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OBSERVATIONAL TESTS OF CONVECTIVE CORE OVERSHOOTING IN STARS OF INTERMEDIATE TO HIGH MASS IN THE GALAXY

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ABSTRACT

Fourteen tests for the presence of convective overshooting in large convecting stellar cores are discussed for stars with masses of 4–17 $M_\odot$ which are members of detached close binary systems and of open clusters in the Galaxy. Nine of these tests, including all the best tests, are either new or newly applied. A large amount of theoretical and observational data has been scrutinized and subjected to averaging in order to minimize accidental and systematic errors. Defining a ratio of the effective convective overshoot distance beyond the classical Schwarzschild core boundary to the local pressure scale height, the present investigation finds a conservative upper limit of $d/H_P < 0.4$ from at least four tests and a tighter upper limit of $d/H_P < 0.2$ from one good test that is subject to only mild restrictions and is based on the maximum observed effective temperature of evolved blue supergiants. All the best test results, however, are formally consistent with the assumption $d/H_P = 0$ and are not sensitive to uncertainties about the initial chemical composition (or, probably, the opacities). Further analysis shows why the values of $d/H_P = 0.25$–2 that were derived in previous studies should probably be regarded as only upper limits.

Three additional conclusions can be drawn specifically concerning the B-type main-sequence stars. First, any current uncertainty about the distance scale for these stars is unimportant in conducting the present tests for convective core overshooting. Second, the correct effective temperature scale for the B0.5-B2 stars is almost certainly close to one of the proposed hot scales. Third, the value of the initial metals abundance that is required in stellar models in order to reproduce the observations is $Z_e \approx 0.04$ independently of overshooting; this exceptionally high value, which is twice the directly observed atmospheric metals abundance in most of these stars, may indicate the need for larger envelope opacities than the standard Los Alamos opacities used here.

Subject headings: convection — stars: abundances — stars: evolution — stars: interiors

1. INTRODUCTION

From the standpoint of stellar evolution, the fundamental characteristic of an upper main-sequence star is the size of its convective core. Turbulent convection in the core must include overshooting of convective elements beyond the classical Schwarzschild core boundary, where the convective buoyancy force, but not the velocity of rising elements, vanishes. Unfortunately, theory is not yet able to reliably predict the amount of convective overshooting or the efficiency of the mixing of material (Eggleton 1983; Baker & Kuhfuss 1987; Renzini 1987), and so recourse must be had to parameterized descriptions, the most common one being to specify d/H_P, the ratio of the overshoot distance for effective mixing to the local pressure scale height. In cases where a more sophisticated theory is employed in the stellar models, an equivalent value of d/H_P can usually be deduced from the derived model structure. Comparisons of stellar models computed with different assumed or calculated values of d/H_P are then made with observed stars to infer the true distance of overshooting.

By using only Galactic stars with intermediate to high masses, d/H_P has been found to be ~ 0 (Vanbeveren 1983, 1987, 1989; Schulte-Ladbeck 1989; Popova & Tutukov 1990), 0.2–0.3 (Maeder & Mermilliod 1981; Grenier et al. 1985; Mermilliod & Maeder 1986; Maeder & Meynet 1987, 1989; Wolff 1990), 0.4–0.8 (Bressan, Bertelli, & Chiosi 1981; Bertelli, Bressan, & Chiosi 1984, 1985; Hilditch & Bell 1987; Nasi & Forieri 1990; Napiwotzki, Schönberner, & Weidemann 1991), ~ 0.7 (Stothers & Chin 1985), 0.3–1.7 (Doom 1982a, b), and ~ 2 (Doom 1985). In addition, an upper limit of $d/H_P < 1.7$ can be placed from the results of one other study of these stars (Stothers & Chin 1990). In view of the fact that in many of these studies the same, or similar, observational material has been used, the wide range of deduced values of d/H_P strikingly illustrates the difficulty of the problem. On the other hand, the previous studies utilized not always the best observational data and failed to evaluate possible errors for all observed quantities. These studies were also restricted by the use of very limited grids of theoretical evolutionary sequences of models and by the use of only a few kinds of comparisons with observations, which in most cases were confined to the computed and observed differences between terminal-age main-sequence (TAMS) stars and zero-age main-sequence (ZAMS) stars. The critical role of the effective temperature calibration of early B-type stars was also never recognized.

To lessen the potential impact of accidental and systematic errors lurking in the evolutionary sequences, the present paper makes use of all relevant published sequences in a new, comprehensive survey of the problem. State-of-the-art stellar models, which might be thought preferable, have no real advantage over standard models, because the envelope metal opacities as well as the theory of turbulent convection (and semiconvection) still have uncertainties that are far larger than the incremental improvements that have been incorporated into state-of-the-art models.

In keeping with the present philosophy of reducing potential error in the finally accepted data by making use of abundant

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input material, an average of all accessible measurements of each quantity for each observed star is taken whenever possible. The stars analyzed here belong to well-measured detached close binary systems and open clusters of the Galaxy. Although OB associations certainly contain far more stars, their Hertzsprung-Russell (H-R) diagrams suffer from long-standing problems of interpretation that have defied even the most recent efforts to resolve. The present selection of star clusters may seem rather restrictive, but it is prudent to err on the side of safety.

Stars considered in this paper fall mainly into the range of intermediate to high masses, roughly 4–17 $M_\odot$. Not counting the stars in Wolf-Rayet binary systems, only three stellar masses exceed $\sim 17\ M_\odot$. More massive stars than this are excluded from most of the discussion because they may lose significant amounts of material in stellar winds, which confuses the interpretation of their evolutionary status. Less massive stars, on the other hand, possess much smaller convective cores and so are often classed in a category of their own. Inasmuch as d/H_P may well be a function of stellar mass and even of evolutionary phase, we can only hope to establish here some average property of d/H_P in a limited mass interval. We consider separately two subranges: the intermediate masses (4–9 $M_\odot$) and the high masses (10–17 $M_\odot$).

When these stars appear as intermediate-mass giants or high-mass supergiants in open clusters, the clusters are often designated as "intermediate-age" and "young," respectively. Although only Galactic clusters are considered in this paper, some relevant work has also been done on the far richer clusters in the Magellanic Clouds. Conflicting conclusions about convective overshooting, however, have been reached in the case of the intermediate-age clusters like NGC 1866 in the Large Cloud, overshooting being thought to be either substantial (Barbaro & Pigatto 1984; Bertelli et al. 1985; Chiosi et al. 1986, 1989; Chiosi & Pigatto 1986; Mazzei & Pigatto 1988, 1989; Maeder & Meynet 1989; Alongi et al. 1991; Vallenari et al. 1991) or still unproven (Brocato & Castellani 1988; Brocato et al. 1989). Some very new results for NGC 330 and NGC 458 in the Small Cloud will be discussed briefly in § 7. Despite the advantage in supergiant numbers accruing to these Magellanic Cloud clusters, the Clouds are very far away, and some important stellar data (for example, spectral types) are usually either unavailable or of poor quality.

For the convenience of readers with varying interests, this paper is organized largely in modular form, so that the applications sections (§§ 4, 5, and 6) and the main conclusions with the summary table of results (§ 7) may be read independently of the other sections, including the documented presentation of the theoretical data (§ 2) and observational data (§ 3). For the time-pressed reader, the paper's bottom line is that $d/H_P < 0.2$.

2. THEORETICAL DATA

A large and homogenous grid of over 150 theoretical evolutionary tracks for stars in the mass range 3–120 $M_\odot$ has been computed and published by Stothers & Chin (1975, 1976, 1979) and Chin & Stothers (1990, 1991). These tracks run from the zero-age main sequence to the end of core helium burning. Standard Cox-Stewart opacities were adopted, together with six initial chemical compositions, consisting of $X_e = 0.602$, 0.650, and 0.739 (hydrogen) and $Z_e = 0.021$ and 0.044 (metals). Although many of the thermonuclear-reaction rates are uncertain, the only *major* uncertain reaction rate remains that for $^{12}C(\alpha, \gamma)^{16}O$, which was chosen for most of the evolutionary tracks to lie midway between the current experimental extremes. Wherever the uncertainty in this rate has an important effect on the stellar models, this will be noted.

Convection in the stellar models, including also semi-convection, was handled according to either the Schwarzschild (temperature-gradient) criterion or the Ledoux (density-gradient) criterion, whereas convective overshooting from both the central convective core and the outer convection zone was simply parameterized by an assigned ratio of overshoot distance to local pressure scale height: d/H_P(core) or D/H_P(outer envelope). The convective mixing length in the outer envelope was taken to be a constant multiple, α, of the local pressure or density scale height. Ideally, α should be chosen so as to produce a match between the predicted and observed effective temperatures of late-type supergiants for each combination of stellar mass, initial chemical composition, envelope opacities, etc. It turns out, however, that the more fundamental aspects of the stellar models which are to be tested in this paper are not very sensitive to any reasonable variations of the convective parameters in the outer envelope as long as $0 \leq D/H_P \leq 0.4$ and $0.5 \leq \alpha_P \leq 2$. Another simplifying factor is that semi-convection is always relatively unimportant for main-sequence evolution and also is unimportant for post–main-sequence evolution in stars less massive than $\sim 17\ M_\odot$ if $d/H_P > 0.4$.

Stellar wind mass loss was included in the models by using two different observational parameterizations. Since observationally realistic rates of mass loss appear to have little influence on the evolutionary tracks for stellar masses less than $\sim 17\ M_\odot$, the more numerous evolutionary tracks computed without mass loss are adopted here. Rotation and magnetic fields were entirely ignored in the models, in part because too little is yet known about what assumptions are realistic for starting conditions and for evolutionary developments, and in part because these forces are anticipated to have only a relatively minor influence on the stellar models in any comparisons with observations (e.g., Endal & Sofia 1976), at least from the point of view of looking for a significant manifestation of convective core overshooting.

Evolutionary sequences computed by other authors for the purpose of studying convective core overshooting can be found in Cloutman & Whitaker (1980), Becker & Cox (1982), Matraka, Wassermann, & Weigert (1982), Huang & Weigert (1983), Bertelli et al. (1984, 1985), Bertelli et al. (1986), Maeder & Meynet (1987, 1988, 1989), Langer, Arcoragi, & Arnould (1989), Bertelli et al. (1990), and Maeder (1990). Although the grids of evolutionary sequences published in these papers are much smaller than the Chin & Stothers (1991) grid, they are useful because the input physics and computational techniques used are different.

Figure 1 illustrates 10 of the Chin & Stothers sequences in the H-R diagram. All are based on the Ledoux criterion, with convective core overshooting taken to be either absent ($d/H_P = 0$) or moderate ($d/H_P = 0.35$). The initial chemical composition used is solar, $(X_e, Z_e) = (0.739, 0.021)$. It is one of four compositions considered in this paper, the other three being (0.739, 0.044), (0.650, 0.021), and (0.650, 0.044). These four compositions define a rectangle within which most Population I stars in the general solar neighborhood are expected to lie. Except for (0.739, 0.021), these compositions are extreme cases and are not necessarily physically realistic.

