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RR LYRAE STARS: A THEORETICAL STUDY OF BAILEY TYPE *c* VARIABLES

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ABSTRACT

New theoretical models of stars pulsating in the first overtone have been constructed to simulate RR Lyr variables of Bailey type *c*. Despite the use of different opacities, these new models agree very well with earlier models built by Christy and Stellingwerf. Quantitative comparisons using empirical light curves and velocity curves of metal-poor type *c* variables confirm the validity of the models. Masses of 0.55–0.65 $M_\odot$ and luminosities of 40–50 $L_\odot$, derived here for the type *c* variables, are consistent with previous results obtained for type *ab* variables. A Christy echo of the kind normally associated with fundamental-mode pulsators was detected in the interior velocity structure of one first-overtone model that happens to have a large velocity amplitude.

Subject headings: stars: pulsation — stars: RR Lyrae

I. INTRODUCTION

First-overtone pulsators are found in great number among the Bailey-type *c* RR Lyrae stars (Schwarzschild 1941). Except for the work of Christy (1966) and Stellingwerf (1975), however, no systematic theoretical investigation of nonlinear oscillations of these stars has been published. In a previous paper (Stothers 1981), nonlinear models for Bailey-type *ab* RR Lyr stars, which pulsate in the fundamental mode, were computed and found to give better agreement with actual observations than the original fundamental-mode models of Christy and Stellingwerf. Therefore, we have proceeded to calculate a set of first-overtone models with the same physical assumptions that were used for the successful fundamental-mode models.

The main departure from the older assumptions of Christy and Stellingwerf has consisted of a change of opacities, which previously were those produced at Los Alamos. The newer opacities belong to a set computed by Carson and have been tabulated by Carson, Stothers, and Vemury (1981). Recently, Carson *et al.* (1984) have discovered that a "bump" in these opacities in the region of ultimate ionization of the CNO elements is due to an error in the computation for highly ionized metals. On the other hand, the pulsations of Cepheid-like stars are influenced very little by the physical conditions in layers of such high temperature, where the pulsation amplitudes are very small. Furthermore, the opacity in the fully pulsating layers arises almost wholly from the ionization of hydrogen and helium. Therefore, our present and previous nonlinear models for RR Lyr stars (and Cepheids) are believed to be physically valid. The irrelevance of the metallic abundance was expressly shown for two models of very long-period RR Lyr stars (BL Her stars) in a previous paper (Carson, Stothers, and Vemury 1981).

In the present paper, nonlinear models of type *c* RR Lyr stars will be presented and used to infer the masses and luminosities of these short-period variables. Our results turn out to accord well with what has already been derived for type *ab* variables.

II. NONLINEAR MODELS

The helium and metal abundances used for the present models are $Y = 0.25$ and $Z = 0.005$. Other basic parameters include mass, $M/M_\odot = 0.578$ and 0.679; luminosity, $L = 1.50 \times 10^{35}$ and 1.95×10^{35} ergs s^{-1}; and effective temperature, $T_e = 6500$, 7100, and 7400 K. As before, convection has been ignored, since Deupree (1977) and Stellingwerf (1984) have shown that it will be unimportant for RR Lyr stars of such high effective temperature. Practical methods of achieving a nearly pure first-overtone pulsation in nonlinear models have been described by Christy (1964) and Hubickyj (1983).

Special notation in our paper includes: K.E., peak kinetic energy; Δ, full (not half) amplitude; *Asymmetry*, time spent on the descending branch of the surface velocity (or luminosity) curve divided by time spent on the ascending branch; $\phi_l^2 - \phi_l^1$, phase of the second peak (the true light maximum) minus phase of the first peak (the premaximum hump) on the surface light curve; $\delta\phi$(mean), phase of mean luminosity minus phase of mean velocity, during rising light at the surface; and $r_{\rm node}$, radius of the first-overtone's outer pulsation node. Smoothing of the surface light and velocity curves has been performed in the manner suggested by Carson, Stothers, and Vemury (1981). Our model with $M/M_\odot = 0.578$, $\log (L/L_\odot) = 1.585$, and $\log T_e = 3.869$ will be designated, for reference purposes, as the "standard" model. Note that the effective temperature of this model is *not* the same as that of the "standard" model for a type *ab* RR Lyr star (Stothers 1981).

