinflation era. The supression is due to the fact that while the temperature and energy densities are roughly the same at the beginning and the end of the inflationary phase, the scale factor expanded by at least 25 orders of magnitude.

Turbulence generated at the reheating time will not be erased, but at this time x_H corresponds to a scale which at the present epoch has a size of $\sim 10^3$ cm. Such a scale was a tiny $\sim 10^{-17}$ fraction of the Hubble radius at the time when the temperature was 10^{10} K as to have been completely dissipated by neutrino viscosity (Weinberg 1971, 1972).

4. COUPLED EQUATIONS FOR TURBULENCE AND DENSITY FLUCTUATIONS

The universe emerges from the inflationary era with no turbulence but with a density-fluctuation spectrum of the form suggested by Harrison (1970b) and Zeldovich (1972; see also Blau & Guth 1987; Narlikar & Padmanabhan 1991). Such a spectrum is consistent with the fluctuations in the temperature of the microwave background radiation recently detected by *COBE* (Silk 1992).

In the following, we consider the possibility that these density fluctuations can generate turbulence which in turn can affect the evolution of the density fluctuations. Turbulence can also be important in the determination of the transport properties of the cosmic fluid as well as to excite magnetic fields that could serve as a seed for galactic magnetic fields. To explore these topics, one must consider the coupled equations describing density and velocity fluctuations. Given the small value of the density fluctuations they are treated in the linear approximation. As for the velocity fluctuations, we include the nonlinear velocity interaction terms, thus allowing for turbulence. These terms are important, regardless of the amplitude of the velocity fluctuations, as long as the eddy interaction time scale is shorter than the time scales of viscous dissipation and of cosmic expansion. Qualitatively, density fluctuations can generate longitudinal velocity fluctuations which in turn, through the nonlinear terms, can generate longitudinal turbulence.

Following Weinberg (1972), we consider small perturbations in the metric, the energy density, the pressure, and the four-velocity. The dissipative terms proportional to the heat conduction and to bulk viscosity are neglected and only terms proportional to the shear viscosity are retained. The unperturbed energy density and pressure are denoted by ρ and p, respectively. The corresponding perturbations are ρ_1 and p_1, respectively. The metric is given as

$$g_{\mu\nu} = g^0_{\mu\nu} + h_{\mu\nu} \,, \tag{23}$$

where $g^0_{\mu\nu}$ is the unperturbed metric of equation (1) and $h_{\mu\nu}$ is the metric perturbation. The perturbation in the velocity is denoted by u^i, as in § 2. It can be shown (Weinberg 1972) that it is possible to apply coordinate transformations that do not change the unperturbed quantities but yield $h_{0\alpha} = 0$. In the perturbation equations we retain only linear terms except for the velocity equation in which we retain, as explained above, the nonlinear term in the velocities that can give rise to turbulence. The resulting equations are

$$\frac{\partial \rho_1}{\partial t} + 3\frac{\dot{R}}{R}(\rho_1 + p_1) = -(\rho + p)\left[\frac{\partial}{\partial t}\left(\frac{h^*}{2R^2}\right) + \frac{\partial u^j}{\partial x^j}\right] , \tag{24}$$

$$\frac{\partial^2 h^*}{\partial t^2} - 2\frac{\dot{R}}{R}\frac{\partial h^*}{\partial t} + 2\left[\left(\frac{\dot{R}}{R}\right)^2 - \frac{\ddot{R}}{R}\right]h^* = -8\pi\frac{G}{c^2}(\rho_1 + 3p_1)R^2 \,, \tag{25}$$

$$\begin{aligned} \frac{\partial u^i}{\partial t} + u^j\frac{\partial u^i}{\partial x^j} &= -c^2R^{-2}(\rho + p)^{-1}\frac{\partial p_1}{\partial x^i} - \frac{[\partial \ln(\rho + p)R^5]}{\partial t}u^i \\ &\quad - 16\pi\frac{G}{c^4}v(\rho + p)u^i + vR^{-2} \\ &\quad \times\left[\frac{\partial^2 u^i}{\partial x^j \partial x^j} + \frac{1}{3}\frac{\partial^2 u^j}{\partial x^i \partial x^j} + \frac{4}{3}\frac{\partial^2}{\partial t\,\partial x^i}\left(\frac{h^*}{2R^2}\right)\right] , \end{aligned} \tag{26}$$

where $h^* = R^2 h^k_k$.