As mentioned above, a standard version of the Los Alamos opacities was adopted for the Chin & Stothers sequences. At low densities, the newer Los Alamos Opacity Library tables

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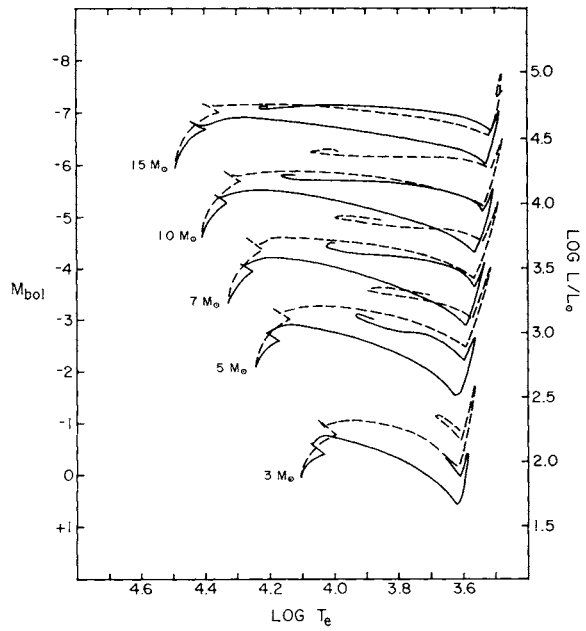

FIG. 1.—Theoretical H-R diagram showing evolutionary tracks running from the ZAMS stage to the end of core helium burning. The initial chemical composition is $(X_e, Z_e) = (0.739, 0.021)$. Convective core overshooting is either neglected (*continuous lines*) or included with the overshoot parameter set equal to $d/H_P = 0.35$ (*dashed lines*). In all cases semiconvection is effectively omitted.

are not very different from the older tables, as can be seen from the opacity values themselves and from a simple comparison of ZAMS stellar models based on the old and new opacities. The stellar models of Maeder & Meynet (1988) and Maeder (1990) for $(X_e, Z_e) = (0.700, 0.020)$ and (0.640, 0.040) are based on the new opacities and have compositions close to those used in two sets of the Chin & Stothers models. After adjustment for the composition differences, agreement in the quantities log $(L/L_\odot)$, log $(R/R_\odot)$, and log T_e occurs within 0.03 dex over the mass range 4–17 $M_\odot$, except for an occasional difference of up to 0.08 dex in log $(L/L_\odot)$ for masses below 12 $M_\odot$. All of these differences are smaller than current observational errors.

Larger opacity revisions are possible, however (Carson 1976; Rozsnyai 1989; Iglesias & Rogers 1991). In the present state of imperfection of the metal opacities in particular, the best one can do is to use Z_e as a free parameter to match theoretical models of ZAMS stars to the observations, much as one uses α_P for convective envelopes in the case of the late-type supergiants. The derived "effective" value of Z_e, of course, need not represent the true metals abundance.

2.1. *Main-Sequence Stars*

Models of main-sequence stars show the following characteristics when convective core overshooting is extensive: (1) a more massive helium core; (2) a fainter luminosity and a smaller radius near the ZAMS, but a brighter luminosity and a larger radius later; (3) a reduced effective temperature; and (4) a prolonged lifetime of core hydrogen burning.

To test various aspects of these predictions, theoretical isochrones on the H-R diagram are needed for making comparisons with cluster H-R diagrams. The main-sequence turnup of a cluster isochrone rises nearly vertically in the H-R diagram, with a slight curvature toward cooler effective temperatures near the top. Consequently, the maximum effective temperature along the turnup is an excellent indicator of isochronic age. Sets of theoretical isochrones are constructed here by using the evolutionary tracks of Chin & Stothers (1991) with various overshoot parameters. As mentioned above, mass loss has been ignored, but its effect on theoretically computed isochrones is known to be unimportant (Paerels, Lamers, & de Loore 1980).

Results for the basic computed isochrone parameters are presented in Table 1. These parameters include: log T_e, logarithm of the effective temperature at the hottest point along the turnup; Sp, the corresponding spectral type; log τ, logarithm of the isochronic age; $M_{\rm bol}$(tip), bolometric absolute magnitude at the tip of the turnup; and Δ log T_e(width), logarithmic difference between the effective temperature at the hottest point and the effective temperature at the tip of the turnup. Conversions from effective temperature to spectral type have been made by using the relations of Morton (1969) for O9-B0 stars and of Morton & Adams (1968) for B0.5-B7 stars. Within the narrow spectral subrange B0.5-B2 a considerable uncertainty still exists about the calibration, but Morton & Adams's relation lies approximately midway between the cooler relations of Humphries, Nandy, & Kontizas (1975) and Nandy & Schmidt (1975) and the hotter relations of Code et al. (1976), Underhill et al. (1979), Böhm-Vitense (1981), Remie & Lamers (1982), Theodossiou (1985), and Kilian et al. (1991). Figure 2 displays all nine relations.

For any choice of the (spectral type, effective temperature) relation, it is very simple to interpolate in Table 1, because most of the quantities there are locally nearly linear in the logarithm of the effective temperature of the hottest point along the turnup. Figure 3 displays log τ as a function of log T_e in the case of $(X_e, Z_e) = (0.739, 0.021)$ for three values of d/H_P.

As in our earlier set of isochrones (Stothers 1972), the tip of the turnup is assigned to lie at the point of lowest effective temperature, after which the evolution is very rapid. The tip therefore falls along the TAMS.

2.2. *Post–Main-Sequence Stars*

After the end of central hydrogen burning, a massive star rapidly expands its envelope and crosses the H-R diagram on the Helmholtz-Kelvin time scale of the contracting core. For most stellar masses, the onset of slow helium depletion typi-

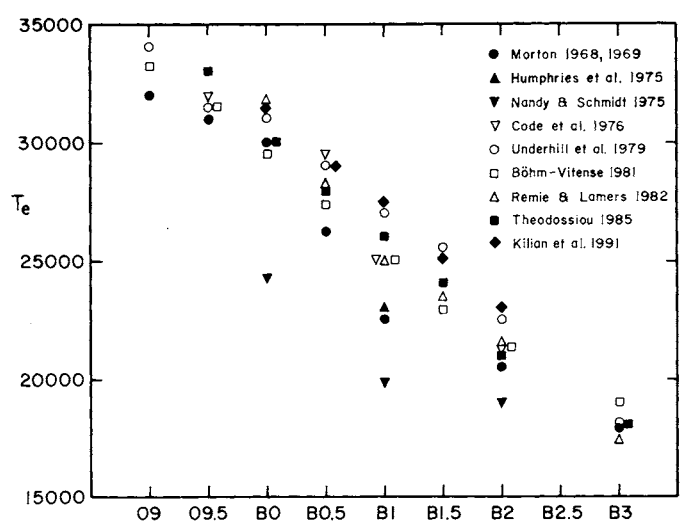

FIG. 2.—Effective temperature calibrations for O9-B3 main-sequence stars

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TABLE 1

THEORETICAL MAIN-SEQUENCE TURNUPS

Morton Sp	log T_e	$X_e = 0.739$, $Z_e = 0.021$		$X_e = 0.739$, $Z_e = 0.044$		$X_e = 0.650$, $Z_e = 0.021$		$X_e = 0.650$, $Z_e = 0.044$	
		$d/H_P = 0$	$d/H_P = 0.70$	$d/H_P = 0$	$d/H_P = 0.70$	$d/H_P = 0$	$d/H_P = 0.70$	$d/H_P = 0$	$d/H_P = 0.70$
					log τ (yr)				
O9	4.505	6.80	6.92	6.72	6.82	6.77	6.89	6.70	6.79
O9.5	4.491	6.84	6.97	6.76	6.87	6.81	6.94	6.74	6.84
B0	4.477	6.88	7.03	6.80	6.92	6.86	6.99	6.78	6.89
B0.5	4.418	7.07	7.23	6.98	7.13	7.05	7.20	6.95	7.10
B1	4.354	7.29	7.46	7.18	7.36	7.26	7.44	7.16	7.33
B1.5	4.333	7.36	7.54	7.26	7.44	7.33	7.51	7.23	7.40
B2	4.312	7.43	7.62	7.32	7.51	7.40	7.59	7.30	7.48
B2.5	4.283	7.53	7.73	7.43	7.62	7.51	7.70	7.40	7.58
B3	4.253	7.64	7.85	7.54	7.73	7.62	7.81	7.51	7.70
B4	4.223	7.76	7.96	7.64	7.85	7.73	7.93	7.62	7.81
B5	4.193	7.87	8.08	7.76	7.96	7.85	8.05	7.73	7.93
B6	4.164	7.98	8.19	7.87	8.07	7.96	8.17	7.84	8.04
B7	4.134	8.10	8.31	7.98	8.19	8.07	8.28	7.96	8.17
					M_{bol} (tip)				
O9	4.505	−8.3	−8.6	−8.9	−9.1	−8.0	−8.2	−8.9	−8.8
O9.5	4.491	−8.0	−8.3	−8.6	−8.8	−7.7	−8.0	−8.5	−8.5
B0	4.477	−7.8	−8.1	−8.3	−8.6	−7.4	−7.7	−8.2	−8.3
B0.5	4.418	−6.6	−7.0	−7.2	−7.6	−6.3	−6.7	−6.9	−7.2
B1	4.354	−5.4	−6.0	−6.0	−6.5	−5.1	−5.7	−5.7	−6.2
B1.5	4.333	−5.1	−5.6	−5.6	−6.1	−4.8	−5.3	−5.3	−5.8
B2	4.312	−4.7	−5.3	−5.3	−5.8	−4.4	−5.0	−5.0	−5.5
B2.5	4.283	−4.2	−4.8	−4.8	−5.3	−3.9	−4.5	−4.5	−5.0
B3	4.253	−3.7	−4.3	−4.3	−4.8	−3.4	−4.1	−4.0	−4.6
B4	4.223	−3.2	−3.9	−3.8	−4.4	−3.0	−3.6	−3.5	−4.1
B5	4.193	−2.7	−3.4	−3.3	−3.9	−2.5	−3.1	−3.0	−3.6
B6	4.164	−2.3	−3.0	−2.8	−3.4	−2.0	−2.7	−2.5	−3.1
B7	4.134	−1.8	−2.5	−2.3	−3.0	−1.6	−2.2	−2.1	−2.7
					Δ log T_e (width)				
O9	4.505	0.029	0.117	0.048	0.202	0.013	0.063	0.037	0.099
O9.5	4.491	0.026	0.110	0.042	0.156	0.012	0.058	0.033	0.093
B0	4.477	0.023	0.103	0.038	0.146	0.010	0.053	0.028	0.087
B0.5	4.418	0.013	0.076	0.023	0.106	0.004	0.046	0.009	0.061
B1	4.354	0.009	0.065	0.014	0.081	0.002	0.041	0.005	0.049
B1.5	4.333	0.009	0.064	0.012	0.078	0.001	0.038	0.004	0.046
B2	4.312	0.008	0.062	0.010	0.075	0.001	0.035	0.003	0.044
B2.5	4.283	0.007	0.059	0.009	0.072	0.001	0.035	0.003	0.042
B3	4.253	0.007	0.059	0.009	0.069	0.001	0.036	0.002	0.041
B4	4.223	0.007	0.058	0.008	0.066	0.001	0.038	0.001	0.040
B5	4.193	0.007	0.058	0.008	0.065	0.001	0.038	0.001	0.040
B6	4.164	0.007	0.059	0.007	0.064	0.001	0.039	0.001	0.039
B7	4.134	0.007	0.060	0.006	0.064	0.000	0.039	0.000	0.041

cally begins when the star has evolved into a red giant or supergiant (as illustrated in Fig. 1). The only exception to this rule in the case of stars with normal to high metallicities occurs when the stellar mass exceeds $\sim 15\ M_\odot$, convective core overshooting is insignificant or at most moderate, and semiconvection is treated according to the Schwarzschild criterion (Stothers & Chin 1976). In that case, the star halts its expansion and begins to deplete core helium as a blue supergiant, for reasons explained in Stothers & Chin (1968) and Chiosi & Summa (1970). Otherwise, the star becomes blue only by looping out of the red-supergiant region. Although many physical factors influence whether such a blue loop occurs, the highest effective temperature attained on the loop is most strongly influenced by the amount of convective core overshooting in the previous main-sequence phase.