Full-amplitude properties of seven first-overtone models are presented in Table 1. The corresponding luminosity and velocity curves at the surface appear in Figures 1 and 2. Just as Christy (1966) found for his models, the light curves of most of our first-overtone pulsators show a broad double peak and very little asymmetry, in contrast to fundamental-mode pulsators. The dip between the two peaks occurs at the time of minimum radius, when the gas in the visible part of the atmosphere becomes shock-heated and ionized, with a consequent large increase of the atmospheric opacity.

For our hotter models with $\log T_e > 3.84$, surface amplitudes lie within the ranges $\Delta R/R = 0.06$–0.09, $\Delta V = 40$–55 km s^{-1}, and $\Delta M_{\rm bol} = 0.5$–0.8. These are about half of the derived values for fundamental-mode pulsators, and agree well with Christy's (1966) and Stellingwerf's (1975) results. The relative amplitudes of the two light peaks change with basic model parameters as follows: if the luminosity-to-mass ratio is raised, the second peak becomes higher and broader; while if the effective temperature is increased, the first peak becomes lower,

TABLE 1

FULL-AMPLITUDE PROPERTIES OF THE THEORETICAL MODELS OF TYPE *c* RR LYRAE STARS

	MODEL						
PARAMETER	1	2	3	4	5	6	7
$M/M_{\odot}$	0.578	0.578	0.578	0.578	0.578	0.679	0.679
$\log(L/L_{\odot})$	1.585	1.585	1.585	1.700	1.700	1.585	1.700
$\log T_e$	3.813	3.851	3.869	3.851	3.869	3.813	3.851
$R/R_{\odot}$	4.93	4.14	3.81	4.73	4.35	4.93	4.73
P(day)	0.378	0.298	0.262	0.371	0.322	0.363	0.338
K.E. (10^{40} ergs)	0.197	0.092	0.061	0.093	0.053	0.276	0.149
$\Delta R/R$	0.14	0.07	0.07	0.08	0.06	0.10	0.09
V_{out} (km s^{-1})	37	28	25	25	22	30	30
V_{in} (km s^{-1})	−34	−22	−19	−22	−17	−29	−24
ΔV (km s^{-1})	71	50	44	47	39	59	54
L_{max} (10^{35} ergs s^{-1})	1.9	2.1	2.0	2.4	2.4	1.8	2.8
L_{min} (10^{35} ergs s^{-1})	1.0	1.0	1.1	1.5	1.5	1.1	1.3
ΔM_{bol}	0.71	0.81	0.61	0.51	0.50	0.53	0.83
Asymmetry (vel)	3.2	2.8	2.8	2.1	2.1	3.2	3.2
Asymmetry (lum)	4.3	1.7	1.9	1.4	2.0	1.2	3.2
$\phi_l^2 - \phi_l^1$	0.12	0.16	0.12	0.22	0.12	0.29	0.14
$\delta\phi$(mean)	0.00	−0.10	−0.12	−0.12	−0.14	+0.01	−0.10
r_{node}/R	0.86	0.82	0.81	0.82	0.82	0.83	0.82

with little change occurring in the second peak. Examination of Christy's light curves confirms that these trends do not depend on the opacities used. Although we have not tested the effect of changing the helium abundance, Christy's models show that an increase of Y elevates the first peak and brings it closer in phase to the second peak. Unrealistically large changes in Y, however, are needed to achieve an observable effect.

