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# SECOND-OVERTONE MODELS OF RR LYRAE STARS

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## ABSTRACT

Linear models of RR Lyrae stars pulsating in the second overtone indicate that this mode is self-excited only for rather low luminosities if the masses are normal. An unstable second-overtone model has been selected from the linear survey and has been followed up to limiting amplitude by using a nonlinear hydrodynamical computer program. Deep splitting of the main light peak in the theoretical light curve occurs but does not resemble any observed feature. Moreover, the distribution of periods of RR Lyrae stars in globular clusters can be accounted for adequately by fundamental-mode and first-overtone pulsation. It is concluded that second-overtone pulsators probably do not exist among RR Lyrae stars.

*Subject headings:* stars: pulsation — stars: RR Lyrae

## I. INTRODUCTION

It is still an open question whether second-overtone pulsators exist among RR Lyrae stars. Observationally, such stars would be expected to show effective temperatures comparable to or higher than those of known first-overtone pulsators, just as the latter are generally hotter than recognized fundamental-mode variables. Owing to a small radius and high angular oscillation frequency, the periods should be among the shortest observed. Van Albada and Baker (1973) suggested that as many as four type *c* variables in the globular cluster M68 having periods about 0.8 times the periods of the other type *c*'s in the same cluster might be second-overtone pulsators. Similar suggestions have been made for one variable in NGC 4833 (Demers and Wehlau 1977) and two in IC 4499 (Clement, Dickens, and Bingham 1979).

Little is known theoretically about the possible existence of second-overtone pulsators. In his nonlinear survey of RR Lyrae models, Christy (1966) discovered no evidence of unstable second overtones, but he did not make a systematic search for them. Other published models so far have been linear ones. In a follow-up study of Christy's principal model sequence, characterized by a mass of $M/M_{\odot} = 0.578$ and a luminosity of $\log (L/L_{\odot}) = 1.585$, Castor (1971) found that at all effective temperatures the second overtone was in fact stable; on the other hand, Stellingwerf (1976) obtained instability over a wide range of temperatures. Moreover, the use of $M/M_{\odot} = 1.0$ produced a large instability at $\log (L/L_{\odot}) = 1.70$ and 1.89 in the models of Baker (1965) and Deupree and Hodson (1977), respectively. Hubickyj (1983) subsequently showed that the second overtone always becomes less stable with increasing mass or with decreasing luminosity, and that metals abundance is not an important factor although the hydrogen and helium opacities can be critical.

The most probable mass and luminosity for an RR Lyrae star are $M/M_{\odot} = 0.6 \pm 0.1$ and $\log (L/L_{\odot}) = 1.7 \pm 0.1$ (Stothers 1981, 1983; Cox, Hodson, and Clancy 1983; Hubickyj 1983; Hubickyj and Stothers 1986). The main purpose of the present paper is to determine whether unstable second-overtone RR Lyrae stars can, in principle, exist and, if so, whether their light and velocity curves are distinctive in any way.

## II. LINEAR MODELS

Linear nonadiabatic pulsation theory has been applied to static models of radiative stellar envelopes composed of hydrogen, helium, and metals with relative abundances $(X, Y, Z) = (0.745, 0.250, 0.005)$. By using the Carson opacities (Carson, Stothers, and Vemury 1981) or the revised "King" Los Alamos opacities (Stothers 1981), the second-overtone models are found to be stable with standard values of $M/M_{\odot} = 0.6$ and $\log (L/L_{\odot}) = 1.7$ (Hubickyj 1983).

Within the estimated uncertainties of these values, however, $M/M_{\odot} = 0.679$ and $\log (L/L_{\odot}) = 1.585$ may be selected as being marginally realistic. Second-overtone models constructed for this special case with revised "King" Los Alamos opacities exhibit stability, but with the Carson opacities the models are quite unstable. The difference between the models depends only on the hydrogen and helium opacities, because the metallic contribution to the total opacity is negligible in the main pulsating layers of these stars. Therefore, the well-known errors in Carson's metallic opacities are irrelevant for RR Lyrae stars, as was pointed out previously (Hubickyj and Stothers 1986). However, the division between stability and instability must actually be rather delicate, because, for example, different published versions of the Los Alamos opacities have sometimes given different results (Castor 1971; Stellingwerf 1976). The Carson opacities are adopted here since they have led to better agreement between theoretical models and observations in the case of BL Her stars and type *ab* RR Lyrae stars, and at least equally good agreement in the case of first-overtone type *c* RR Lyrae stars (Carson, Stothers, and Vemury 1981; Stothers 1981; Carson and Stothers 1982; Hubickyj and Stothers 1986).