Equation (26) differs from equation (2) by terms containing the divergence of u^i, by the appearance of the metric perturbation in the viscosity term, and by the appearance of the term proportional to the viscosity which multiplies the velocity. The latter did not appear in equation (2), taken from Tomita et al. (1970), since these authors did not consider perturbations to the metric in the covariant derivatives of the energy-momentum tensor of the cosmic fluid. The additional viscosity term is scale independent. Its magnitude relative to the ordinary viscosity term which depends on the second spatial derivatives of the velocity, for a scale of physical size l, is of the order of

$$16\pi\frac{G}{c^4}(\rho + p)l^2 = 6\left(1 + \frac{p}{\rho}\right)\left(\frac{l}{l_H}\right)^2 = (6\text{–}8)\left(\frac{l}{l_H}\right)^2 , \tag{27}$$

© American Astronomical Society • Provided by the NASA Astrophysics Data System

1993ApJ...409..495G

where the dynamical equation

$$H^2 = \frac{8\pi G}{3c^2}\rho - \frac{k}{R^2} \tag{28}$$

was used (with the curvature term neglected—which is justified for times earlier than the decoupling time). The coefficients 6 and 8 correspond to matter-dominated and radiation-dominated epochs, respectively. In the next section we shall find that for turbulence generated from density perturbations, the maximal l is two orders of magnitude smaller than the Hubble radius; thus, the new term is not important.

For adiabatic perturbations we have

$$p_1 = \left(\frac{v_s}{c}\right)^2 \rho_1 \tag{29}$$

with the sound velocity v_s given by

$$v_s^2 = \frac{\dot{p}}{\dot{\rho}} c^2 , \tag{30}$$

where a dot stands for a time derivative.

Applying the conservation equation for the unperturbed energy-momentum tensor

$$\dot{\rho} = -3\frac{\dot{R}}{R}(\rho + p) , \tag{31}$$

and using equations (29), (30), (31) in equation (24) yields

$$\frac{\partial\delta}{\partial t} = -\frac{\partial}{\partial t}\left(\frac{h^*}{2R^2}\right) - \frac{\partial u^j}{\partial x^j} , \tag{32}$$

with

$$\delta = \frac{\rho_1}{\rho + p} . \tag{33}$$

Equation (32) can be used to substitute for h^* in equations (25) and (26), resulting in coupled equations for the density and velocity perturbations.

$$\frac{\partial^2\delta}{\partial t^2} + 2\frac{\dot{R}}{R}\frac{\partial\delta}{\partial t} = 4\pi\frac{G}{c^2}(\rho + p)\left(1 + 3\frac{v_s^2}{c^2}\right)\delta - \left(\frac{\partial}{\partial t} + 2\frac{\dot{R}}{R}\right)\frac{\partial u^j}{\partial x^j} , \tag{34}$$

$$\frac{\partial u^i}{\partial t} + u^j\frac{\partial u^i}{\partial x^j} = -R^{-2}v_s^2\frac{\partial\delta}{\delta x^i} - \frac{\partial\ln[(\rho + p)R^5]}{\partial t}u^i - 16\pi\frac{G}{c^4}\nu(\rho + p)u^i + \nu R^{-2} \times\left(\frac{\partial^2 u^i}{\partial x^j\partial x^j} - \frac{\partial^2 u^j}{\partial x^i\partial x^j} - \frac{4}{3}\frac{\partial^2\delta}{\partial t\partial x^i}\right) . \tag{35}$$

In terms of the physical velocity $v^i = R(t)u^i$, but using the same comoving spatial coordinates, equations (34) and (35) take the form

$$\frac{\partial^2\delta}{\partial t^2} + 2\frac{\dot{R}}{R}\frac{\partial\delta}{\partial t} = 4\pi\frac{G}{c^2}(\rho + p) \times\left(1 + 3\frac{v_s^2}{c^2}\right)\delta - R^{-1}\left(\frac{\partial}{\partial t} + \frac{\dot{R}}{R}\right)\frac{\partial v^j}{\partial x^j} \tag{36}$$

$$\frac{\partial v^i}{\partial t} + R^{-1}v^j\frac{\partial v^i}{\partial x^j} = -R^{-1}v_s^2\frac{\partial\delta}{\partial x^i} - \frac{\partial\ln[(\rho + p)R^4]}{\partial t}v^i - 16\pi\frac{G}{c^4}\nu(\rho + p)v^i + \nu R^{-2} \times\left[\frac{\partial^2 v^i}{\partial x^j\partial x^j} - \frac{\partial^2 v^j}{\partial x^i\partial x^j} - \frac{4}{3}R(t)\frac{\partial^2\delta}{\partial t\partial x^i}\right] . \tag{37}$$

In the general case, the nonlinear term in the velocities couples and mixes longitudinal and vortical velocities. However, in the present case, the density fluctuations excite purely longitudinal velocity fluctuations. For longitudinal velocity fluctuations, the nonlinear term in equation (37) that generates the turbulence can be shown to have a zero curl. Thus, also the generated turbulence will be purely longitudinal. Only if some seed of vortical velocity fluctuations is present could it be amplified by the nonlinear interactions with the longitudinal turbulence, so that a component of vortical turbulence could be generated also. In what follows, the term turbulence will be meant to denote the above longitudinal turbulence.