The phase lag between mean light and mean velocity behaves very much as it does for fundamental-mode pulsators, decreasing (in an algebraic sense) as the effective temperature or, to a lesser extent, the luminosity-to-mass ratio is raised. The lag attains zero when $T_e \approx 6500$ K, as Christy also found in most of his models with a similar helium abundance. Since the first-overtone period depends on mass and radius as

$$P_1 \approx 0.0165(R/R_{\odot})^{7/4}(M/M_{\odot})^{-3/4} \text{ days},$$

with $R = (L/4\pi\sigma T_e^4)^{1/2}$, the phase lag is clearly not a monotonic function of period.

Zonal velocity curves for our "standard" model and for a cooler model are shown in Figures 3 and 4. In the hotter model, the velocity reversal at the node is clearly evident, and sinusoidal motion appears above and below the node, except very close to the surface and very deep in the interior. On the other hand, the zonal velocity plot for the cooler model resembles in many ways the complex structure found for the fundamental mode (Stothers 1981, Fig. 3). Most noticeable is the Christy echo. Apparently, the echo phenomenon—whatever its cause—is strong enough in this model to alter the typical internal velocity structure found for a first-overtone pulsator. Careful examination of the zonal velocity plot shows a very gradual change of the reflection of the maximum in velocity above the node into a minimum below the node, in contrast to the abrupt phase shift seen in the more typical case. None of our other models shows this peculiarity. The explanation probably lies in the abnormally large velocity amplitude of this model, since the surface echo is known to be a threshold phenomenon that occurs when the velocity amplitude exceeds ~60 km s^{-1}, according to the results obtained for fundamental-mode pulsators (Vemury and Stothers 1978; Carson, Stothers, and Vemury 1981).

Since the echo is either absent or weak in a normal first-overtone pulsator, no diagnostic bump that might reveal the mass and mean molecular weight of the star is seen on the surface velocity curve. Nevertheless, a small, unrelated velocity perturbation does occur at, or shortly after, the time of maximum inward velocity, although its diagnostic potential seems to be very slight.

III. COMPARISON WITH TYPE *c* VARIABLES

Four type *c* RR Lyr stars have been observed in sufficient detail to determine the phase lag with some precision. These stars are listed in Table 2. All are metal-poor stars with a spectroscopic indicator $\Delta S \geq 5$ according to Kemper (1982). Although DH Peg and YZ Cap were previously assigned ΔS values in the range 0–3 (Preston 1959; Kinman 1961; Butler 1975), their space velocities are fairly large and both lie at high galactic latitude. We assume, therefore, that all four variables are field counterparts of the numerous variables found in globular clusters.

On compiling the data in Table 2, we have multiplied the measured radial velocity amplitudes by 24/17 to convert them to proper astrocentric velocity amplitudes. The visual light amplitudes, however, need no correction to change them into bolometric light amplitudes. Effective temperatures have been taken from Kemper (1982), who used Kurucz's (1979) model atmospheres. Kemper's measurement of $\log T_e = 3.869$ for T Sex agrees well with an older, independent determination by Preston and Paczyński (1964).

Since these four variables are rather homogeneous in all of their properties, average quantities will be formed for them: $\langle P \rangle = 0.291$ day, $\langle \Delta V \rangle = 45$ km s^{-1}, $\langle \Delta M_{vis} \rangle = 0.50$, $\langle \phi_l^2 - \phi_l^1 \rangle = 0.11$, $\langle \delta\phi(\text{mean}) \rangle = -0.15$, and $\langle \log T_e \rangle = 3.873$. In addition, it is known that DH Peg and RZ Cep display splitting and emission in their Hα lines near maximum light, at the time of the temporary luminosity dip (Garbuzov 1983). This supports the idea of a shock wave transiting the atmosphere during that interval of time. Two of our theoretical