## III. A NONLINEAR MODEL

To predict the observable light and velocity curves for a typical second-overtone pulsator, the nonlinear computer program described by Vemury and Stothers (1978) was used together with input parameters $M/M_{\odot} = 0.679$, $\log (L/L_{\odot}) = 1.585$, $\log T_e = 3.851$, and the Carson opacities. Regular pulsations were initiated from a relaxed static model, to which was applied a velocity profile approximating closely that given by a linearized-pulsation calculation. The initial surface velocity

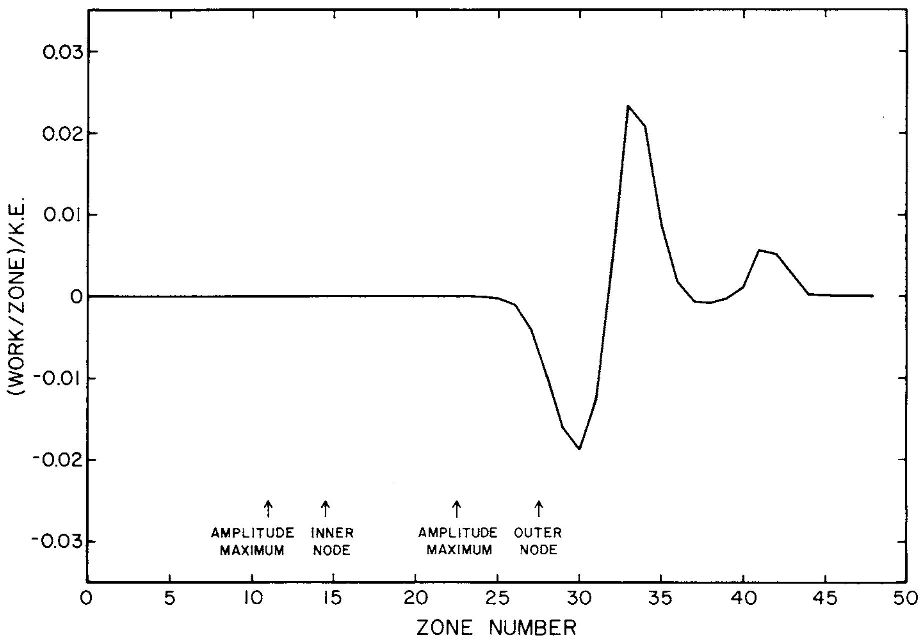


FIG. 1.—Work integral per zone (in units of the peak kinetic energy) for the second-overtone model

was specified to be a small value ($3 \text{ km s}^{-1}$, directed inward) so as not to depart too much from the linear approximation. Care was necessary to avoid introducing significant pulsational contributions from the (unstable) first overtone and fundamental mode. The growing pulsations were followed until the model's limit cycle was attained.

At full amplitude, the model had the following characteristics: $P = 0.218$ day, K.E. $= 3.25 \times 10^{38}$ ergs, $\Delta R/R = 0.042$, $V_{\text{out}} = 19 \text{ km s}^{-1}$, $V_{\text{in}} = -18 \text{ km s}^{-1}$, $\Delta V = 37 \text{ km s}^{-1}$, $L_{\text{max}} = 1.79 \times 10^{35} \text{ ergs s}^{-1}$, $L_{\text{min}} = 1.26 \times 10^{35} \text{ ergs s}^{-1}$, $\Delta M_{\text{bol}} = 0.38$, and pulsational nodes at $r/R = 0.91$ and 0.70. Amplitudes below the outer node, however, were almost negligible. The work curve is plotted in Figure 1, demonstrating that no significant driving or damping occurs below zone 25, where the temperature averages $1 \times 10^5$ K. Consequently, the important oscillations are confined to a shallower region than in the case of the fundamental mode or even the first overtone; however, the deeper zones in the model are needed to fix the precise locations of the nodes and secondary maxima that determine the value of the period. The surface pulsation amplitudes turn out to be somewhat smaller than in a typical first-overtone pulsator, just as the amplitudes of the first overtone typically fail to reach fundamental-mode levels (Christy 1966; Stellingwerf 1975; Hubickyj and Stothers 1986).