The published literature has accordingly been searched to determine, for each stellar mass and for various overshoot parameters, the maximum effective temperature that can occur during the core helium-burning phase. For $d/H_P = 0$, the relevant sequences turn out to be those published by Chin & Stothers (1991) for 5, 7, and 10 $M_\odot$ and by Maeder (1981) for 15 $M_\odot$. None of these sequences included semiconvective mixing, and therefore all were, in effect, based on the Ledoux criterion, which seems to produce little or no semiconvection in stars with these masses (Stothers & Chin 1975). For $d/H_P > 0$, the Chin & Stothers (1991) sequences without semiconvective mixing turn out to be the relevant ones at all masses. All of these sequences were selected from among sequences that covered the full ranges of presently allowable values of α_P, $^{12}C(\alpha, \gamma)^{16}O$ reaction rate, and initial chemical composition.

Some sequences that reach the red stage never develop blue loops. In that case, the highest effective temperature achieved during core helium burning is simply that of a red giant or

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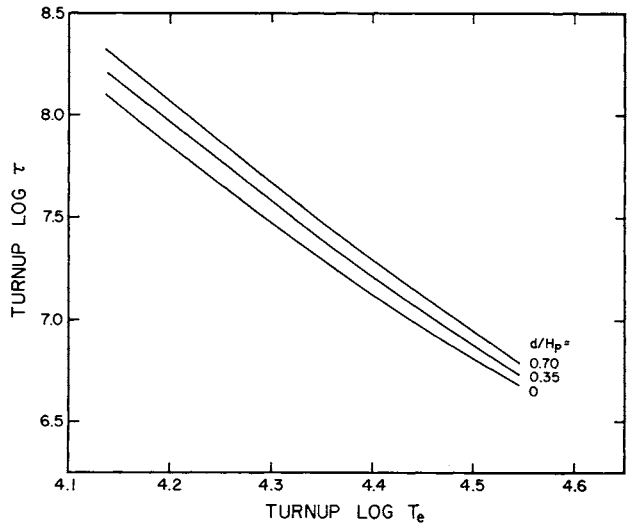

FIG. 3.—Theoretical relation between the age of a cluster main-sequence isochrone and the effective temperature of the hottest point on the turnup in the H-R diagram.

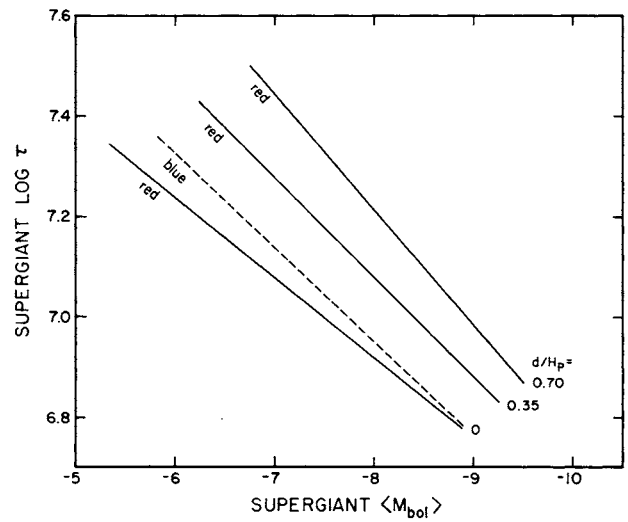

FIG. 4.—Theoretical relation between the age of a massive blue or red evolved supergiant and its mean bolometric absolute magnitude.

supergiant. If, however, a blue loop does form, the point of its maximum extension always lies well away from the Hayashi line. The coolest derived maximum effective temperature for a blue loop (or a blue halt) in the case $d/H_P = 0$ is also a useful limiting quantity and has been determined from the published literature (5 $M_\odot$, Meyer-Hofmeister 1972; 7 $M_\odot$, Iben 1972; 10 $M_\odot$, Stothers & Chin 1976; 15 $M_\odot$, Simpson 1971).

The luminosity of a massive supergiant, especially a blue one, displays relatively little evolutionary change as it burns core helium (Fig. 1). Because of this, its age as reckoned from the ZAMS is fairly tightly correlated with its luminosity. The theoretically predicted relation, which is affected little by stellar wind mass loss, is given in Table 2 for both blue and red supergiants with initial masses of 10–30 $M_\odot$ as based on data from Chin & Stothers (1990, 1991). Figure 4 exhibits the correlation as a function of d/H_P in the case $(X_e, Z_e) = (0.739, 0.021)$.

There are, altogether, five potentially observable consequences of main-sequence convective core overshooting that can be looked for in evolved giants and supergiants: (1) a reduced maximum effective temperature along the blue loop; (2) an increased luminosity relative to nonovershooting models of the same mass; (3) a smaller ratio of mean luminosity in the blue phase to mean luminosity in the red phase; (4) a smaller ratio of the amounts of time spent in the blue and red phases, although this ratio is extremely sensitive to the choices of other physical input parameters; and (5) a smaller ratio of core helium-burning lifetime to core hydrogen-burning lifetime.

TABLE 2

THEORETICAL AGES OF CORE-HELIUM-BURNING SUPERGIANTS WITH INITIAL MASSES OF 10–30 $M_\odot$

			log τ (yr)	
X_e	Z_e	d/H_P	Blue Tip	Red Branch
0.739	0.021.....	0	$8.55 + 0.20M_{bol}$	$8.25 + 0.17\langle M_{bol}\rangle$
		0.70	...	$9.09 + 0.24\langle M_{bol}\rangle$
0.739	0.044.....	0	$8.47 + 0.19M_{bol}$	$8.22 + 0.17\langle M_{bol}\rangle$
		0.70	...	$9.09 + 0.24\langle M_{bol}\rangle$
0.650	0.021.....	0	$8.48 + 0.20M_{bol}$	$8.17 + 0.17\langle M_{bol}\rangle$
		0.70	...	$9.06 + 0.24\langle M_{bol}\rangle$
0.650	0.044.....	0	$8.37 + 0.19M_{bol}$	$8.12 + 0.17\langle M_{bol}\rangle$
		0.70	...	$9.05 + 0.24\langle M_{bol}\rangle$

3. OBSERVATIONAL DATA

3.1. *Detached Close Binary Systems*

Main-sequence stars that are members of detached close binary systems can be considered as having evolved in an undisturbed manner. Their orbital masses and orbital radii therefore provide valuable information about the internal structure of normal stars. Hilditch & Bell (1987), in an update of Popper's (1980) survey, have tabulated the most comprehensive observational data for such systems. Luminosities of the components were computed by Popper and by Hilditch & Bell from Stefan's law by using the observed radii and observed spectral types in conjunction with Hayes's (1978) effective temperature scale. Hayes's scale is essentially that of Code et al. (1976) and is therefore a relatively hot scale (cf. Fig. 2).

Very recent work (Popper & Hill 1991; Terrell 1991) has shown that Hilditch & Bell's estimated errors are somewhat too small in some cases. However, the general pattern of stars in the (mass, luminosity) and (mass, radius) planes is not expected to change very much.

3.2. *Young Clusters*

To avoid the confusing observational problems posed by large stellar associations, the analysis of the present paper is restricted to open star clusters. The main selection criterion is the observation of a reasonably populous, well-defined main-sequence turnup in the H-R diagram. For the youngest clusters, the best accuracy is achieved through the use of high-resolution spectral types. Although open clusters in the Galaxy are notoriously poor in bright stars, their member stars can be usefully pooled in a composite H-R diagram in order to minimize the effect of stochastic fluctuations of the initial mass function for individual clusters (even relatively rich ones). By choosing only clusters with turnup spectral types later than B0, the turnup can be fairly well resolved without confusion from a large number of O-type blue stragglers. Additional selection criteria used here include individual stellar reddening correc-

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TABLE 3

CLUSTER DISTANCE MODULI AND REDDENINGS BASED ON *UBV*, *uvby*, Hβ, AND Hγ PHOTOMETRY AND MK SPECTRAL TYPES

Cluster	$(m-M)_0$	$\langle E_{B-V} \rangle$	References
NGC 581	11.6	0.38	McCuskey & Houk 1964
	12.0	0.41	Hoag & Applequist 1965
	11.9	0.39	Moffat 1972
	12.2	0.38	Sagar & Joshi 1978
	11.7	0.41	Mermilliod 1981
	11.8	0.40	Osman et al. 1984
NGC 3293...	12.1	...	Hoffleit 1956
	12.1	0.3:	Feast 1958
	12.3	...	Schild 1970
	12.3	...	Balona & Crampton 1974
	12.1	0.31	Feinstein & Marraco 1980
	12.0	0.33	Turner et al. 1980
	12.0	0.3:	Shobbrook 1980, 1983
NGC 3766...	10.8	0.16	Ahmed 1962
	11.4	0.19	Sher 1962, 1965
	11.6	0.22	Schild 1970
	10.9	0.16	Winnenburg 1973
	11.4	0.19	Mermilliod 1981
	11.4	0.20	Shobbrook 1985, 1987
NGC 4755...	9.6[a]	0.31[a]	Arp & Van Sant 1958
	11.6	0.44	Hernández 1960
	11.9	0.48	Feast 1963
	11.8	0.40	Graham 1967
	11.6	0.42	Schild 1970
	11.8	...	Balona & Crampton 1974
	11.8	0.40	Perry et al. 1976
	11.4	0.35	Mermilliod 1981
	11.8	0.44	Dachs & Kaiser 1984
	11.3	0.36	de Waard et al. 1984
	11.4	0.38	Shobbrook 1984
	11.8	0.40	Kjeldsen & Frandsen 1991
IC 2581.....	11.9[a]	0.51[a]	Schmidt-Kaler 1961
	11.1[a]	0.37[a]	Fernie 1963
	12.0	0.42	Lloyd Evans 1969
	12.3	0.45	Turner 1973, 1978
	12.1	0.41	Mermilliod 1981
	12.2	0.41	Eggen 1982
χ Per.....	11.8	...	Morgan et al. 1953
	11.8	0.53	Johnson & Hiltner 1956
	12.0	0.56	Becker 1961
	11.7	...	Blaauw 1963
	12.0	0.56	Borgman & Blaauw 1964
	11.9	0.5:	Wildey 1964
	11.4	0.59	Crawford et al. 1970
	11.7	0.61	Vogt 1971
	11.8	...	Lloyd Evans 1972
	11.5	...	Balona & Crampton 1974
	11.8	0.53	Mermilliod 1981
Per OB2.....	7.4	0.2:	Blaauw 1952
	7.8	...	Morgan et al. 1953
	7.7	...	Johnson 1957
	8.0	0.3:	Seyfert et al. 1960
	7.6	0.3:	Borgman & Blaauw 1964
	7.9	0.31	Guetter 1977

[a] Not used to form the average in Table 4.

