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VERY SLOW CLASSICAL CEPHEIDS: THEORETICAL MODELS WITH PERIODS LONGER THAN 50 DAYS

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ABSTRACT

Systematics of the light curves of classical Cepheids with the longest known periods have been investigated with the help of full-amplitude models of pulsating stellar envelopes. For periods exceeding ~60 days, flat-topped light curves of the S Vul type are found to replace the smooth, asymmetric light curves characteristic of the slightly faster Cepheids. Predicted light and velocity amplitudes (although not the predicted radius amplitudes) agree well with observations. Variables with fluctuating light minima are observed to lie well off the mean period-luminosity relation, as are a few other (more stable?) variables with similarly long periods. The explanation for the long periods is probably low effective temperature rather than a low stellar mass. Because of the abnormal slowness of the classical Cepheids with periods longer than ~100 days, it is recommended that these variables not be used to calibrate the mean period-luminosity relation. Analogies between the slow classical Cepheids and the slow Population II Cepheids are drawn.

Subject headings: stars: Cepheids — stars: interiors — stars: pulsation

I. INTRODUCTION

Classical Cepheids with periods longer than 50 days are very rare in our Galaxy and in other spiral galaxies, although they are found somewhat more frequently in irregular galaxies. Their slowness and rarity probably explain why they have not formed the subject of more observational and theoretical studies. Two papers presenting full-amplitude models of these variables have been published so far: one by King *et al.* (1973), who assumed for the stellar models a normal evolutionary mass-luminosity relation, and the other by Christy (1975*a*, *b*), who adopted masses that were ~60% smaller. Both sets of authors adopted Los Alamos opacities. The theoretically calculated bolometric light amplitudes for the fundamental mode (the only unstable mode at such long periods) were 0.7–2.3 mag in the period range 57–148 days. These values disagree strongly with the observed amplitudes, which do not exceed 0.8 mag (bolometric).

It is now generally recognized that the Los Alamos opacities do not lead to models for classical Cepheids (of any period) that agree well with observations unless Cepheid masses of ~0.5 the expected "evolutionary" masses are used (e.g., Simon and Davis 1983). The Carson opacities, on the other hand, seem to produce more nearly (although certainly not perfectly) successful models at least in the period range 5–50 days (Vemury and Stothers 1978; Carson and Stothers 1984). Therefore, it is worth investigating whether the longest period classical Cepheids can also be modeled better with these alternative opacities.

II. MODEL ASSUMPTIONS

Methods and assumptions used in the present investigation follow the procedures of our earlier work. Specifically, our models obey a normal evolutionary mass-luminosity relation applicable to core helium-burning stars, given approximately by

$$\log (L/L_\odot) = 0.66 + 3.50 \log (M/M_\odot) .$$

This assumes an envelope chemical composition of $(X, Y, Z) =$ (0.739, 0.240, 0.021) but would be different for other compositions and for other structural and evolutionary assumptions (Carson and Stothers 1984). Radiative equilibrium is taken to prevail throughout the pulsating layers of the envelope, convection being entirely ignored.

Opacities are the same as those published in tabular form by Carson and Stothers (1984). The unusually high luminosities in the present models lead to such low envelope densities that the low-density opacity "bump," which happens to coincide with the ultimate ionization of the CNO elements at temperatures of $\sim 10^6$ K but is now thought to be unphysically large, lies well above the opacity background (Carson 1976). This causes the local luminosity limit (Eddington 1921) to fall below the star's actual luminosity, and prevents a dynamically stable envelope structure in purely radiative equilibrium from being calculated. Rather than introduce the complication of convection into deep envelope layers that can have no large effect on the pulsational properties of the models, we have modified the opacity table slightly so as to reduce the size of the "bump." Specifically, we substituted the following ($\log T$, $\log \rho$, $\log \kappa$) values: (5.2, −9, −0.40), (5.4, −9, −0.45), (5.4, −8, −0.35), (5.6, −7, −0.20), (5.6, −6, 0.00), (5.8, −7, −0.30), (5.8, −6, −0.20), (6.0, −7, −0.40), and (6.0, −6, −0.30). Justification for this simplifying procedure is that the contribution of *all* the metals to the opacity is known to influence the light and velocity curves of Cepheid models very little (Fricke, Stobie, and Strittmatter 1971).