5. GENERATION OF TURBULENCE IN THE PREDECOUPLING EPOCH

We wish to use the above coupled equations to explore the possibility that the density fluctuations, after reentry through the Hubble radius, can generate turbulence. Specifically, it is of interest to find the epoch at which generation occurs, the size of the generated scales, the amplitude of the generated turbulent velocity, and the evolution in time of the two coupled spectra. In order to obtain a detailed and quantitative answer, the equations should be solved numerically. However, one can gain quite a considerable insight from an analytic study based on the time scales of the competing physical processes which will yield the necessary conditions for the generation of turbulence. These conditions provide the answer to the first three questions above.

Before deriving these constraints, it is useful to prepare some expressions and state the assumptions made. The decoupling of photons from baryons occurs at a temperature of ~ 3000 K, corresponding to a redshift $z_d \sim 10^3$. The time t_{eq} when the energy densities of matter and radiation were equal, corresponds to a redshift

$$1 + z_{eq} = 4.3 \times 10^4\Omega h^2(1 + 0.227N_\nu)^{-1} , \tag{38}$$

where Ω is the present density in units of the closure density, h is the present value of the Hubble constant in units of 100 km s^{-1} Mpc^{-1}, and N_ν is the number of neutrino families with masses $\lesssim 0.3$ eV. The numbers are normalized to a temperature of 2.7 K for the microwave background radiation. In what follows we adopt $N_\nu = 2$ to allow for the existence of a tau-neutrino more massive than the above limit. This yields

$$1 + z_{eq} = 2.95 \times 10^4\Omega h^2 . \tag{39}$$

Inflation implies $\Omega = 1$, a value that requires the existence of dark matter (DM). Standard nucleosynthesis requires (Narlikar & Padmanabhan 1991)

$$\Omega_b h^2 \leq 0.026 , \tag{40}$$

where Ω_b is the present epoch baryon density in units of the closure density. Since $h \gtrsim \frac{1}{2}$, it follows that $\Omega_b \lesssim 0.1$, so that most of the DM on cosmological scales must be nonbaryonic.

© American Astronomical Society • Provided by the NASA Astrophysics Data System

1993ApJ...409..495G

One should note that nonstandard nucleosynthesis may allow for values of Ω_b up to ~ 1, alleviating the need for nonbaryonic DM (Rees 1987b). In what follows we assume that any nonbaryonic DM present consists of particles that are nonrelativistic at the epoch relevant for turbulence generation. In this case, one deals with cold DM (CDM) which is decoupled, except gravitationally, from the baryons and photons. The cosmic fluid considered for the generation of turbulence, consists of all particles that are coupled and in thermal equilibrium. The uncoupled components obey separate conservation equations, and although decoupled, their energy density contributes to the cosmological expansion. For redshifts $10^{10} \gtrsim z \gtrsim z_d$, the fluid consists of photons and baryonic matter while for $z < z_d$, it consists of baryonic matter only. Neutrinos that were part of the coupled fluid in the early universe, decoupled at $z \lesssim 10^{10}$.

The time scale of the cosmological expansion is $H^{-1}(z)$, with $H(z)$ the Hubble parameter at redshift z. From equation (28) it follows that

$$H(z) = H_0 \Omega^{1/2}(1+z)^{3/2} \times \left[\left(1+\frac{1+z}{1+z_{\rm eq}}\right)\left(1+\frac{1}{1+z_{\rm eq}}\right)^{-1} + \frac{1-\Omega}{\Omega}\frac{1}{1+z}\right]^{1/2} . \tag{41}$$

Inflation requires $\Omega = 1$ so that the last term in the parentheses, which stems from the curvature term in equation (28), vanishes. Even if $\Omega > 0.1$, the above term is smaller than 10^{-2} for $z > z_d$. Therefore,

$$H(z) \simeq H_0 \Omega^{1/2}(1+z)^{3/2}\left(1+\frac{1+z}{1+z_{\rm eq}}\right)^{1/2} . \tag{42}$$

Four time scales must be considered. The first is τ_b, the time scale for buildup of longitudinal velocity from the density fluctuation. From equation (37) it follows that

$$\tau_b^{-1} = \frac{1}{v}\frac{\partial v}{\partial t} \sim \frac{v_s^2 \delta}{l v(l)} , \tag{43}$$

where l is the physical size of the scale generated and $v(l)$ is the amplitude of the generated velocity; both values are considered at the time of generation corresponding to the redshift z.