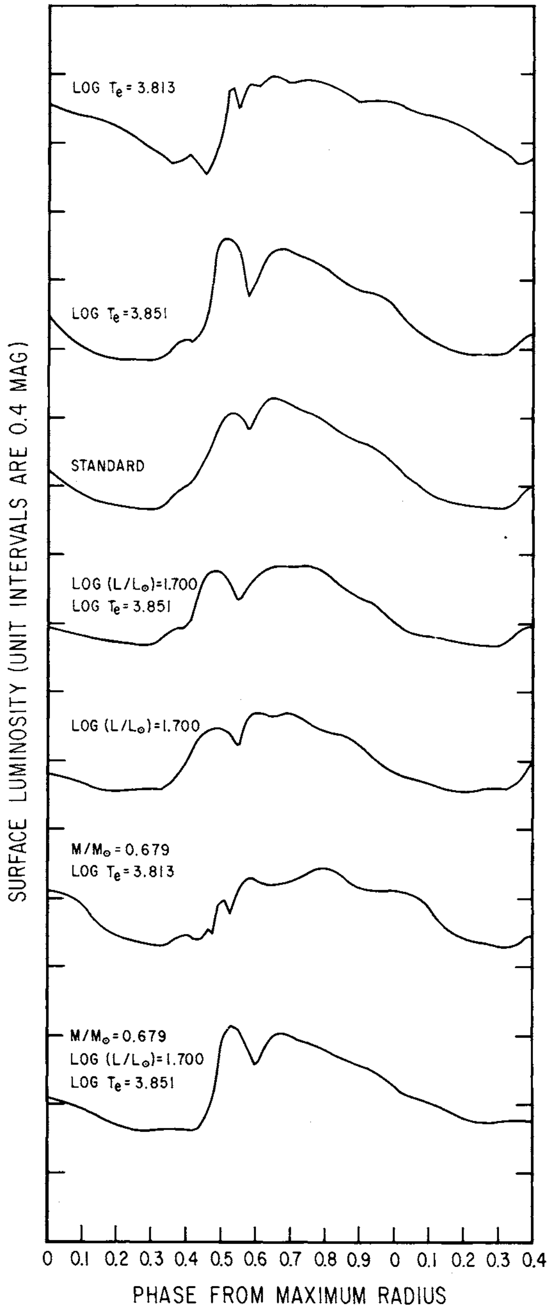

FIG. 1.—Surface luminosity curves for the seven type *c* RR Lyr models. "Standard" type *c* model refers to $M/M_\odot = 0.578$, log $(L/L_\odot) = 1.585$, and log $T_e = 3.869$; the other models have parameter values that are different as indicated.

models (numbered 3 and 5 in Table 1) show light curves that resemble those actually observed. In particular, both models display a premaximum hump that is comparable to, but does not exceed, the main light maximum. Their pulsational elements average $\langle P \rangle = 0.292$ day, $\langle \Delta V \rangle = 42$ km s^{-1}, $\langle \Delta M_{\rm bol} \rangle = 0.55$, $\langle \phi_l^2 - \phi_l^1 \rangle = 0.12$, and $\langle \delta\phi(\text{mean}) \rangle = -0.13$. Both models have masses of 0.578 $M_\odot$ and effective temperatures of log $T_e = 3.869$. Since the light curve of the fainter of the two models drops somewhat more steeply after light maximum, this model is preferred observationally. Its luminosity is log $(L/L_\odot) = 1.585$.

A possible ambiguity in inferring the correct mass arises from the fact that models with approximately the same luminosity-to-mass ratio, but with different masses, exhibit similar pulsational properties, apart from a small shift in period (compare, for example, models 2 and 7). Our selected sample of type *c* variables, therefore, can only properly be said to display $L/M \approx 70$ solar units, together with log $T_e \approx 3.87$.