Effective temperatures of very bright Cepheids are somewhat uncertain (Pel 1980; van Genderen 1983*b*), and we have simply adopted $\log T_e = 3.69$–3.75. Thus, these stars seem to be considerably cooler than the theoretical blue edge of the instability strip, which for the Carson opacities lies at $\log T_e \approx 3.87$.

III. RESULTS

The properties of five models of very slow classical Cepheids are summarized in Table 1. The periods of these models range

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TABLE 1

FULL-AMPLITUDE PROPERTIES OF THE MODELS OF VERY SLOW CLASSICAL CEPHEIDS

Parameter	Model 1	2	3	4	5
$M/M_\odot$	13	13	13	15	20
log $(L/L_\odot)$	4.6	4.6	4.6	4.8	5.1
log T_e	3.69	3.72	3.75	3.72	3.72
$R/R_\odot$	282	246	214	309	437
P (days)	83	64	50	96	157
K.E. (10^{42} ergs)	23.0	39.0	51.2	33.5	20.8
$\Delta R/R_\odot$	0.15	0.22	0.28	0.20	0.11
V_{out} (km s^{-1})	15	21	34	13	8
V_{in} (km s^{-1})	−22:[a]	−27	−34	−21	−8
ΔV (km s^{-1})	37:[a]	48	68	34	16
L_{max} (10^{37} ergs s^{-1})	19.6	18.6	21.5	29.5	55.2
L_{min} (10^{37} ergs s^{-1})	11.3:[a]	11.6	8.6	20.1	43.9
ΔM_{bol}	0.6:[a]	0.5	1.0	0.4	0.2
Asymmetry (vel.)	4.9	3.8	4.6	1.1	0.5
Asymmetry (lum.)	1.9	1.6	2.0	1.0	0.5

[a] Variable light and velocity minima.

from 50 to 157 days and, except at the longest period, adhere well to the relation

$$P = 0.014(R/R_\odot)^2(M/M_\odot)^{-1}$$

derived previously for 13–50 day models (Carson and Stothers 1984). Amplitudes of the models decline as the equilibrium luminosity increases or as the equilibrium effective temperature decreases. This result supports the trends already evident in our previously published models having log $(L/L_\odot) > 4.15$ and log $T_e < 3.73$.

The relatively low bolometric light amplitudes of the four slowest models are corroborated by actual observations (Fig. 1). Table 2 lists all known very slow classical Cepheids in different galactic systems. Although estimates of effective temperature, and hence of bolometric correction, are unavailable for Cepheids outside our Galaxy and the Magellanic Clouds, the periods, blue light curves, and visual light amplitudes of more distant Cepheids resemble so closely those of the nearer objects that their bolometric light amplitudes are doubtless also comparable in size. The theoretical radius amplitudes, however, appear to be only about half as large as the photometric radius amplitudes measured by van Genderen (1983*b*). To resolve this discrepancy between theory and observation, radial velocity measurements of the variables should be made.

Light curves and velocity curves of our new models fall into two categories, defined by symmetrical and asymmetrical shapes, as exemplified in Figure 2. In plotting the light curves, however, a considerable amount of smoothing has been performed to rid them of nonphysical "bumps" that were caused by the combined effects of coarse mass zoning and low pulsation amplitude (Keller and Mutschlecner 1971).