The second time scale is the one characterizing the buildup of longitudinal turbulence, namely the time scale of the nonlinear eddy interaction $l/v(l)$. The other two time scales are that of cosmic expansion H^{-1}, and τ_v, that of viscous dissipation on the scale l:

$$\tau_v^{-1} = \nu\left(\frac{\pi}{l}\right)^2 . \tag{44}$$

In order for density fluctuations on a scale l to generate a longitudinal velocity fluctuation it is necessary that

$$\tau_b^{-1} > H , \tag{45}$$

which, upon using equation (43), yields

$$l v(l) < v_s^2 \delta H^{-1} . \tag{46}$$

For the generated velocity to excite turbulence, the time scale of the nonlinear eddy interaction must be shorter than the cosmic expansion time scale. Thus,

$$\frac{v(l)}{l} > H \tag{47}$$

which, when combined with equation (46), results in an upper limit on the scale l for which turbulence can be excited at a given redshift z, namely,

$$\frac{l}{l_H} < \frac{v_s}{c}\delta^{1/2} . \tag{48}$$

An additional necessary condition follows from the requirement that the turbulence generated at a scale l not be dissipated by viscosity. This requires an eddy interaction time scale shorter than the viscous time scale τ_v

$$\frac{v}{l} > A\tau_v^{-1} = A\nu\left(\frac{\pi}{l}\right)^2 . \tag{49}$$

where $A > 1$ is a dimensionless number. In the following, we normalize A to 10 so that $A = 10A_{10}$. Combining equations (46) and (49), yields

$$\nu\pi^2 < 0.1 A_{10}^{-1} v_s^2 \delta H^{-1} , \tag{50}$$

which is scale independent. This is the necessary condition for the generation of turbulence.

Let us consider first turbulence generation in the predecoupling era. The dominant viscosity for $z > z_d$, is the radiative viscosity resulting from scattering of photons by electrons, given by (Weinberg 1972)

$$\nu = \frac{4}{15} l_{\rm T} c \frac{\rho_\gamma}{\rho_b + 4/3\rho_\gamma} ; \tag{51}$$

where $l_{\rm T}$ is the Thomson mean free path

$$l_{\rm T} = (\sigma_{\rm T} n_e)^{-1} \tag{52}$$

and $\sigma_{\rm T} = 6.65 \times 10^{-25}$ cm^2 is the Thomson electron scattering cross section. The electron number density n_e is taken to be

$$n_e = \frac{\rho_b}{m_p c^2}(X + 0.5Y) \sim 0.88\frac{\rho_b}{m_p c^2} , \tag{53}$$

where X and Y denote the cosmological abundances of hydrogen and helium, respectively, m_p is the mass of the proton, and ρ_b is the energy density of baryonic matter. In equation (53) it is assumed that matter is fully ionized which is valid for cosmic times earlier than the decoupling time t_d. From equations (42), (52), and (53), it follows that

$$\frac{l_{\rm T}}{l_H} = 16.4(1+z)^{-3/2}\left(1+\frac{1+z}{1+z_{\rm eq}}\right)^{1/2}(\Omega h^2)^{-1/2}\left(\frac{\Omega}{\Omega_b}\right) . \tag{54}$$

Let us turn now to the necessary condition equation (50), and consider first the predecoupling era $z > z_d$. The sound velocity in this epoch is given by

$$\left(\frac{v_s}{c}\right)^2 = \frac{1}{3}\frac{\rho_\gamma}{\rho_\gamma + \rho_b} . \tag{55}$$

Using equations (51) and (55), one finds from equation (50)

$$\frac{l_{\rm T}}{l_H} \lesssim 1.7 \times 10^{-6}\delta_4 A_{10}^{-1}\frac{\rho_\gamma + 3/4\rho_b}{\rho_\gamma + \rho_b} = (1.3\text{–}1.7) \times 10^{-6}\delta_4 A_{10}^{-1} , \tag{56}$$

© American Astronomical Society • Provided by the NASA Astrophysics Data System

1993ApJ...409..495G

where $\delta_4 = 10^4\,\delta$. Using equation (54) in equation (56) results in

$$1 + z \gtrsim 4.6 \times 10^4 \left(\frac{A_{10}\Omega}{\Omega_b \delta_4}\right)^{2/3} (\Omega h^2)^{-1/3} \left(1 + \frac{1+z}{1+z_{\rm eq}}\right)^{1/3} . \quad (57)$$

This relation sets a lower limit on the redshift at which the process of turbulence generation can take place. In terms of the parameter

$$\zeta = \frac{1+z}{1+z_{\rm eq}} . \quad (58)$$

we have

$$\frac{\zeta}{(1+\zeta)^{1/3}} \gtrsim 1.38 \left(\frac{\Omega A_{10}}{\Omega_b \delta_4}\right)^{2/3} (\Omega h^2)^{-4/3} . \quad (59)$$