A slightly smaller L/M ratio has been obtained from the analysis of a much cooler, slower type *c* variable, RU Psc. This star shows $P = 0.390$ day, $\Delta M_{\rm vis} = 0.47$, $\phi_l^2 - \phi_l^1 = 0.21$, and log $T_e = 3.827$ (Paczyński 1965; Kemper 1982). Like other slow, metal-poor type *c* variables, RU Psc has a highly sym-

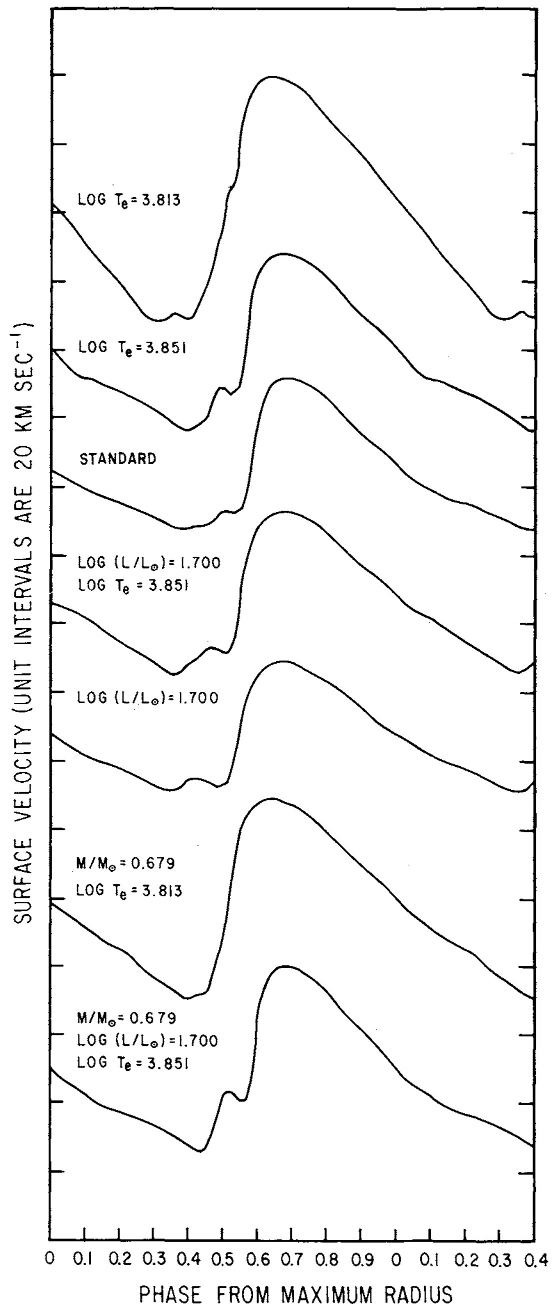

FIG. 2.—Surface velocity curves for the seven type *c* RR Lyr models. Same labeling as in Fig. 1.

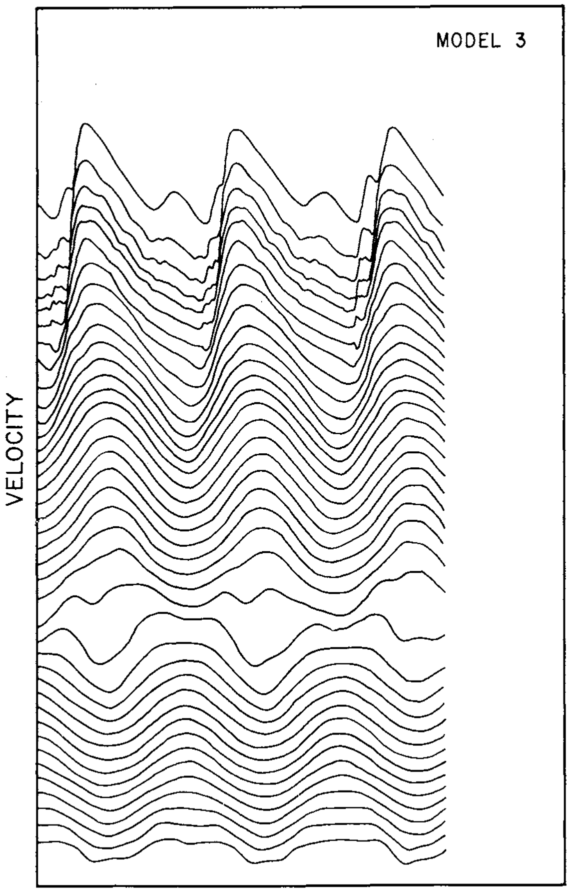

FIG. 3.—Velocity curves for the various zones in model 3, which is the "standard" type *c* RR Lyr model. Vertical scale is different for each zone.