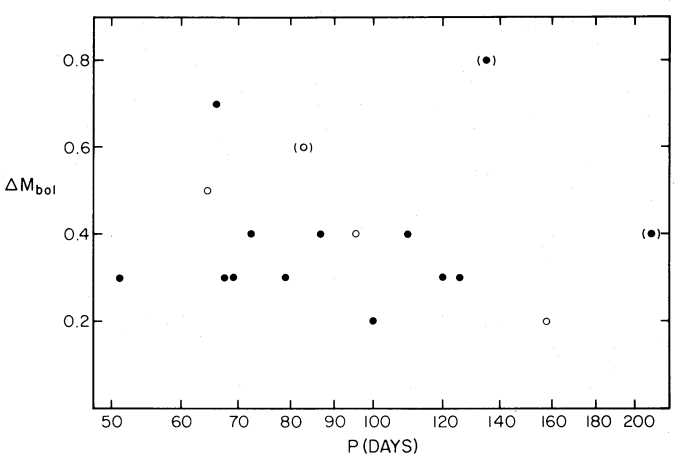

FIG. 1

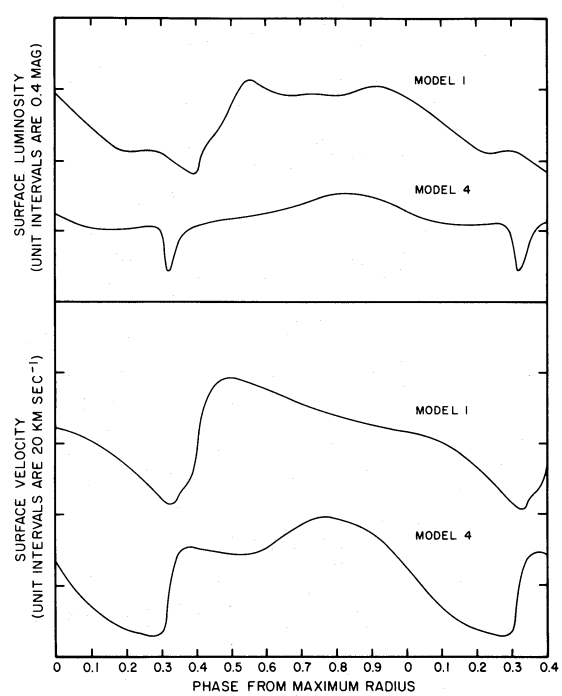

FIG. 2

FIG. 1.—Luminosity amplitude vs. period for the coolest theoretical models (*open circles*) and the observed classical Cepheids (*filled circles*) with $P > 50$ days. Parentheses indicate stars with fluctuating luminosity minima.

FIG. 2.—Surface luminosity and surface velocity curves for two typical models of very slow classical Cepheids.

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TABLE 2

VERY SLOW CLASSICAL CEPHEIDS

Galaxy	Variable	P (days)	ΔM_V	ΔM_{bol}	$\langle M_V \rangle$	References
Milky Way	GY Sge	51	0.8	0.3	−6.34	1
	S Vul	67	0.6	0.3	−6.89	2,3,4
LMC	HV 2827	79	0.6	0.3	−7.01	5
	HV 5497	99	0.5	0.2	−7.25	5,6,7,8,9
	HV 2883	110	1.2	0.4	−6.58	5,6,8
	HV 2447	119	0.6	0.3	−7.39	5,7,8,10
	HV 883	134	1.3:[a]	0.8:[a]	−7.35	5,7,8,10,11
SMC	HV 824	66	1.0	0.7	−6.77	7,10,12
	HV 11157	69	0.4	0.3	−6.47:	7,8,13
	HV 834	73	0.7	0.4	−6.86	5,7,8,10,14
	HV 829	87	0.7	0.4	−7.13	5,7,8,10,12
	HV 821	127	0.7	0.3	−6.93	7,8,10,12
	HV 1956	210	0.7:[a]	0.4:[a]	−7.41:	7,13,14
NGC 6822	V7	65	0.5			15
	V13	91	0.8			15
M31	H 42	177	1.1:[a,b]			16
M33	V19	55	0.8[b]			17,18
	V10	69	0.6[b]			17,18
IC 1613	V22	146	1.6:[a,b]			19
NGC 2403	V33	56	0.5			20
	V46	58	0.8[b]			20
	V19	81	0.7			20
	V3	87	0.7			20

[a] Variable light minima.