Note that in equations (57) and (59) δ_4 is the value at the time of generation of the turbulence. Inflation produces a scale-invariant density spectrum suggested by Harrison (1970b) and Zeldovich (1972) in which the amplitude of δ is the same for any scale when this scale reenters the Hubble radius. A scale that enters the Hubble radius before $t_{\rm eq}$ grows by a factor $S = O(10)$ up to $t_{\rm eq}$ (Narlikar & Padmanabhan 1991). After this time, it grows linearly with the cosmic scale factor $R(t)$. Thus one has

$$\delta_4(\zeta) = S\delta_4^0 , \quad \zeta > 1 , \quad (60)$$

$$\delta_4(\zeta) = \frac{S}{\zeta}\,\delta_4^0 , \quad \zeta < 1 , \quad (61)$$

where δ_4^0 is the value at reentry through the Hubble radius. The fluctuations in the microwave background radiation detected by *COBE*, are consistent with a value of $\delta_4^0 \sim 0.3$.

Applying equations (60) and (61) to equation (59) yields lower limits on the redshift (expressed by ζ) at which turbulence generation can take place,

$$\frac{\zeta}{(1+\zeta)^{1/3}} \gtrsim 0.34 \left(\frac{\Omega A_{10}}{\Omega_b \delta_4^0 S_{10}}\right)^{2/3} (\Omega h^2)^{-4/3} , \quad \zeta > 1 \quad (62)$$

$$\frac{\zeta}{1+\zeta} \gtrsim 3.9 \times 10^{-2} \left(\frac{\Omega A_{10}}{\Omega_b \delta_4^0 S_{10}}\right)^{2} (\Omega h^2)^{-4} , \quad \zeta < 1 . \quad (63)$$

Using the constraint on Ω_b, equation (40), derived from nucleosynthesis and normalizing δ_4^0 to 0.3, equations (62) and (63) become

$$\frac{\zeta}{(1+\zeta)^{1/3}} \gtrsim 8.8 \left(\frac{\delta_4^0 S_{10}}{0.3 A_{10}}\right)^{-2/3} (\Omega h^2)^{-2/3} , \quad \zeta > 1 \quad (64)$$

$$\frac{\zeta}{1+\zeta} \gtrsim 640 \left(\frac{\delta_4^0 S_{10}}{0.3 A_{10}}\right)^{-2} (\Omega h^2)^{-2} , \quad \zeta < 1 . \quad (65)$$

If the dimensionless quantities in the various parantheses are taken equal to unity, equation (65) has no solution. If one considers nonstandard nucleosynthesis then $\Omega_b \sim 1$ for $h \sim 0.5$ (Rees 1987b). In this case equation (63) yields

$$\frac{\zeta}{1+\zeta} \gtrsim 110 \left(\frac{\delta_4^0 S_{10}}{0.3 A_{10}}\right)^{-2} , \quad \zeta < 1 \quad (66)$$

which, again, has no solution for $\zeta < 1$. Therefore, we conclude that $\zeta > 1$ in which case equation (64) implies that $\zeta \gtrsim 27 A_{10}$. For the case of nonstandard nucleosynthesis, equation (62) yields $\zeta \gtrsim 11 A_{10}$. In what follows we use $\zeta \gtrsim 27 A_{10}$.

Having found that turbulence generation is possible for redshifts $z \gtrsim 27 A_{10} z_{\rm eq}$, let us find the size of the generated scales. The maximal scale that can be generated is given by equation (48). Note that v_s in equation (55) depends on the photon energy density ρ_γ and the baryon energy density ρ_b. For $N_\nu = 2$ these are given by

$$\rho_\gamma = 0.69 \rho_r , \quad (67)$$

$$\rho_b = \frac{\Omega_b}{\Omega}\,\rho_m , \quad (68)$$

with ρ_r and ρ_m denoting the total radiation and matter energy densities, respectively. Use of equations (67) and (68) in equation (55) yields

$$\left(\frac{v_s}{c}\right)^2 = \frac{1}{3}\,\frac{\zeta}{\zeta + 1.45(\Omega_b/\Omega)} . \quad (69)$$

The maximal scale is then obtained from equations (48) and (69), that is,

$$\frac{l}{l_H} < 10^{-2} \left[\frac{\delta_4}{3}\,\frac{\zeta}{\zeta + 1.45(\Omega_b/\Omega)}\right]^{1/2} . \quad (70)$$

Since $\zeta \gtrsim 27 A_{10}$

$$\frac{l}{l_H} \lesssim 10^{-2} \left(\frac{\delta_4^0}{0.3}\,S_{10}\right)^{1/2} . \quad (71)$$