The surface velocity and light curves of the present model are shown in Figure 2. Despite the rather low pulsation amplitudes, the curves are markedly nonsinusoidal. In particular, the light curve exhibits a prominent early rise, which has the same physical cause as it does in a first-overtone pulsator (Christy 1966). Compared to the first overtone, however, the second overtone shows a much greater depth in its secondary light minimum, which can be regarded as the characteristic feature of second-overtone pulsation. This feature did not show up in the simple one-zone model of Stellingwerf, Gautschy, and Dickens (1987).

To check the physical reality of this feature, our model was recomputed with about half the number of mass zones (25 vs. 47 before). Full-amplitude characteristics were qualitatively the same as before, except that, in the more crudely zoned model,

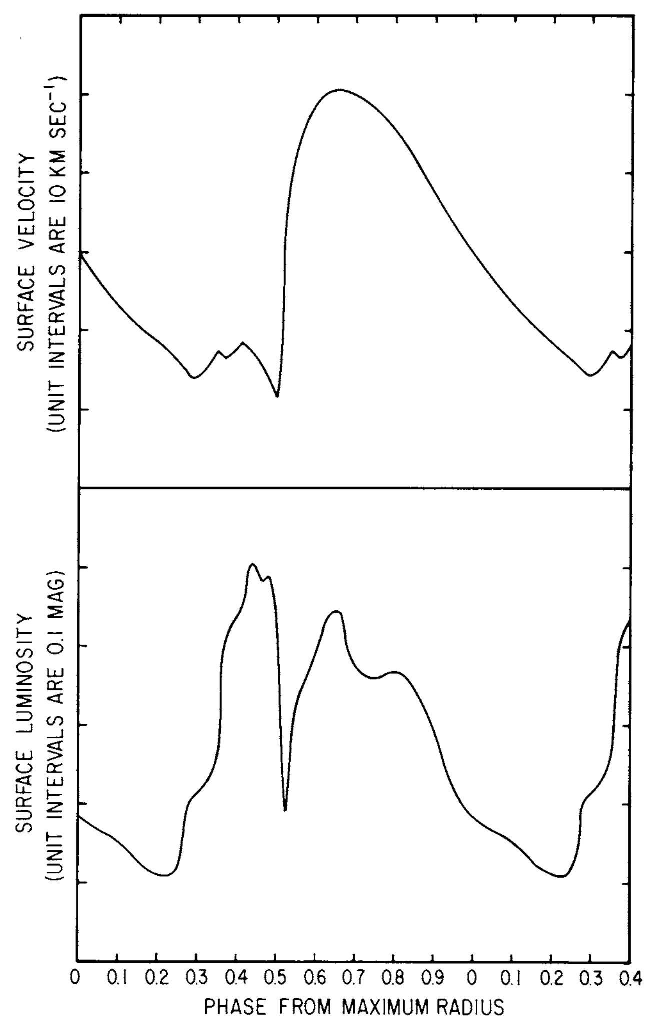


FIG. 2.—Surface velocity and surface luminosity curves for the second-overtone model.

the luminosity dip reached only halfway and the asymmetry of the light and velocity curves was also less. These differences demonstrate that the large number of mass zones in the more finely zoned model resolves the dip better and establishes the overall shape of the light curve more reliably. The 47 zones in this model, in fact, are comparable to the 38–49 zones used in the successful first-overtone and fundamental-mode models.

Since the most essential features of the theoretical first-overtone and fundamental-mode light curves vary relatively little from model to model, and since the range of possible input parameters for unstable second-overtone models is so restricted, we may regard the present second-overtone model as representative of its class.