[b] Based on $\Delta M_V = 0.6\Delta M_B$.

REFERENCES.—(1) Forbes 1982. (2) Turner 1980. (3) Fernie 1970. (4) Joy 1952. (5) van Genderen 1983*a*. (6) van Genderen 1983*b*. (7) Eggen 1977. (8) Madore 1975. (9) Woolley *et al.* 1962. (10) Gascoigne and Kron 1965. (11) Butler 1978. (12) van Genderen 1977. (13) van Genderen 1983*c*. (14) Butler 1976. (15) Kayser 1967. (16) Baade and Swope 1965. (17) Hubble 1926. (18) Sandage 1983. (19) Sandage 1971. (20) Tammann and Sandage 1968.

Bumpiness of the type encountered here has been illustrated and discussed, for example, by Davis, Moffett, and Barnes (1981).

In general, the asymmetry of the light curves is found to decrease with advancing period. For periods longer than ~60 days, flat-topped light maxima are also encountered. These features correspond to what is actually observed, except in the case of four variables (GY Sge, HV 2883, HV 834, and HV 821) where the maxima display no perceptible flattening at all. Judging from some of our faster models published earlier (Carson and Stothers 1984, Fig. 8), we suspect that these four variables have slightly brighter luminosities or slightly lower masses than the values assumed here.

A well-observed Cepheid of very long period in our Galaxy that can be regarded as an archetype of the group is S Vul. This variable has a period of 67 days, a bolometric light amplitude of 0.3 mag, and a flat-topped light maximum covering 0.3 of a cycle (Fernie 1970). Its radial velocity amplitude (Joy 1952), multiplied by 24/17 to correct for geometric projection and limb darkening (Getting 1934), is 34 km s^{-1}. Our model 1, shown in Figure 2, represents the class of slow variables typified by S Vul quite satisfactorily.

All four Cepheids known to have periods longer than ~130 days show some variability of their light minima from cycle to cycle. Van Genderen (1983*a*) explicitly pointed out this curious behavior in the case of HV 883, but it can be seen in the published light curves of the three other variables as well. The amplitude of minimum-light variability increases with period and seems to be a bolometric, not a color, effect. Our coolest model (the one with log $T_e = 3.69$) shows the same behavior. While the model maxima remain nearly constant from cycle to cycle, the minima fluctuate by up to 0.2 mag; this fluctuation has the character of a slow wave covering several pulsation cycles and does not represent a regular alternation of shallow and deep minima. It is not yet known observationally how rapidly the minima fluctuate in actual Cepheids.

Luminosities of all the very slow Cepheids with well-determined distances are listed in Table 2, where van Genderen's (1983*b*, *c*) luminosities for Cepheids in the Magellanic Clouds have been adjusted to conform with the most recent redetermination of the Clouds' mean distances (Stothers 1983). These luminosities are plotted against period in Figure 3; the straight-line relation represents an extrapolation of the mean period-luminosity relation for galactic Cepheids that have very recently been calibrated by four independent methods (Stothers 1983). Up to a period of 100 days, very slow Cepheids cling rather closely to this extrapolated relation, although at longer periods most fall below the mean line and exhibit a lot of scatter. While it is possible that the adopted extinction corrections, and hence derived luminosities, of these discrepant variables are too small, this explanation is not likely to be correct (Madore 1982; McGonegal *et al.* 1982). A more realistic interpretation is that the effective temperatures are low enough to produce the long periods that are observed (since $P \propto LT_e^{-4}M^{-1}$). In our view of our theoret-