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tions and individual stellar radial velocities to help assess the membership of the very rare evolved supergiants. It has been known for a long time that these evolved supergiants consist almost entirely of B, A, and M types, with only a scattering of intermediate F, G, and K types.

Young clusters that satisfy these selection requirements are listed in Table 3. The distance moduli and average reddenings tabulated there are variously based on *UBV*, *uvby*, Hβ, and Hγ photometry, and MK spectral types. Original data sources alone are cited; secondary sources (as well as a few sources that used *RGU* photometry) are omitted. Of the listed data, it was decided not to use Arp & Van Sant's (1958) for NGC 4755, because their published photometry is suspect (Feast 1963); Schmidt-Kaler's (1961) for IC 2581, because the stars he measured actually belong to the surrounding association (Turner 1978); and Fernie's (1963) for IC 2581, because his fit to the ZAMS seems to be faulty (Lloyd Evans 1969). Mean values of the accepted distance modulus and average reddening for each cluster are entered in Table 4.

Characteristics of the tips of the main-sequence turnups are revised somewhat from our earlier tabulation (Stothers & Chin 1985) in order to take advantage of the improved cluster distance moduli as well as of the opportunity to redefine the location of the tip of the turnup from the brightest actual stars (not from the *estimated* termination level) among the very few stars that are located along the upper part of the turnup. Table 5 lists the revised data. Assignments of the turnup spectral types (Stothers & Chin 1985) agree within a quarter of a decimal subdivision with earlier assignments (Stothers 1972; Mermilliod 1981). To transform visual absolute magnitudes into bolometric ones, the scale of bolometric correction (BC) as a function of effective temperature will be taken from Flower (1977), whose scale for B stars is similar to that of Habets & Heintze (1981). However, two different relations between spectral type and effective temperature will be considered: the intermediate Morton & Adams (1968) temperature scale and the hot Underhill et al. (1979) temperature scale.

TABLE 4

ADOPTED CLUSTER PROPERTIES

Cluster	$(m-M)_0$	$\langle E_{B-V}\rangle$
NGC 581	11.9	0.40
NGC 3293	12.1	0.32
NGC 3766	11.3	0.19
NGC 4755	11.6	0.41
IC 2581	12.2	0.42
χ Per	11.8	0.56
Per OB2	7.8	0.30

A list of evolved supergiants belonging to the young clusters used in this paper is contained in Table 6. Schild (1970), however, did not regard the bright luminosity class Ia star HD 100943 and the outlying Ib supergiant HD 100826 as likely members of NGC 3766. Yet the evidence from their apparent visual magnitudes and reddenings as well as from their radial velocities (Buscombe & Kennedy 1969; Lloyd Evans 1980) strongly suggests membership, and they are accepted here as bona fide members. For similar reasons, we include HD 111613 in NGC 4755 (Humphreys 1978), which was also rejected by Schild. On the other hand, we agree with Schild's proposed rejections of HD 91969 (B0 Ib) and HD 91943 (B0.5 Ib) from membership in NGC 3293 on the grounds of their significantly discrepant apparent visual magnitudes, spectral types, and radial velocities (Feast 1963). Radial velocity measurements for all of the accepted stars, except HD 100826 (above), have been provided by Humphreys (1978), Mermilliod & Maeder (1986), and Sowell (1987).

Sources of spectroscopic and photometric data for our final selection of supergiants are listed in Table 3. Additional sources are: for *spectral types*, Bidelman (1947), Mermilliod (1976), Humphreys (1978), Keenan & Pitts (1985), and Sowell (1987); for *UBV photometry*, Johnson & Mendoza (1966), Blanco et al. (1968), Lee (1970), Eggen (1971), and Humphreys (1978). The total range of the published V-magnitudes for each of the M-type supergiants is much too large to be due to photometric errors alone, and these stars are undoubtedly intrinsic variables of modest amplitude like Betelgeuse (α Orionis). The median magnitude, V, and the full range of the published magnitudes, δV, are listed for each M-type supergiant in Table 6. The exceptionally large range of spectral types shown by RS Persei, M3-M5.5 (Keenan 1942; Bidelman 1947; Blanco 1955; Wildey 1964), makes this star's mean bolometric properties too uncertain to be used here.

The V-magnitude of each supergiant in Table 6 can be corrected for interstellar extinction by assuming that the supergiant's intrinsic color, $(B-V)_0$, has a one-to-one relation with its spectral type and luminosity class. The total extinction correction, $A_V = R_V E_{B-V}$, can then be found from the supergiant's measured reddening $E_{B-V} = (B-V) - (B-V)_0$. Sources of transformation data are as follows. In the case of the B-type and A-type supergiants, it was decided to take $(B-V)_0$ from Schmidt-Kaler (1965) and FitzGerald (1970); $R_V = 3.2$ from Schmidt-Kaler (1965); BC as a function of effective temperature from Flower (1977); and effective temperatures from Fitzpatrick & Garmany (1990). In the case of the M-type supergiants, the choice was to take $(B-V)_0$ and BC from Elias, Frogel, & Humphreys (1985), and $R_V = 3.6$ and effective temperatures from Lee (1970).

TABLE 5

CLUSTER MAIN-SEQUENCE TURNUPS

Cluster	Turnup	M_V (tip)	Sp (tip)	Blue Stragglers
NGC 581	B2	−4.3	B2 III	...
NGC 3293	B0.5-B1	−5.4	B0.5 II-III, B1 Ib-II	O7 V
NGC 3766	B1.5-B2	−3.7	B2 III	...
NGC 4755	B0.5-B1	−5.9	B1.5 Ib,[a] B2 Ib[a]	...
IC 2581	B0.5	−5.3	B1 II-III	O7 V[b]
χ Per	B1:	−5.5	B2 Ib-II	...
Per OB2	B1:	−5.9	B1 Ib	O8 III, O9.5ep

[a] B2 Ib, B3 Ib according to Bidelman (Hernández 1960).
[b] Not in the cluster nucleus.

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TABLE 6

EVOLVED SUPERGIANTS IN YOUNG CLUSTERS

Cluster	Star	Sp	V	δV	$B-V$	E_{B-V}	M_V	M_{bol}	log T_e
NGC 581	HD 9311	B5 Ib	7.20	...	0.29	0.39	−6.0	−6.8	4.13
	BD +59°274	M0.5 Ib	8.42	0.16	2.07	0.39	−4.9	−6.2	3.55
NGC 3293	CPD−57°3502	M1 Iab-Ib	7.38	0.37	2.05	0.36	−6.0	−7.3	3.55
NGC 3766	HD 100943	B4.5 Ia	7.14	...	0.12	0.22	−4.9	−5.8	4.14
	HD 100826	A0 Ib	6.29	...	0.09	0.11	−5.4	−5.7	4.00
	CD−60°3621	M0.5 Iab-Ib	7.21	0.13	1.94	0.26	−5.0	−6.3	3.55
	CD−60°3636	M0 Iab-Ib	7.50	0.24	1.84	0.16	−4.4	−5.6	3.56
NGC 4755	HD 111973	B4 Ia	5.95	...	0.23	0.34	−6.7	−7.7	4.15
	HD 111904	B9 Ia-Iab	5.77	...	0.32	0.33	−6.9	−7.3	4.04
	HD 111613	A2 Ia-Iab	5.73	...	0.37	0.35	−7.0	−7.2	3.96
	CPD−59°4549	M2 Iab-Ib	7.51	0.30	2.20	0.51	−5.9	−7.4	3.54
IC 2581	HD 90706	B3 Iab	7.06	...	0.47	0.60	−7.1	−8.2	4.18
	HD 90772	A8 Ia	4.67	...	0.50	0.37	−8.7	−8.6	3.91
χ Per	RS Per	M4 Iab	8.55	0.48	2.26	0.56	−5.3	−7.7:	3.47

The locations of these supergiants in the bolometric H-R diagram are fairly accurate. The estimated mean error of M_{bol} is ± 0.25 mag, while for log T_e it is ± 0.03 dex. The total error estimate for M_{bol} is compounded from the following contributions: A_V, ± 0.1 mag; BC, ± 0.1 mag; fitting of the cluster's observed main sequence, ± 0.15 mag; and use of the standard ZAMS for B stars, ± 0.15 mag (Blaauw 1963). It is uncertain whether the revised distance modulus of the Hyades cluster (3.3 as compared to the older 3.0) should be reflected in a commensurate increase in the absolute magnitudes of ZAMS B stars; independent evidence suggests that the traditional distance scale for B stars (Johnson 1963; Blaauw 1963) is very nearly correct as it stands (see § II*a* of Stothers 1983 and § 3 of Feast 1991).

The total error estimate for log T_e has been derived in part by assuming that the assigned spectral type of a supergiant is not in error by more than one decimal subclass and in part by considering the small scatter in the published effective temperature estimates for supergiants of known spectral type. For B-type and A-type supergiants, the basic effective temperature estimates are due to Humphries et al. (1975), Nandy & Schmidt (1975), Code et al. (1976), Flower (1977), Underhill et al. (1979), Kontizas & Theodossiou (1980), Böhm-Vitense (1981), and Theodossiou (1985), while Fitzpatrick & Garmany (1990) provide an effective temperature scale that lies somewhere near the middle of the others. For M-type supergiants, the effective temperature scales of Johnson (1966) and Lee (1970) agree very closely with each other.

3.3. *Intermediate-Age Clusters*

Evolved giants and supergiants that are possible members of intermediate-age clusters have been listed by Harris (1976), Mermilliod (1976), Schmidt (1984a, b), and Sowell (1987). Composite H-R diagrams for these evolved stars (Carson & Stothers 1976; Harris 1976; Mermilliod 1981; Grenon & Mermilliod 1984; Schmidt 1984b) indicate that the bluest objects have yellow colors and F or G spectral types, while the reddest are of type K, with an occasional exceptional type M. Schmidt (1984a, b) and Sowell (1987) have critically reexamined the memberships of all yellow candidates. We here adopt Schmidt's (1984b) final list, which is contained in his Tables 1 and 2, but we reject two stars in NGC 2546 following Sowell (1987).

Transformation data that were used by Schmidt to obtain bolometric absolute magnitudes included $R_V = 3.3$ from Crawford & Mandewewala (1976) and BC from Sandage & Gratton (1963). Schmidt also adopted the Crawford (1978) distance scale for middle and late B-type main-sequence stars, but this scale is close to Blaauw's (1963) and Johnson's (1963). The cluster distance moduli are in any case not very accurate, because, with few exceptions, only *UBV* (or other three-color) photometry is available for the main-sequence stars, and consequently unavoidable difficulties in fitting the ZAMS crop up (Mermilliod, Mayor, & Burki 1987). These and other problems limit the usefulness of the intermediate-age clusters to only a few of our proposed tests for convective core overshooting. Nevertheless, the estimated formal mean errors of the (M_{bol}, log T_e) coordinates of the yellow giants provided by Schmidt are similar to those for the evolved blue supergiants in the young clusters.