## IV. COMPARISON WITH OBSERVATIONS

The periods of observed RR Lyrae stars in globular clusters display a short-period cutoff at 0.23 day (Sawyer Hogg 1973), which is consistent with the 0.219 day period of HX Ara, the fastest field RR Lyrae star having adequate observations (Duerbeck and Walter 1976). The light curves of these rapid type *c* variables, however, do not resemble the light curve of the present second-overtone model, but rather they look like typical first-overtone model light curves.

Furthermore, all known double-mode RR Lyrae stars in globular clusters and the field (Jerzykiewicz and Wenzel 1977; Andrews 1980; Sandage, Katem, and Sandage 1981; Goranskij 1981; Cox, Hodson, and Clancy 1983; Nemec 1985; Clement *et al.* 1986) have a period ratio around 0.745, which is characteristic of mixed first-overtone and fundamental-mode pulsation rather than of mixed second-overtone and first-overtone pulsation, which would produce a period ratio of about 0.800.

In the globular clusters NGC 4833, M68, and IC 4499, the type *c* variables with the shortest periods have been suspected by some authors of being second-overtone pulsators because their periods appear to be isolated from the others by a factor of about 0.8 (van Albada and Baker 1973; Demers and Wehlau 1977; Clement, Dickens, and Bingham 1979). To this list Wehlau (1986) now adds the fastest variables in NGC 1851. The period distributions for the variables in these four clusters are shown in Figure 3, for which the data sources are the following: NGC 4833 (Demers and Wehlau 1977), M68 (van Agt and Oosterhoff 1959; Andrews 1980), IC 4499 (Clement, Dickens, and Bingham 1979; Clement *et al.* 1986), and NGC 1851 (Wehlau *et al.* 1978, 1982). In the case of the double-mode variables, both components are shown (in black). There seems to be little doubt that an accident of small number statistics can account for the apparent isolation of the one or two variables with the shortest periods in NGC 4833, M68, and IC

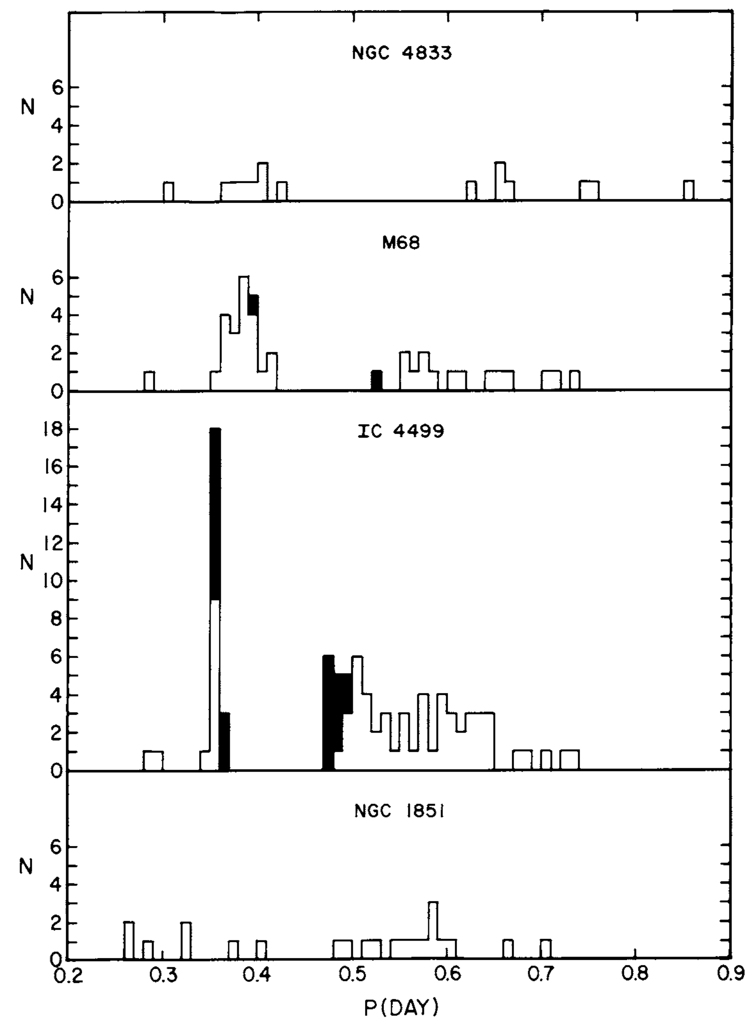


FIG. 3.—Period distribution of the RR Lyrae stars in four globular clusters. Both components of the double-mode variables are indicated (in black)