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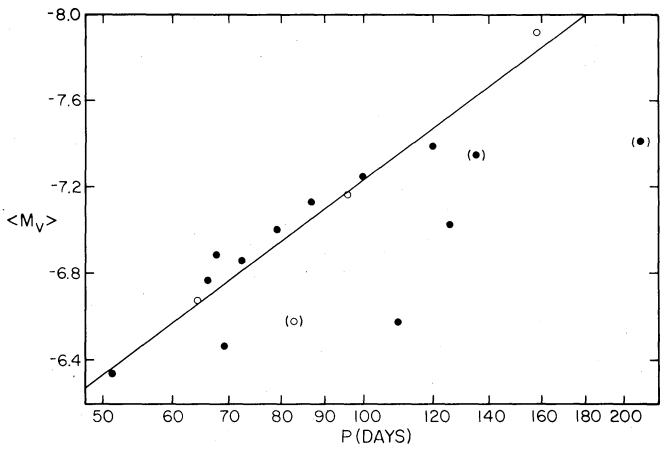

FIG. 3.—Period-luminosity relation for the coolest theoretical models (*open circles*) and the observed classical Cepheids (*filled circles*) with $P > 50$ days. Parentheses indicate stars with fluctuating luminosity minima. The straight line represents an extrapolation of the mean period-luminosity relation for galactic Cepheids of 3–67 day period.

ical results, it is perhaps significant that two of the four discrepant variables (HV 883 and HV 1956) show fluctuating light minima; the two other variables (HV 2883 and HV 821) deserve more extensive observation. In all cases, an effort should be made to obtain improved effective temperatures for the variables. Van Genderen's (1983*b*) preliminary effective temperatures are only suggestive of a temperature effect. Certainly, mass loss of the amount required to explain the long periods (up to 50% of the original mass) seems to be an unrealistic assumption for these stars of only $\sim 15\ M_\odot$ (Stothers and Chin 1979).

The four slowest theoretical models discussed above are also plotted in Figure 3 (bolometric corrections have been adopted from Flower 1977). Formal agreement with the empirical mean period-luminosity relation is perhaps not very significant, however, not only on account of the limited scope for comparison afforded by the somewhat featureless light curves of these slow variables, but also in view of the known brightness excess (~ 0.5 mag) of the best-fitting theoretical models for $P < 40$ days (Stothers 1983). Nevertheless, we may infer from the observed luminosities that none of the masses of the known variables exceeds $\sim 17\ M_\odot$, while probably all of these stars are burning core helium, contrary to an earlier conjecture by Becker, Iben, and Tuggle (1977).

IV. CONCLUSION

Most of the very slow classical Cepheids display low bolometric light amplitudes, small light asymmetries, and some degree of flattening of the maxima of their light curves. These characteristic features are satisfactorily reproduced by our full-amplitude models constructed with a normal evolutionary mass-luminosity relation and with the Carson radiative opacities. Theoretical velocity curves are also consistent with what is known for S Vul, the archetype of this class and the only member with a measured radial velocity curve. Radius amplitudes of the models, however, fall systematically below van Genderen's photometrically determined values.

Theory and observation concur that the transition from smooth, asymmetric light curves (typical of classical Cepheids of somewhat shorter period) into the flat-topped variety occurs at a period of ~ 60 days. (This transition is the counterpart of the switch from κ Pav–type light curves to W Vir–type light curves among Population II Cepheids.) Many of the classical Cepheids whose periods exceed ~ 100 days exhibit light minima that fluctuate from cycle to cycle (somewhat like RV Tau behavior among the slowest Population II Cepheids). Our models suggest that this fluctuation is produced by a relatively low mean effective temperature, which would also make the period excessively long at a given luminosity, as observed. We therefore recommend that classical Cepheids with periods greater than ~ 100 days not be used to calibrate the mean period-luminosity relation.

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