4. TWELVE TESTS BASED ON DETACHED CLOSE BINARY STAR SYSTEMS

4.1. *ZAMS Stars (Three Tests)*

According to theory, the luminosity and the radius of a ZAMS star are decreased by convective core overshooting. This prediction was used by Stothers & Chin (1990) to test for overshooting in the (mass, luminosity) and (mass, radius) diagrams of accurately measured O and B stars in detached close binary systems listed by Hilditch & Bell (1987). An upper limit of $d/H_P < 1.5$ was found, at least for the effective distance over which convective overshooting reduces the local temperature gradient. The (luminosity, effective temperature) relation, however, was determined to be insensitive to d/H_P.

An associated result from fitting in the (mass, radius) plane was that the "effective" interior metals abundance must be fairly high, $Z_e \approx 0.04$. This result is not really new (Popper et al. 1970; Lacy 1979; Popper 1982) and can also be inferred from fitting more generally in the (luminosity, effective temperature) plane (Stothers & Chin 1990) or in the (surface gravity, effective temperature) plane (Wolff 1990). Figures 5, 6, and 7 illustrate the fits.

4.2. *TAMS Stars (Two Tests)*

In Figures 5 and 6 containing the most reliable observational data of Hilditch & Bell (1987), the superposed lines are theoretical predictions for both the ZAMS and TAMS (Chin & Stothers 1991). Two values of the overshoot parameter, $d/H_P = 0$ and 0.70, are represented on the figures, but only for the TAMS, because the corresponding lines for the ZAMS

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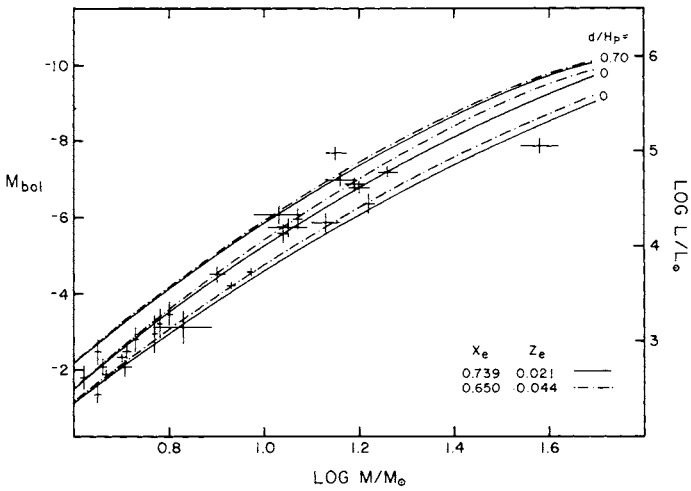

FIG. 5.—Mass vs. luminosity for upper–main-sequence stars. Observed members of detached close binary systems are plotted with their error estimates indicated. The theoretical ZAMS is shown for $d/H_P = 0$; the theoretical TAMS appears for $d/H_P = 0$ and 0.70.

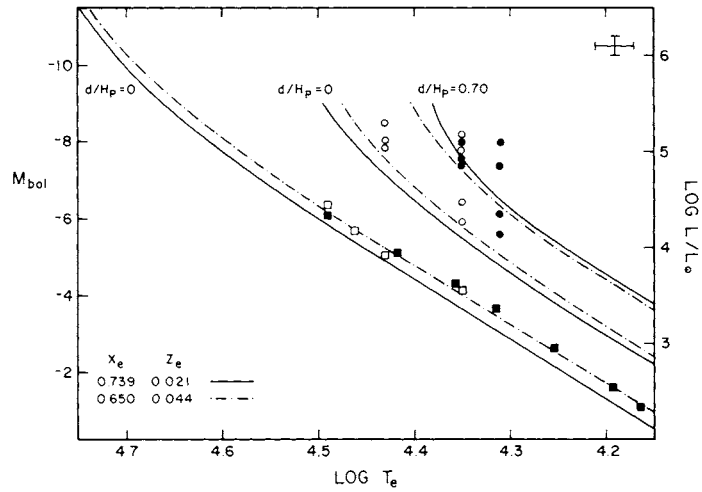

FIG. 7.—H-R diagram for upper–main-sequence stars. The observed standard ZAMS is indicated by the squares, and observed tips of the main-sequence turnups in young clusters by the circles. Conversion from spectral type to effective temperature was made by using the Morton & Adams (1968) relation (*filled symbols*) or the Underhill et al. (1979) relation (*open symbols*). The theoretical ZAMS is shown for $d/H_P = 0$; the theoretical TAMS appears for $d/H_P = 0$ and 0.70.

virtually coincide on the scale of the figures. In analyzing the observational data, it must be kept in mind that stellar evolution accelerates as the star approaches close to the TAMS, so that the expected population of stars near the TAMS is correspondingly less dense than near the ZAMS. Accordingly, the apparent location of the observational TAMS may be a slight underestimate of the true TAMS, although the effect ought to be less in luminosity than in radius.

Notice that the observational data, especially the stellar radii, are entirely consistent with the assumption of no overshooting. By using only the luminosities published by Popper (1980) earlier, Popova & Tutukov (1990) arrived at the same conclusion. An upper limit of $d/H_P < 0.4$ may therefore be set with some confidence. Hilditch & Bell (1987), however, came to an opposite conclusion, mostly because they included members of contact and semidetached systems in their diagnostic plots. Since the radii of such stars are probably enlarged by tidal action as well as by either mass loss or mass accretion, the apparent main-sequence band may well be artificially widened by including these perturbed stars.

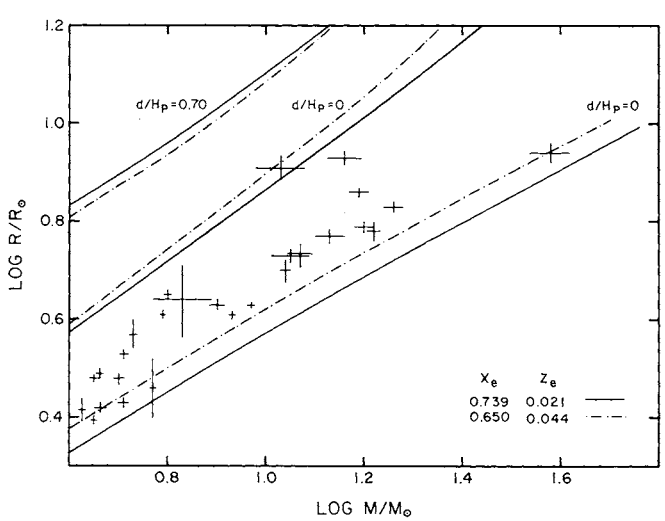

FIG. 6.—Mass vs. radius for upper–main-sequence stars. Same notation as for Fig. 5.

4.3. *Wolf-Rayet Stars (Six Tests)*

Schulte-Ladbeck (1989) concluded that binary-system Wolf-Rayet stars, which are believed to be the exposed helium cores of very massive O stars involved in a prior evolutionary mass transfer, possess normal core masses with respect to the masses of their OB star companions. Her detailed comparisons with Vanbeveren's (1987) nonovershooting stellar models and with Doom & De Greve's (1983) overshooting stellar models based on $d/H_P \approx 1.7$ would seem to imply that d/H_P must be considerably less than 1.7. Vanbeveren (1983) suggested that the short orbital periods observed for these systems also imply little or no overshooting.

Four other tests for overshooting that are based on Wolf-Rayet stars involve the surface chemical abundances of these stars and have yielded mutually contradictory results for d/H_P: ~ 0 (Vanbeveren 1983, 1989; Langer 1988), ~ 0.25 (Maeder & Meynet 1987), and ~ 2 (Prantzos et al. 1986). All of these tests, however, depend critically on the authors' differing assumptions about stellar wind mass loss and duplicity in a rare, and poorly understood, class of stars.

4.4. *Apsidal-Motion Stars*

Apsidal-motion constants for many well-observed O and B main-sequence stars have traditionally been found to be too small in comparison with the values expected from standard stellar models. Nonstandard stellar models with convective core overshooting (Odell 1974; Claret & Giménez 1991) or with increased metals opacities (Odell 1974; Stothers 1974; Monet 1980; Jeffery 1984) help to reduce or eliminate this discrepancy. Until the metals opacities are better known, however, no useful limit can be set on d/H_P by using this approach.

5. A TEST BASED ON MAIN-SEQUENCE TURNUP STARS

For the present set of young clusters with main-sequence turnups between B0.5 and B2, the tips of the turnups (Table 5)

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are plotted in the theoretical H-R diagram in Figure 7. Conversions from spectral type to effective temperature were made by using two temperature scales: the intermediate Morton & Adams (1968) temperature scale and the hot Underhill et al. (1979) temperature scale. The tips are much too bright if the Morton & Adams scale is used, but they are more or less as theoretically expected for the Underhill et al. scale, especially if the stellar models have a high metals abundance. The range of scatter above the TAMS line, however, is somewhat larger than the internal error and may originate in part, (1) from the discreteness of the spectral classification system, which lowers the resolution in effective temperature and bolometric correction; and (2) from the possible presence of some nonmembers among the stars, from unrecognized binary systems with nearly equal components, from massive stars that have been rejuvenated through a binary mass exchange, and from the observed circumstance of continuing massive-star formation in these clusters (Herbst & Miller 1982; Stothers 1985), all of which act to increase the apparent length of the turnup.

Only a weak limit on overshooting, $d/H_P < 0.7$, can be inferred from Figure 7. An earlier inference of very extensive overshooting from a diagram of this type (Stothers & Chin 1985) was made strictly on the basis of the Morton & Adams (1968) temperature scale. By using the Böhm-Vitense (1981) temperature scale, which falls between the Morton & Adams and Underhill et al. (1979) scales, Mermilliod & Maeder (1986), not surprisingly, derived an intermediate value for the overshoot parameter.

The essential equality of the tip and turnup spectral types in most of the clusters (Table 5) also suggests little or no overshooting (cf. Table 1). The apparent tip in NGC 4755, however, is anomalously cool and bright. It is defined by two stars, HD 111934 and HD 111990, that may well be in the helium core contraction stage or else burning core helium. If so, the revised tip of the turnup becomes normal (Schild 1970). The same explanation may account for the anomalously cool and bright location of HD 14443 in χ Per (Schild 1965), unless this star is hotter than its assigned spectral type of B2.

In the case of the still younger clusters and associations, the main-sequence band appears to merge continuously with the domain of more highly evolved stars for reasons that are still not understood (e.g., Fitzpatrick & Garmany 1990; Tuchman & Wheeler 1990) but are probably in part related to mass loss (e.g., Doom, De Greve, & de Loore 1986). The situation at present is much too confused to accept the apparent merger as evidence of a huge main-sequence widening, let alone as a demonstration of convective core overshooting, as some authors have proposed (Massevitch et al. 1979; Bressan et al. 1981; Meylan & Maeder 1982, 1983; Doom 1982a, b, 1985; Bertelli et al. 1984; Stothers & Chin 1985; Beech 1988; Nasi & Forieri 1990). Although relative star counts seem to point to some convective core overshooting, all of these counts are seriously incomplete among the O stars that feed the B and A supergiants (Humphreys & McElroy 1984), and, in fact, the same star counts can be interpreted as indicating no overshooting (Vanbeveren 1987).