The corresponding present-epoch size $l(z = 0)$,

$$l(z = 0) = l(z)(1 + z) , \quad (72)$$

is obtained from equations (42) and (71),

$$l(z = 0) \lesssim 6.3 A_{10}^{-1}\ {\rm kpc} \left(\frac{\delta_4^0}{0.3}\,S_{10}\right)^{1/2} (\Omega h^2)^{-1} . \quad (73)$$

The mass corresponding to this scale is $\sim 3.6 \times 10^4 A_{10}^{-1}\, M_\odot$, which is seven orders of magnitude smaller than a typical galactic mass. Note that the above scale is the maximal comoving scale that can be generated. Smaller comoving scales started to be generated at earlier times (larger ζ values) and continued to be generated up to the time when the maximal scale was generated. For each such scale, part of the turbulent energy is cascaded to smaller scales and part is dissipated by viscosity. As already stated, numerical solutions are required to find the detailed evolution of the turbulence spectrum. In particular, since the radiative viscosity is $\propto (1+z)^{-3}$, it is not clear whether part of turbulence can survive the dissipation up to the decoupling time.

We can also estimate the amplitude of the generated turbulent velocity. At any given cosmic time, equation (48) defines the largest scale for which turbulence can be generated. From equations (46) and (48) the amplitude of the turbulent velocity at the largest scale is given by

$$v \sim v_s \delta^{1/2} . \quad (74)$$

For this velocity amplitude, the time scale for buildup of velocity, τ_b, equation (43), coincides with the eddy interaction time scale $l/v(l)$. This is to be expected since the largest scales generated at a given time do not receive energy cascaded from even larger scales. As the buildup of the longitudinal velocity continues, τ_b increases while the eddy time scale decreases. The

© American Astronomical Society • Provided by the NASA Astrophysics Data System

1993ApJ...409..495G

buildup saturates when the two time scales become equal. Note that from equations (33) and (74) it follows that the kinetic energy density associated with the turbulence is roughly in equipartition with the thermal energy density associated with the density perturbations.

From equation (74) it follows that at generation, the turbulent velocity is subsonic since $\delta < 1$. For generation epochs at redshift $z \gtrsim 27A_{10}z_{\rm eq}$ ($\zeta \gtrsim 27A_{10}$), equations (69) and (74) imply that

$$\frac{v}{c} \sim 10^{-2}\left(\frac{\delta_4^0}{0.3}S_{10}\right)^{1/2} . \tag{75}$$

Despite this large amplitude, the turbulent velocity on these scales does not pose any problems concerning the isotropy of the background radiation. The scales that can be generated are more than three orders of magnitude smaller than those on which the isotropy was observed. Therefore, the problem facing the original treatments of cosmic turbulence (Anile et al. 1976; Kurskov & Ozernoi 1978) does not exist in the present case.

6. TURBULENCE IN THE POSTDECOUPLING EPOCH

Could turbulence be generated at times later than t_d? At this time the relevant fluid consists of baryonic matter only and the sound velocity is given by

$$\frac{v_s}{c} \sim \left(\frac{5}{3}\frac{kT_b}{m_p c^2}\right)^{1/2} \sim 2 \times 10^{-5}\frac{1+z}{1+z_d} , \tag{76}$$

where

$$T_b \sim \left(\frac{1+z}{1+z_d}\right)^2 T(z_d) \tag{77}$$

is the baryon temperature at redshift $z \leq z_d$, $T(z_d) \sim 3000$ K is the temperature of the photons and baryons just before decoupling, and m_p is the mass of the proton.

For $t > t_d$, the baryonic matter is neutral to a high degree and the universe is transparent to radiation because $l_T > l_H$. The viscosity at this epoch is due to scattering of neutral atoms, that is

$$\nu = \frac{1}{3}v_s l_b , \tag{78}$$

with l_b the mean free path given by

$$l_b = \frac{1}{n_b \sigma_b} . \tag{79}$$

where $\sigma_b \sim 10^{-15}$ cm^2 is the relevant cross section (Allen 1973) and n_b is the number density of atoms

$$n_b = \frac{\rho_b}{m_p c^2}(X + 0.25Y) \sim 0.82\frac{\rho_b}{m_p c^2} , \tag{80}$$

Therefore,

$$\frac{l_b}{l_H} = 3.6 \times 10^{-13}\left(\frac{1+z}{1+z_d}\right)^{-3/2}(\Omega h^2)^{-1/2}\left(\frac{\Omega}{\Omega_b}\right) . \tag{81}$$

As will be seen below, the generated scales are so small that they reentered the Hubble radius prior to $t_{\rm eq}$. Thus, the density perturbation δ_4 increased from the first time it entered through the Hubble radius until the time corresponding to the redshift $z \leq z_d$, so that

$$\delta_4(z) = 3\frac{\delta_4^0}{0.3}S_{10}\frac{1+z_{\rm eq}}{1+z} = 89\frac{\delta_4^0}{0.3}S_{10}\Omega h^2\frac{1+z_d}{1+z} . \tag{82}$$

From equation (82) it follows that the assumption of small density perturbations is valid only if z is sufficiently large. Thus, demanding that $\delta < 0.1$ implies that $z \gtrsim 90$.