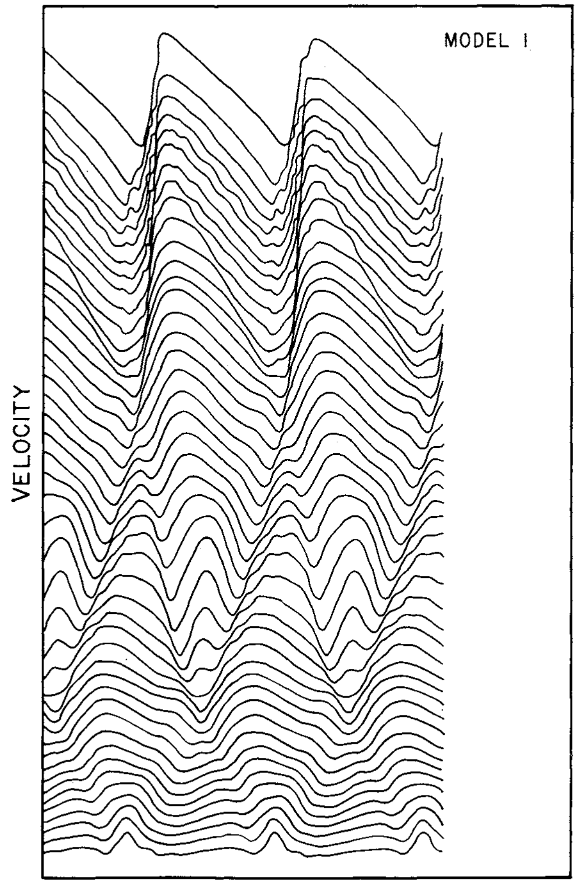

FIG. 4.—Velocity curves for the various zones in model 1, which is a large-amplitude type *c* RR Lyr model. Vertical scale is different for each zone.

metrical light curve (asymmetry of 1.2). Our models 1 and 6, both with log $(L/L_\odot) = 1.585$ and log $T_e = 3.813$, resemble RS Psc in some ways, but differ sufficiently from each other as to suggest that the squatter light curve of the more massive model provides a superior fit. This model's mass is 0.679 $M_\odot$. Consequently, $L/M \approx 60$ solar units.

Further comparison between our models and observed type *c* variables can be made by Fourier-analyzing the light curves. Amplitudes and phase angles of the lowest Fourier components were previously determined for the variable RZ Cep by Martin and Plummer (1917), for this star and 10 other type *c* field variables by Simon and Teays (1982), and for numerous type *c* variables in the globular cluster ω Cen by Payne-Gaposchkin (1947) and Petersen (1984). Although it is not certain what the individual Fourier components specifically measure in these stars, a formal comparison between Fourier

TABLE 2

OBSERVED PROPERTIES OF FOUR TYPE *c* RR LYRAE VARIABLES

PARAMETER	VARIABLE			
	DH Peg	YZ Cap	RZ Cep	T Sex
P(day)	0.256	0.273	0.309	0.325
(24/17) $\Delta V_{\rm rad}$ (km s^{-1})	40	60	40	40
$\Delta M_{\rm vis}$	0.49	0.52	0.54	0.45
Asymmetry (vel.)	2	3	2	2
Asymmetry (lum.)	1.6	1.9	1.6	1.6
$\phi_l^2 - \phi_l^1$	0.10	0.10	0.11	0.13
$\delta\phi$(mean)	−0.18	−0.14	−0.09	−0.19
log T_e	3.860	3.885	3.879	3.869
References	1, 4–6, 9, 10	3, 5, 9, 10	6–8, 10	2, 5, 9, 10

REFERENCES.—(1) Bonsack 1957; (2) Tifft and Smith 1958; (3) Kinman 1961; (4) Tifft 1964; (5) Preston and Paczyński 1964; (6) Paczyński 1965; (7) Epps and Sinclair 1973; (8) Cester and Todoran 1976; (9) Lub 1977; (10) Kemper 1982.

coefficients of the 11 observed field variables and our present models has been made by Simon (1985) and the agreement was found to be "very good."