Intermediate-age clusters suffer from even greater difficulties associated with fitting very poorly populated cluster main sequences to the standard ZAMS, as well as with recognizing nonmembers, assessing duplicity, and allowing for a potentially large range of stellar ages (Schmidt 1984b; Mermilliod et al. 1987). These factors all tend to spread the main-sequence band and to make very difficult the determination of the location of the true TAMS. The observed vertical spread above the theoretical TAMS line for no overshooting is less than 1 mag (Maeder & Mermilliod 1981), which could be accidental error, duplicity alone being able to contribute a spread of up to 0.75 mag. A higher metals abundance in the stellar models would also reduce the discrepancy. Not unexpectedly, a wide range of values for the overshoot parameter have been inferred by using intermediate-age clusters, d/H_P being estimated as 0.2–0.3 (Maeder & Mermilliod 1981; Maeder & Meynet 1989), 0.4–0.8 (Bertelli et al. 1985), and ~ 2 (Doom 1985). Field stars of intermediate age show a similar scatter (Grenier et al. 1985).

To circumvent the problems with determining a star's distance-based luminosity, model atmospheres can be used to infer the star's surface gravity, so that a distance-free comparison can be made with evolutionary tracks in the (surface gravity, effective temperature) diagram. This technique has yielded a d/H_P value of ~ 0 by using main-sequence stars in the Orion, Sco-Cen, and Per OB3 associations, and a value greater than 0.25 based on the stars in Lac OB1, the Pleiades, and NGC 2301 (Wolff 1990; Napiwotzki et al. 1991). Since the inferred surface gravity is very sensitive to the adopted effective temperature, the large scatter in the derived d/H_P values is perhaps not surprising.

Because this approach using effective temperatures and luminosities (or surface gravities) for TAMS stars is so fraught with pitfalls, it is probably safer to use the (mass, luminosity) and (mass, radius) diagrams based on detached close binary systems (§ 4.2), where these serious difficulties do not arise.

6. SEVEN TESTS BASED ON EVOLVED GIANTS AND SUPERGIANTS

6.1. *Highest Effective Temperature of Evolved Stars in Young and Intermediate-Age Clusters*

Hot evolved giants and supergiants with the highest likelihood for membership in both the young and intermediate-age clusters and having the most accurately known luminosities and effective temperatures (§§ 3.2, 3.3) are shown plotted on the H-R diagram in Figure 8. The superposed solid lines for several d/H_P values represent the maximum derived effective temperatures of theoretical models evolving during the core helium-burning phase without mass loss. The single dashed line indicates, for $d/H_P = 0$, the *coolest* derived maximum effective temperature among the published models, excluding the situation where the models burn core helium solely as red giants or supergiants.

No observed star is found to the left of the hottest theoretical extremum predicted for $d/H_P = 0$, while some stars lie to the left of the coolest theoretical extremum in the same case. This demonstrates that the assumption of $d/H_P = 0$ satisfies the observations. The result is unchanged even if one ignores the two brightest stars, members of IC 2581, which have possibly suffered significant mass loss (their luminosities suggest initial masses of $\sim$20–25 $M_\odot$). Using the hottest theoretical extremum for $d/H_P = 0.35$, conservative upper limits of $d/H_P < 0.2$ and $d/H_P < 0.4$ can be placed from the data for stars in the young clusters and in the intermediate-age clusters, respectively. The only two ways to raise the effective temperatures significantly would be to assume either very deep convective envelope overshooting with $D \gg 0.4H_P$ (Stothers & Chin 1991; Alongi et al. 1991) or a critical initial rotational state of the star (Kippenhahn, Meyer-Hofmeister, & Thomas 1970; Meyer-

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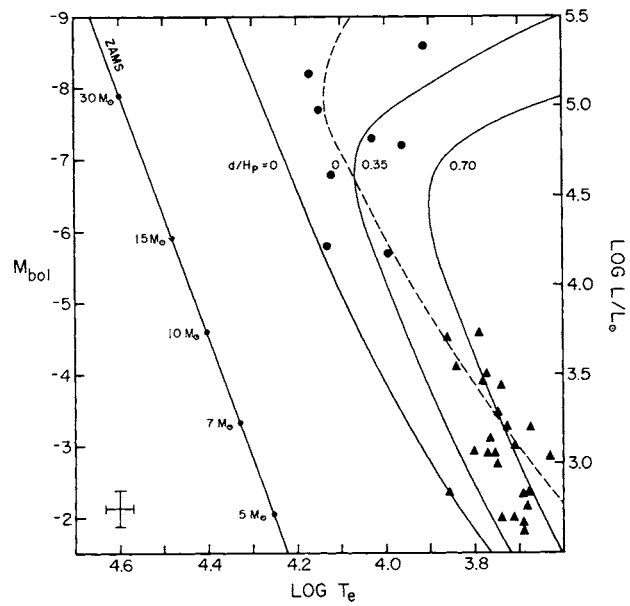

FIG. 8.—H-R diagram for hot evolved stars in young clusters (*filled circles*) and in intermediate-age clusters (*filled triangles*). The continuous lines lying to the right of the ZAMS trace out the locus of the maximum derived effective temperature of stellar models during the core helium-burning phase. The dashed line refers to the coolest derived maximum effective temperature, excluding stellar models that burn helium solely as red giants or supergiants.

Hofmeister 1972). Both assumptions are unrealistically extreme, however.

Using comparable data, Maeder & Meynet (1989) deduced a definite value of $d/H_P \approx 0.25$ by interpreting the median line of the observational points in the H-R diagram to be the true blue edge. In our view, their underlying assumption is incorrect for several reasons. First, the estimated errors of the observational points are relatively small (especially for the more refined data used here), and so the observational scatter must be mostly real. Second, the significant spread of effective temperatures at each luminosity almost certainly represents evolution along the blue loop during core helium burning, especially because the theoretical blue edge is found to be comparatively insensitive to initial chemical composition differences that may exist among the stars (Chin & Stothers 1991); both the observed and predicted evolutionary spreads of log T_e are 0.15 ± 0.05 dex. Third, none of the stars in Figure 8 is known or suspected to be the remnant of a binary mass exchange. Fourth, the incorporation of somewhat inferior data from other young clusters and associations, in order to build up the statistics, confirms the distribution of points in Figure 8 (see the H-R diagrams in Humphreys 1978; Humphreys & Davidson 1979; Humphreys & McElroy 1984).

6.2. *Relative Luminosities of Blue and Red Supergiants in Young Clusters*

Young star clusters in which both blue and red evolved supergiants coexist and have been accurately measured are NGC 581, NGC 3766, and NGC 4755. The mean bolometric absolute magnitude of the blue supergiants in each of these clusters is plotted in Figure 9 against the mean bolometric absolute magnitude of the red supergiants. The estimated error of the absolute magnitudes is very small, only ± 0.15 mag, because *relative* luminosities of stars within the same cluster are being compared.

According to published theoretical models, a core helium-burning star evolves nearly horizontally on the H-R diagram when the star is blue, and nearly vertically when red. To allow for this expected difference in behavior, Figure 9 shows both the upper and lower predicted limits of luminosity based on red-supergiant models in slow stages of evolution plotted against the average luminosity of the corresponding blue-supergiant models. Allowance for equality of the ages (rather than the masses) of the supergiants would theoretically entail a correction of only ~ 0.1 mag or less.

Comparison of the theoretical models with observations suggests that convective core overshooting is probably slight, $d/H_P < 0.4$ being inferred. The present test, however, would be more meaningful if additional clusters with reliable data were available, because some of the scatter in Figure 9 is probably due to a moderate age spread among the supergiants in each cluster.

To reduce the effect of the observational scatter, the mean luminosity of all the blue supergiants in the three clusters can be compared with the mean luminosity of all the red supergiants. The result for the luminosity difference is $\Delta\langle M_{\rm bol}\rangle = -0.4 \pm 0.5$. Although this difference turns out to be not statistically significant, it is worth noting that a larger (but less well observed and possibly biased) sample of supergiants belonging to young clusters and associations also shows a small negative luminosity difference (Stothers & Lloyd Evans 1970). Theoretical stellar models predict $\Delta\langle M_{\rm bol}\rangle = -0.4 \pm 0.1$ for $d/H_P = 0$, and $\Delta\langle M_{\rm bol}\rangle = 0.0 \pm 0.1$ for $d/H_P = 0.35$, over the mass range 10–17 $M_\odot$. Therefore, a value of $d/H_P < 0.4$ seems to be suggested by the observations.

Unlike the other tests for convective core overshooting performed in this paper, this test alone is potentially sensitive to the criterion adopted for convective neutrality in semi-convective zones. The prediction just given for $d/H_P = 0$ was based on the Ledoux criterion, whereas in the case of the Schwarzschild criterion the corresponding prediction would be $\Delta\langle M_{\rm bol}\rangle = -0.1 \pm 0.1$, applicable to stellar masses above $\sim 15\ M_\odot$. However, since both predictions for $d/H_P = 0.35$ are $\Delta\langle M_{\rm bol}\rangle = 0.0 \pm 0.1$ and since masses of the cluster super-

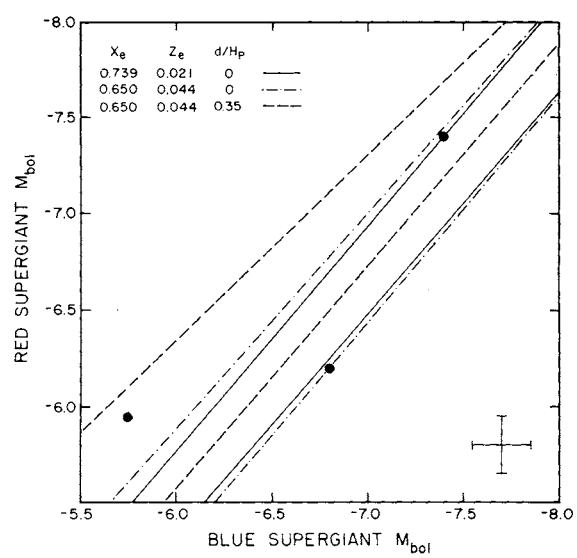

FIG. 9.—Mean bolometric absolute magnitude of red evolved supergiants in a young cluster vs. the same for the cluster's blue evolved supergiants. Theoretical predictions are shown by the series of lines.

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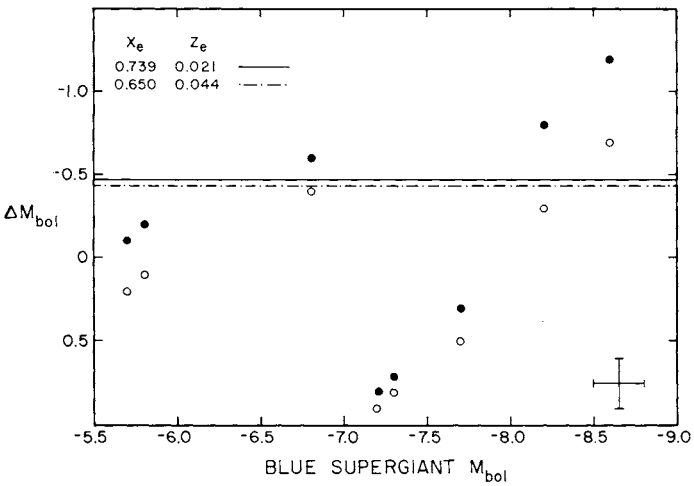

FIG. 10.—Bolometric magnitude difference between the blue evolved supergiants and the stars at the tip of the main-sequence turnup in young clusters, as a function of the bolometric absolute magnitude of the blue supergiant. Conversion from spectral type to effective temperature for the main-sequence turnup stars was made by using the Morton & Adams (1968) relation (*filled circles*) or the Underhill et al. (1979) relation (*open circles*). Theoretical predictions are shown by the two lines; they refer to $d/H_P = 0$, but they shift upward by $0.3d/H_P$ mag when convective core overshooting is present.

giants appear to be less than $\sim 17\ M_\odot$, the practical outcome of this test for convective core overshooting is unchanged.