We wish to apply the necessary condition for turbulence generation, equation (50), to find whether turbulence generation is possible in the postdecoupling era. Use of equations (76), (78), (81), and (82) in equation (50) results in

$$\frac{1+z}{1+z_d} \gtrsim 1.6 \times 10^{-3}\left(\frac{\delta_4^0 S_{10}}{0.3A_{10}}\right)^{-2/3}(\Omega h^2)^{-1}\left(\frac{\Omega}{\Omega_b}\right)^{2/3} , \tag{83}$$

which, for Ω_b given by equation (40), yields

$$1 + z \gtrsim 19\left(\frac{\delta_4^0 S_{10}}{0.3A_{10}}\right)^{-2/3}(\Omega h^2)^{-1/3} . \tag{84}$$

For nonstandard nucleosynthesis, $\Omega_b = 1$ and $h = \frac{1}{2}$, one finds $z \gtrsim 6.4$. Both this and the limit given in equation (84) are smaller than $z = 90$; thus they pose no additional constraint on turbulence generation in the postdecoupling era.

From equations (48), (76) and (82) it follows that the largest scale that can be generated is given by

$$\frac{l}{l_H} \lesssim 1.9 \times 10^{-6}\left(\frac{\delta_4^0}{0.3}S_{10}\right)^{1/2} \times (\Omega h^2)^{1/2}\left(\frac{1+z}{1+z_d}\right)^{1/2} , \quad z \leq z_d . \tag{85}$$

Since for $z < z_d$, $l_H \propto (1+z)^{-3/2}$, it follows from equation (85) that the present epoch size is independent of z:

$$l(z=0) \lesssim 0.2 \text{ kpc}\left(\frac{\delta_4^0}{0.3}S_{10}\right)^{1/2} , \tag{86}$$

which is smaller than that of equation (73) by a factor of ~ 30 and contains a mass of $\sim 1\ M_\odot$.

The amplitude of the generated turbulent velocity is obtained from equations (74), (76), and (82) to be given by

$$\frac{v}{c} \sim 2 \times 10^{-6}\left(\frac{\delta_4^0}{0.3}S_{10}\right)^{1/2}\left(\frac{1+z}{1+z_d}\right)^{1/2} \tag{87}$$

and is maximal for $z = z_d$. On such small scales one expects that local flows will be more important than turbulence. From equations (76) and (87), it follows that for $z \gtrsim 90$, relevant to our discussion, turbulence is subsonic.

The small values of the turbulent velocity and of the largest scales are a consequence of the small value of the sound velocity. On these same scales turbulence has already been generated in the radiation-dominated era but was dissipated by the radiative viscosity prior to t_d. Since the viscosity in the postdecoupling era is many orders of magnitude smaller than the radiative viscosity, equation (50) can be satisfied anew and turbulence can be generated again on the same scales.

7. DISCUSSION

Inflation can provide a natural mechanism for the generation of cosmic turbulence and thus motivates a reexamination of this issue. We have found that by the end of inflation any turbulence generated during the inflationary phase or in the preinflation era will be suppressed by a factor $\gtrsim 10^{100}$. There-

© American Astronomical Society • Provided by the NASA Astrophysics Data System

1993ApJ...409..495G

fore, the constraints imposed on large-scale turbulence would not feed back into constraints on inflation itself. This constitutes yet another demonstration of the consistency of inflation with observations.

The other key result of this work is that the density fluctuations spectrum generated by inflation could excite turbulence at later cosmic epochs. Because the source that generates the turbulence is the gradient of the dimensionless density fluctuations, the turbulence is purely longitudinal. Only if a seed of vortical velocity fluctuations were present, could it have been amplified by the nonlinear interaction with the longitudinal turbulence, resulting in a component of vortical turbulence in addition to the longitudinal turbulence. We find that turbulence generation could have occurred at two epochs. One at redshift $z \gtrsim 27 z_{\rm eq}$ and the other in the postdecoupling era, $z < z_d$. In the first case, the largest generated scale has a present epoch size of ~ 6.3 kpc which includes a mass $\lesssim 3.6 \times 10^4\ M_\odot$. The amplitude of the generated turbulent velocity is $0.01c$ which is smaller than the sound velocity. In the second case, the generated turbulence is much weaker and its scale is much smaller, as a result of the small value of the sound velocity. The amplitude of the turbulent velocity is $\lesssim 2 \times 10^{-6}\ c$ and the largest scale is ~ 0.2 kpc. Turbulence on these small scales does not conflict with the observed isotropy of the background radiation and with the absence of overdense structures on scales of galaxies and clusters of galaxies.