IV. REANALYSIS OF TYPE *ab* VARIABLES

By using similar methods, four Bailey-type *ab* RR Lyr stars were previously compared with theoretical models pulsating in the fundamental mode (Stothers 1981). For these four stars, the phase ϕ_v of a small velocity bump, or inflection, that occurs shortly after the phase of maximum radius was estimated from rough published radial-velocity curves (ϕ_v is defined as the phase of the bump, or inflection, after *minimum* radius plus unity). More recently, an improved radial-velocity curve has been secured for RR Lyr by Benz and Stellingwerf (1985). The secondary bump appears at $\phi_v = 1.8$, not near $\phi_v = 1.6$ as was originally suggested by Sanford (1935) and the diagrams of Struve and Blaauw (1948). Since RR Lyr's measured radial-velocity amplitude is approximately the same in all three studies, ϕ_v cannot be presumed to have varied because of the velocity-modulating Blazhko effect; in fact, the large errors of observation in the two earlier studies can probably be blamed for the whole discrepancy.

The recent revision of ϕ_v is important because it eliminates one serious problem in the previous interpretation of RR Lyr, namely, the fact that the theoretical models predicted $\phi_l \approx \phi_v$ for the phase of the secondary bump on the light and velocity curves, whereas the observed value of ϕ_v seemed to be smaller than the observed value of ϕ_l by 0.2 (Hardie 1955). With the revised value of $\phi_v = 1.8$, the bump mass inferred for RR Lyr is now, uniquely, $\sim 0.65\ M_\odot$. Unfortunately, the pulsation amplitude and asymmetry predicted for theoretical models of this mass (Stothers 1981; Hubickyj 1983) are too large. It is likely that flaws such as these have contributed to the anomalous Fourier phase angles computed by Simon (1985) for all of our models of type *ab* variables. The error must be subtle because similar anomalies have been found in the type *ab* models constructed by other workers (Simon and Aikawa 1986).

Since photometric observations show that $\phi_l = 1.7$–1.8 for the phase of the light-curve bump in many type *ab* variables, a redetermination of the radial-velocity curves for additional RR Lyr stars would be very valuable. Recently, Burki and Meylan (1986) have determined that $\phi_l \approx \phi_v \approx 1.8$ for RR Cet, a star very similar to RR Lyr.

V. CONCLUSION

The present nonlinear models of first-overtone pulsators, constructed with Carson's opacities, closely resemble the analogous models built earlier by Christy (1966) and Stellingwerf (1975) with Los Alamos opacities. Both the new and old models account very well in a qualitative way for the observed properties of metal-poor type *c* RR Lyr stars.

Quantitative comparisons with individual type *c* variables have been made in this paper, and the results suggest masses of 0.55–0.65 $M_\odot$ and luminosities of 40–50 $L_\odot$. These ranges are consistent with previously published results for type *ab* variables (Stothers 1981). A reanalysis of RR Lyr itself (a type *a* variable) has been made from Benz and Stellingwerf's (1985) new radial-velocity measurements, which indicate a mass close to $\sim 0.65\ M_\odot$. A physical range of masses for RR Lyr stars doubtless exists, although the actual spread in luminosities around the independently observed mean value of 45 $L_\odot$ is probably very small (Butler, Dickens, and Epps 1978; Stothers 1983). On the other hand, we hesitate to put a great deal of emphasis on the fidelity of our type *c* models to observations because several flaws are known to exist in the type *ab* models based on the same input physics.

A purely theoretical result of possibly more than incidental value is the discovery of a Christy echo in our one first-overtone model that has a large velocity amplitude. This finding offers a challenge to any interpretation of the echo as a modal resonance phenomenon.

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