6.3. *Relative Luminosities of Evolved Stars and Main-Sequence Stars in Young and Intermediate-Age Clusters*

It is also possible to compare the individual luminosities of evolved blue supergiants in a young cluster with the luminosity of stars at the tip of the main-sequence turnup. Figure 10 shows the observed difference ΔM_{bol} (supergiant minus tip of the turnup) as a function of the supergiant's luminosity. The scatter is very large because it is compounded from the accidental dispersions of two independently measured luminosities. Compared to this observed scatter of ± 1 mag, the theoretically predicted displacements of ΔM_{bol} due to convective core overshooting turn out to be too slight to test for, being only $\delta\Delta M_{bol} \approx 0.3d/H_P$ mag (Chin & Stothers 1991).

For clusters of intermediate age, Schmidt (1984b) detected an equally large scatter in a diagram of this type. However, Maeder & Meynet (1989) argued that their stellar models with $d/H_P = 0.25$ agreed with Schmidt's observations better than did the models of Becker (1981) which omitted convective core overshooting. Even though the initial chemical compositions used in Maeder & Meynet's and Becker's studies were identical, some of the other input physics differed and thereby created a spuriously large theoretical luminosity difference $\delta\Delta M_{bol}$ that Maeder & Meynet mistakenly attributed to overshooting. The fully self-consistent stellar models of Becker & Cox (1982), Matraka et al. (1982), Bertelli et al. (1985), and Chin & Stothers (1991), in fact, demonstrate a negligible theoretical luminosity difference as a result of overshooting.

6.4. *Ages of Evolved Supergiants and Main-Sequence Stars in Young Clusters*

A young star cluster can be dated in two completely independent ways: (1) from the spectral type of its main-sequence turnup and (2) from the absolute magnitudes of its evolved supergiants. A comparison of ages for the present sample of clusters is shown in Figure 11. Each red supergiant in these clusters (omitting the uncertain RS Persei) is plotted eight times, corresponding to the eight adopted combinations of initial chemical composition and convective overshoot parameter. Each blue supergiant, however, appears only four times, because no blue loop develops for the larger of the two adopted overshoot parameters.

It should be observed that with the Morton & Adams (1968) temperature scale, which was the scale actually used to convert the spectral types of the main-sequence stars, the turnup ages are systematically larger than the supergiant ages in the case where no convective overshooting is allowed. This puzzling discrepancy between the two sets of ages has been known for a long time (Stothers 1972). However, the use of the hotter Underhill et al. (1979) temperature scale brings the two sets of ages into satisfactory agreement without the need for convective core overshooting.

Otherwise, we find that with the Morton & Adams (1968) temperature scale a moderate value of $d/H_P \approx 0.4$ is required for agreement, while with the cooler Nandy & Schmidt (1975) scale, d/H_P probably has to exceed 0.7. Since a hot temperature scale is now likely to be correct, Figure 11 seems to imply $d/H_P < 0.4$.

6.5. *Relative Numbers of Evolved Supergiants and Main-Sequence Stars in Young Clusters*

The total number of evolved supergiants observed in a young star cluster can be simply related to the total number of main-sequence stars counted in a fixed interval of luminosity, by introducing the ratio of the post–main-sequence lifetime, τ_{He}, to the core hydrogen-burning lifetime, τ_H (Hayashi & Cameron 1962; Hayashi, Hōshi, & Sugimoto 1962). It is assumed here that the post–main-sequence lifetime is essentially that of core helium burning, owing to the inferred copious emission of neutrinos in the later phases of evolution (Stothers 1969b, 1985).

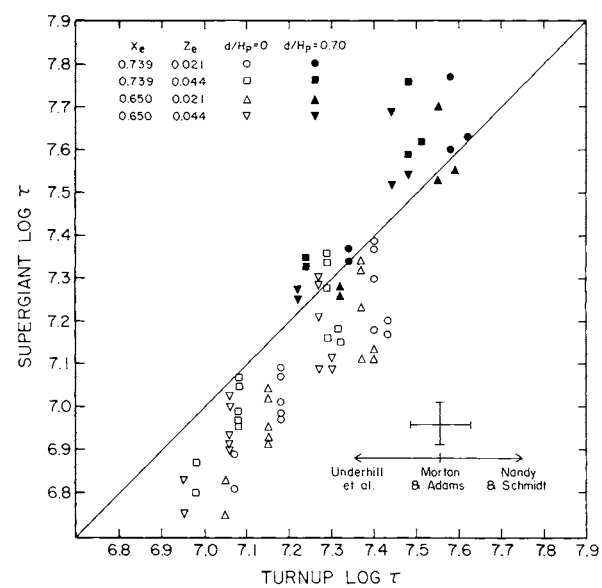

FIG. 11.—Age of the main-sequence turnup vs. ages of blue and red evolved supergiants in young star clusters. Magnitude and direction of the shift of a typical point are indicated for three main-sequence effective temperature scales.

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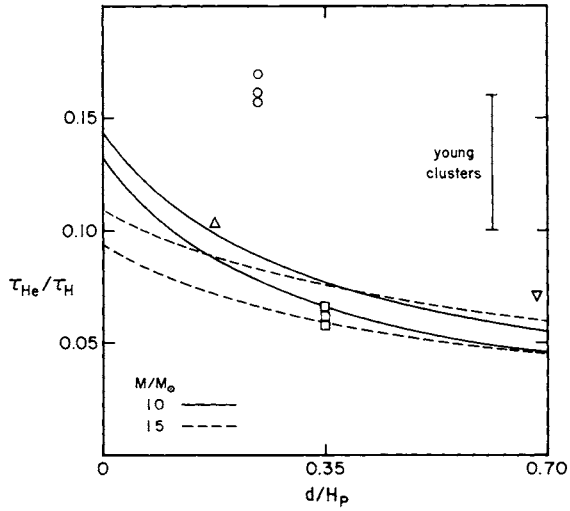

FIG. 12.—Ratio of the core helium-burning lifetime to the core hydrogen-burning lifetime for evolved stars in young clusters. Theoretical predictions come from Chin & Stothers (1991) (*bands*), Bertelli et al. (1984, 1985, 1986) and Nasi & Forieri (1990) (*squares*), Matraka, Wassermann, & Weigert (1982) (*triangle*), Cloutman & Whitaker (1980) (*inverted triangle*), and Maeder & Meynet (1987, 1988, 1989) and Maeder (1990) (*circles*) for stellar masses of 8–20 $M_\odot$.

By pooling the data for several clusters, the statistical significance of the results for any one cluster can be increased. The present set of young clusters contains 11–17 evolved supergiants, depending on whether Schild's (1970) rejection of three supergiants is accepted or not (§ 3.2) and whether one regards any of the three superluminous turnup stars as evolved stars or not (§ 5). Despite this uncertainty in the star numbers, these data together with the main-sequence star counts have already been shown to be roughly consistent with a ratio of $\tau_{He}/\tau_H \approx 0.12$ (Stothers 1985). A somewhat more refined estimate of $\tau_{He}/\tau_H = 0.13 \pm 0.03$ will be used here.

Theoretical predictions of τ_{He}/τ_H are presented in Figure 12 as a function of d/H_P for stars in the mass interval 8–20 $M_\odot$. Note the rapid decline of τ_{He}/τ_H with increasing d/H_P. Two narrow bands shown for the Chin & Stothers (1991) models of 10 $M_\odot$ and 15 $M_\odot$ cover the presently adopted ranges of initial chemical composition. Their models evidently agree very well with those published by Cloutman & Whitaker (1980) for 15.57 $M_\odot$ with $d/H_P \approx 0.7$, by Matraka et al. (1982) for 8 $M_\odot$ with $d/H_P \approx 0.17$, and by Bertelli et al. (1984, 1985), Bertelli et al. (1986), and Nasi & Forieri (1990) for 9, 15, and 20 $M_\odot$ with $d/H_P \approx 0.35$. Why a discrepancy of a factor of ~ 2 is found in the case of the models of Maeder & Meynet (1987, 1988, 1989) and Maeder (1990) for 9, 15, and 20 $M_\odot$ with $d/H_P = 0.25$ cannot be answered at the present time. The fact that stellar wind mass loss was included in their models cannot be the explanation (cf. Bertelli et al. 1984; Nasi & Forieri 1990); nor can any possible choice for the $^{12}C(\alpha, \gamma)^{16}O$ reaction rate affect the core helium-burning lifetime by more than 30% (Stothers & Chin 1973).

If the results of Maeder & Meynet are ignored, a rather firm upper limit of $d/H_P < 0.4$ can be derived from Figure 12. Otherwise, the test has only potential value.

6.6. *Relative Numbers of Hot and Cool Evolved Stars in Young and Intermediate-Age Clusters*

The ratio of the observed numbers of hot and cool evolved stars in a young or intermediate-age cluster is expected to be a close measure of the ratio of the relative amounts of time spent burning core helium at high and low effective temperatures. Table 7 presents the observed number ratios n_b/n_r for the two age groups of clusters, as derived from five published sources.

Correction of these ratios, however, must be made for the possible effects of stellar duplicity. Since main-sequence stars that belong to close binary systems with orbital periods of less than ~ 1 yr cannot evolve into the dimensions of red giants and supergiants, the observed ratios n_b/n_r are necessarily skewed toward somewhat too high values. But in view of the fact that the average correction factor is unlikely to exceed 30% (Stothers 1969a; Burki & Mayor 1984), a correction has not been applied.

For comparison with the observed number ratios, theoretically predicted lifetime ratios τ_b/τ_r are exhibited in Figure 13 as a function of d/H_P. Only nonzero values of τ_b/τ_r, derived from evolutionary tracks containing blue loops, are shown in the figure. Note that the models of Matraka et al. (1982) for 8 $M_\odot$ with $d/H_P \approx 0.17$ and of Bertelli et al. (1985) and Bertelli et al. (1986) for 9 $M_\odot$ with $d/H_P \approx 0.35$ agree very well with the models of Chin & Stothers (1991) for 10 and 15 $M_\odot$, as shown by the dashed lines. However, the models of Maeder & Meynet (1988, 1989) and Maeder (1990) for 9 and 15 $M_\odot$ with $d/H_P = 0.25$ predict ratios τ_b/τ_r that are larger than the others by a factor of ~ 2.