In the postdecoupling epoch, the sound velocity is much smaller than in the earlier cosmic epoch. As a result, if part of the turbulence that was generated in the earlier epoch survived dissipation by the radiative viscosity, it would become supersonic and form shocks resulting in large density enhancements. The latter can grow faster than the density on larger scales for which turbulence was not generated, and serve as seeds for the growth of density on scales of galaxies and clusters of galaxies.

Numerical solutions are required to obtain the detailed evolution of the coupled spectra of the density fluctuations and turbulence. Specifically, they are needed to find if part of the turbulence, generated in the earlier epoch, would survive viscous dissipation. The computation of the dissipation, as function of cosmic time, will allow a quantification of the expected spectral distortion of the background radiation (Sunyaev & Zeldovich 1970) and its comparison with the observational upper limits. Direct numerical simulations are limited by their ability to resolve a sufficient number of scales. The possible approaches are large eddy simulations which use some analytical subgrid model, or spectral models. In future work, we consider using the latter by applying the EDQNM model (Orszag 1977; Lesieur 1990) to the present problem.

We thank David Kraft for a careful reading of the manuscript, and a referee for helpful comments.

REFERENCES

Allen, C. W. 1973, Astrophysical Quantities (London: Athlone)
Anile, A. M., Danese, L., De Zotti, G., & Motta, S. 1976, ApJ, 205, L59
Bardeen, J., Steinhardt, P., & Turner, M. S. 1983, Phys. Rev., D28, 679
Batchelor, G. K. 1973, Theory of Homogeneous Turbulence (Cambridge Univ. Press)
Batchelor, G. K., Canuto, V. M., & Chasnov, J. R. 1992, J. Fluid Mech., 235, 349
Blau, S. K., & Guth, A. H. 1987, in Three Hundred Years of Gravity, ed. S. W. Hawking & W. Israel (Cambridge Univ. Press)
Gamow, G. 1952, Phys. Rev., 86, 251
Harrison, E. R. 1970a, MNRAS, 147, 279
———. 1970b, Phys. Rev., D1, 2726
———. 1973, Phys. Rev. Lett., 30, 188
Kurskov, A. A., & Ozernoi, L. M. 1974a, Soviet Astr., 18, 157
———. 1974b, Soviet Astr., 18, 300
———. 1975, Soviet Astr., 18, 700
———. 1976, Soviet Astr., 18, 569
———. 1978, Ap&SS, 56, 67
Lesieur, M. 1990, Turbulence in Fluids (Boston: Kluwer)
Nariai, H. 1956a, Sci. Rep. Tohoku Univ., Ser. I, 39, 213
———. 1956b, Sci. Rep. Tohoku Univ., Ser. I, 40, 40
Nariai, H., & Tanabe, K. 1978, Prog. Theor. Phys., 60, 1583
Narlikar, J. V., & Padmanabhan, T. 1991, ARA&A, 29, 325
Orszag, S. A. 1977, in Fluid Dynamics, ed. R. Balian & J. L. Peube (New York: Gordon & Breach), 237
Ozernoi, L. M., & Chernin, A. D. 1968a, Soviet Astr., 11, 907
———. 1968b, Soviet Astr., 12, 901
Ozernoi, L. M., & Chibisov, G. V. 1971a, Soviet Astr., 14, 615
———. 1971b, Astrophys. Lett., 7, 201
Peebles, P. J. E. 1980, The Large-Scale Structure of the Universe (Princeton Univ. Press)
Rees, M. J. 1987a, QJRAS, 28, 197
———. 1987b, in Three Hundred Years of Gravity, ed. S. W. Hawking & W. Israel (Cambridge Univ. Press)
Silk, J. 1992, Nature, 356, 746
Sunyaev, R. A., & Zeldovich, Ya. B. 1970, Ap&SS, 7, 20
Tanabe, K. 1981, Prog. Theor. Phys., 65, 190
Tanabe, K., Nariai, H., Matsuda, T., & Takeda, H. 1975, Prog. Theor. Phys., 54, 719
Tomita, K., Nariai, H., Sato, H., Matsuda, T., & Takeda, H. 1970, Prog. Theor. Phys., 43, 1511
von Weizsäcker, C. F. 1951, ApJ, 114, 165
Weinberg, S. 1971, ApJ, 168, 175
———. 1972, Gravitation and Cosmology (New York: Freeman)
Zeldovich, Ya. B. 1972, MNRAS, 160, 1p

© American Astronomical Society • Provided by the NASA Astrophysics Data System