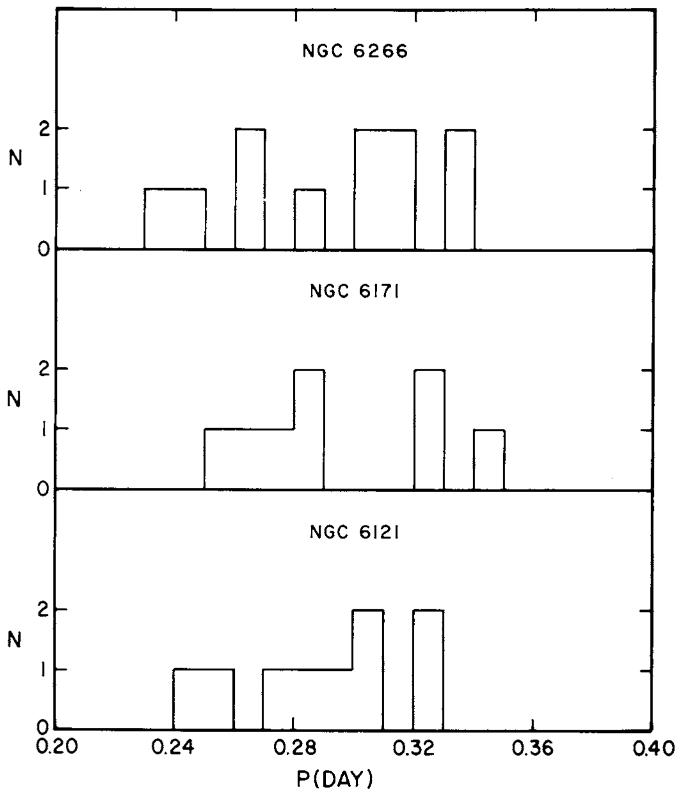


FIG. 4.—Period distribution of type *c* RR Lyrae stars in three globular clusters richest in very short-period type *c*'s.

4499. Justification for such a conclusion rests on the similar isolation of a few type *ab* variables with very long periods in the same clusters, as well as on the remarkably even period distribution of the type *c*'s in NGC 1851, despite the fact that some type *c*'s in this cluster have periods that are *shorter* than those in the other clusters. Periods of type *c* variables in three more clusters (NGC 6266, NGC 6171, and NGC 6121), which contain the largest number of *very* short-period variables in all clusters so far studied (Sawyer Hogg 1973), are shown in Figure 4, and again it can be seen that the distribution is nearly continuous in a statistical sense. Furthermore, the RR Lyrae-rich clusters M15, M3, and M5 show either no gap at all at very short periods or a gap that is too narrow to have any significance (see Fig. 5 of Clement, Dickens, and Bingham 1979).

Nor is it true that exceptionally low amplitude of some of the candidate stars is necessarily a guarantee of second-overtone pulsation. If the second overtone is actually stable in RR Lyrae stars, a first-overtone variable near the blue edge of the instability strip in the H–R diagram would show both a short period and a low amplitude. Cacciari and Renzini (1976) and Sandage (1981) have published observational plots of amplitude versus period that can most easily be interpreted by assuming that all the type *c*'s are first-overtone pulsators and that their amplitudes fall rapidly near the blue edge (Caloi 1979).

## V. CONCLUSION

From the results of the foregoing observational comparisons, it is reasonable to conclude that unstable second-overtone pulsators probably do not exist among RR Lyrae stars. This inference agrees with the predicted high stability of the second overtone against self-excited oscillations in stellar models that are constructed with the most likely values of mass and luminosity for an RR Lyrae star. If, however, an unstable second-overtone RR Lyrae star is ever found, the star would be expected to show a relatively low luminosity for its mass and to display a light curve that is deeply split around the time of light maximum.

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