Ignoring the models of Maeder & Meynet, one would conclude from Figure 13 that for young clusters it is reasonable to infer $d/H_P < 0.4$. Since the observed ratio $n_b/n_r = 1.7 \pm 0.4$ is much closer to the average theoretical ratio $\tau_b/\tau_r = 1.2$ for $Z_e \approx 0.04$ than to $\tau_b/\tau_r = 0.6$ for $Z_e \approx 0.02$, at least in the Chin & Stothers nonovershooting models, this may be further evidence of a large metals abundance (cf. § 4.1). But the ratio τ_b/τ_r is a very sensitive and uncertain quantity for high stellar masses and can exceed unity for $Z_e \approx 0.02$ in other published nonovershooting model sequences.

For intermediate-age clusters nothing definite about d/H_P

TABLE 7

RELATIVE NUMBERS OF HOT AND COOL EVOLVED STARS IN GALACTIC CLUSTERS

Cluster Turnup Spectral Type	Age Groups	n_b	n_r	n_b/n_r	Reference
B0.5-B2	"Young"	8	6	1.3	Present paper
	"II"	12	10	1.2	Harris 1976
	"10, 11, 12"	23	11	2.1	Mermilliod 1981
B3-B7	"Young, Middle"	14	38	0.37	Carson & Stothers 1976
	"III, IV"	27	49	0.55	Harris 1976
	"7, 8, 9"	7	19	0.37	Mermilliod 1981
	"Cepheid-age"	14	36	0.39	Mermilliod et al. 1987

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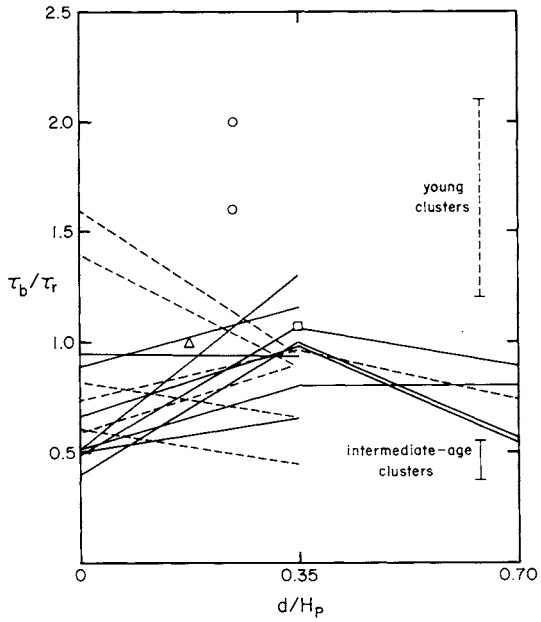

FIG. 13.—Ratio of the amounts of time spent as hot stars and as cool stars during the core helium-burning phase. Data for young and intermediate-age clusters are displayed separately. Theoretical predictions of the ratio (if it is nonzero) are plotted as in Fig. 12, except that here the lines refer to stellar masses of 5 and 7 $M_\odot$ (*solid lines*) and 10 and 15 $M_\odot$ (*dashed lines*). The other symbols refer to masses of 8–20 $M_\odot$.

can be said. Mermilliod et al. (1987), however, reached a more definite conclusion for the intermediate-age clusters by finding that the observed value of $n_b/n_r \approx 0.4$ contradicts the theoretical prediction of $\tau_b/\tau_r > 1$ for $d/H_P = 0$ based on the models of Matraka et al. (1982). However, as Figure 13 shows, they used much too limited a sample of models in reaching this otherwise unsupported conclusion. In a later study, Maeder & Meynet (1989) pooled the data for both the young and intermediate-age clusters, and obtained $n_b/n_r \approx 1.0$. This ratio was noted as being compatible with $1 \leq \tau_b/\tau_r \leq 2$ for $d/H_P = 0.25$, as derived from their own stellar models. Figure 13, however, demonstrates that their models are far from typical and that such extensive pooling of the observational data actually destroys the ability to discriminate d/H_P critically.

6.7. *Classical Cepheids*

Masses for classical Cepheids can be estimated in essentially five independent ways. Orbital, evolutionary, and pulsation-constant masses are in fairly close agreement with each other, and are not highly sensitive to existing uncertainties in the initial chemical composition or opacities within these stars (e.g., Carson & Stothers 1988; Gieren 1989). Bump masses and beat (or double-mode) masses, however, depend sensitively on the envelope opacities. Both types of masses are much too small if the Los Alamos opacities are adopted for the stellar models (Cox 1980), but only the beat masses are too small if Carson's opacities are used (Carson & Stothers 1976, 1988); both types of masses turn out to be essentially normal when the new Iglesias & Rogers opacities are applied (Moskalik, Buchler, & Marom 1991). Although convective core overshooting has previously been suggested as a way to ease or remove some of the mass discrepancies (Becker & Cox 1982; Matraka et al. 1982; Huang & Weigert 1983; Bertelli et al. 1985; Carson & Stothers 1988), this remedy may no longer be necessary. Or if it is, the amount of overshooting cannot be adequately assessed until the opacity situation clears, and, even then, entirely different ways of explaining the mass discrepancies have been proposed (Cox 1980; Stothers 1982; and references therein).

7. CONCLUSION

Twenty different (and mostly independent) tests for convective core overshooting have been discussed in this paper. Fourteen of these tests refer primarily to main-sequence turnup stars and evolved giants and supergiants of 4–17 $M_\odot$ that are observed in the Galaxy. Six additional tests are based on Galactic Wolf-Rayet stars. Final results for all the tests (except four very weak tests among the six tests based on Wolf-Rayet stars) are collected in Table 8. The upper limits on d/H_P listed there are very conservative estimates.

Several conclusions may be drawn from this table and from the discussion in the preceding sections.

1. All the best test results are actually consistent with the assumption $d/H_P = 0$. The sharpest test in the set, based on the maximum effective temperature of evolved blue supergiants, although subject to some mild restrictions, yields $d/H_P < 0.2$, while at least four other tests point to $d/H_P < 0.4$. These test results are not sensitive to uncertainties about the initial chemical composition, and therefore are likely to be robust against further revision of the stellar opacities. The best evidence favoring some convective core overshooting is based on the location of TAMS stars in the H-R diagram, but we have provided arguments why the positive-detection results of Maeder, Mermilliod, and Meynet ($d/H_P = 0.2$–0.3), Bertelli et al. ($d/H_P = 0.4$–0.8), Stothers and Chin ($d/H_P \approx 0.7$), and Doom ($d/H_P \approx 2$) should probably be regarded as only upper limits.

2. Uncertainty about the correct criterion to use for convective neutrality in semiconvective zones is not an important issue in the present tests.

3. The current uncertainty in the distance scale for B-type main-sequence stars also has no significant influence on any of the present tests.

4. The effective temperature scale, where it is most uncertain for B-type main-sequence stars—in the spectral subinterval B0.5-B2—should probably be taken to be the hottest scale among those so far proposed. This conclusion rests mainly on a large number of consistency checks.

5. Stellar models that are most successful in reproducing the observations of B-type main-sequence stars seem to require an unusually high metals abundance, $Z_e \approx 0.04$. This result may indicate an inadequacy of the currently adopted envelope opacities, since the average metals abundance that is directly observed in the atmospheres of Population I stars in the local Galactic neighborhood is very close to being solar, $Z_e \approx 0.02$ (Nissen 1988; Clariá, Lapasset, & Minniti 1989; Luck & Bond 1989; Fitzsimmons et al. 1990). Although Carson's (1976) large metals opacities, which at one time led to some successful predictions, are now known to be incorrect (Carson et al. 1984), more recent theoretical calculations do support the reality of larger metals opacities at the relevant temperatures and densities (Rozsnyai 1989; Iglesias & Rogers 1991). These proposed opacity changes, at some temperatures, dwarf the rather incremental improvements that have been applied at Los Alamos to the standard Cox-Stewart opacities over the years (e.g., Cox & Tabor 1976).

Several other outstanding problems that could be alleviated

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TABLE 8

RESULTS OF TESTS FOR CONVECTIVE CORE OVERSHOOTING IN STARS OF INTERMEDIATE TO HIGH MASS

Age Group	Test	Main-Sequence Temperature Scale	d/H_P
All ages	Masses and luminosities of ZAMS stars	Hot	<1.5[a]
	Masses and radii of ZAMS stars	...	<2[a]
	Luminosities and effective temperatures of ZAMS stars	Hot	Indeterminate[a]
	Masses and luminosities of TAMS stars	Hot	<0.4
	Masses and radii of TAMS stars	...	<0.4
	Apsidal-motion stars	...	Indeterminate[b]
Very young	Masses and orbital periods of Wolf-Rayet stars	...	$\ll 1.7$[c]
Young	Luminosities and effective temperatures of TAMS stars	Hot	<0.7
	Highest effective temperature of evolved supergiants	...	<0.2
	Relative luminosities of blue and red supergiants	...	<0.4
	Relative luminosities of evolved stars and turnup stars	Hot	Indeterminate
	Ages of evolved stars and turnup stars	Hot	<0.4
	Relative numbers of supergiants and main-sequence stars	...	<0.4:
	Relative numbers of blue and red supergiants	...	<0.4:
Intermediate	Luminosities and effective temperatures of TAMS stars	...	<2[d]
	Highest effective temperature of evolved giants	...	<0.4
	Relative luminosities of evolved stars and turnup stars	...	Indeterminate
	Relative numbers of yellow and red giants	...	Indeterminate
	Masses of classical Cepheids	...	Indeterminate

[a] From Stothers & Chin 1990.
[b] From Claret & Giménez 1991.
[c] From Schulte-Ladbeck 1989 and Vanbeveren 1983.
[d] From sources cited in § 5.3.

by introducing larger metals opacities into stellar models include the puzzlingly slow apsidal motions of a number of massive close binary systems (Odell 1974; Stothers 1974; Claret & Giménez 1991), the anomalously small period ratios of the double-mode classical Cepheids and δ Scuti stars (Simon 1982; Andreasen & Petersen 1988; Moskalik et al. 1991), and the apparently excessive radii of normal A-type main-sequence stars (Lemke 1989; Maeder & Meynet 1989; Andersen, Nordström, & Clausen 1990). It is also conceivable that such an opacity increase would help explain, in part, the extended distribution of very massive blue supergiants on the H-R diagram, possibly along the lines of related earlier suggestions (Stothers 1976; Stothers & Chin 1977, 1978; Bertelli et al. 1984; Nasi & Forieri 1990). Finally, there have recently been two detailed theoretical studies of the populous young metal-poor cluster NGC 330 in the Small Magellanic Cloud (Carney, Janes, & Flower 1985; Stothers & Chin, in preparation) that show, using Los Alamos opacities, a best fit to the cluster H-R diagram with an assumed metals abundance that is 2–10 times the spectroscopically inferred value for the brightest cluster stars. The intermediate-age cluster NGC 458 in the Small Cloud shows the same effect.

The studies of NGC 330 and NGC 458 by Stothers & Chin also found $d/H_P < 0.2$ by fitting the many evolved hot and cool supergiants in these clusters to theoretically calculated evolutionary tracks for core helium-burning stars. Their result for d/H_P does not depend sensitively on the choice of initial chemical composition or on the uncertainties about interior rotation or the depth of convective envelope penetration. Regardless of metallicity, Galactic and Small Magellanic Cloud star clusters suggest that d/H_P is probably less than 0.2.

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