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THE 3 K BLACKBODY RADIATION, DIRAC'S LARGE NUMBERS HYPOTHESIS, AND SCALE-COVARIANT COSMOLOGY

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ABSTRACT

In the present paper we present the physical basis for a set of new cosmological ideas as well as the derivation of the relevant dynamic equations. The 3 K blackbody radiation, so far judged irreconcilable with Dirac's ideas, is shown to be not only consistent with the Large Numbers Hypothesis, but actually indispensable in predicting the scale factor $R(t)$ and the curvature k. The results indicate an open universe with $k = 0$ without having to resort to an m versus z relation, or to any other of the classical cosmological tests.

Subject headings: cosmic background radiation — cosmology

I. INTRODUCTION

Forty years ago, Dirac (1937) first pointed out that the existence of several large dimensionless numbers, constructed using e, m, $\hbar$, G, H_0, etc., is very difficult to understand if one views them as built-in features in any physical theory, independently of whether it is at the atomic or cosmological level. A more logical scenario can be presented if one accepts the Dirac suggestion (1973) that such numbers are large simply because we measure them today when the Universe is 20 billion years old. In the past they would have been much smaller: their value is a function of the age of the Universe. Since a variety of such numbers can be constructed involving all the known types of interactions (Canuto and Lodenquai 1977), Dirac's suggestion implies, among other things, that the gravitational constant and perhaps the weak-interaction coupling constant should vary with time, the age of the Universe serving as a time scale.

It is very remarkable that 40 years after Dirac's suggestion, a very similar, though entirely independent, set of ideas is discussed within the context of gauge fields and broken symmetries (Weinberg 1977*a*, *b*; Linde 1974, 1976; Dreitlein 1974; Canuto and Lee 1977). Dirac discussed only gravity, and gauge field people discuss only weak and electromagnetic interactions: however, the similarity of ideas is, in our opinion, impressive.

The Weinberg-Salam theory (see, e.g., Weinberg 1977*a*), the first successful scheme that unifies electromagnetism and weak interactions, implies that the diversity of the two types of interactions is not a built-in *a priori* feature, but rather a consequence of our observing and performing measurements today instead of a cosmological yesterday. The basic idea of such a change with time and therefore the necessity of considering cosmology intertwined with microphysics is back again. The Weinberg-Salam model does not include gravity; but if electromagnetism and weak interactions were at one time of equal strength and if we believe in the existence of a unified theory, then we are forced to admit and conclude that gravity itself must have been of comparable strength way back in the past. The success of the broken symmetry approach therefore forces upon us a wider scenario concerning all types of forces by proposing the equality of all of them way back in the past, the observed diversity of today being due to the aging and expanding of the universe.

Dirac's original paper contained *in nuce* a suggestion very much along these modern lines of thought. Because of the primitive understanding of weak interactions back in 1937, Dirac confined himself to gravity, and the focus of his and most of the subsequent papers was therefore centered on the possibility of G varying, a theme that over the years has been frequently discussed. However, we feel that most of the discussions have been misleading at best. In fact, the Einstein equations were constructed so as to ensure energy conservation by having G strictly constant. No variation, on whatever time scale, is allowed. This is clear if one remembers that the Einstein tensor has zero divergence; and since $T^{\mu\nu}{}_{;\nu}$ is also zero by energy conservation, it then follows that G must be strictly a constant. Any other equation, derived from the Einstein equation, carries the same built-in property, and the common practice of just letting G vary in equations so derived is clearly incorrect, as are the deductions from such a procedure.

All this shows that variations of G can be meaningfully investigated only if one has gravitational equations that allow such a variation. Meaningful cosmological considerations based upon the variation of G are also very difficult, if not impossible, to carry out without dynamic equations. The use of the Large Numbers Hypothesis alone to do cosmology is an unconvincing and dubious procedure with no assurance that the final answer is either unique or

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the most general. Dirac's original ideas have not yet been forged into a full theory, and the use of them alone to construct the scale factor $R(t)$ for cosmological models is much like trying to guess the exact wavefunction of the hydrogen atom from bits and pieces of asymptotic behaviors instead of using Schrödinger's equation.

To search for possible generalizations of the asymptotic behavior is certainly not the way to ameliorate the situation: it is in our opinion a maladroit endeavor (Roxburgh 1976 and reply in Canuto *et al.* 1977*a*; Roxburgh 1977) inherently unable to provide the much needed generalization and deeper understanding.

Several questions come to mind when dealing with the large numbers alone.

1. How many of them is one entitled to use? In principle, we would like to use only a few of them as constraints on a set of gravitational equations, which in turn would predict the cosmological models. The use of all of them at different stages leaves the impression of a patchwork.

2. Even allowing the use of all of them as observational input, one's effort is still limited by the fact that they all refer to the present and no conclusion can be legitimately extrapolated either into the past or into the future, unless one has a dynamical set of gravitational equations upon which the large numbers can be imposed as boundary conditions.

In this paper, we shall present the results of the program intended (1) to derive a generalized system of gravitational equations that allow (but do not require) G to vary, (2) to use the 3 K blackbody radiation to fix the relation between G and the gauge function $\beta(t)$, (3) to use the Large Numbers Hypothesis to derive the geometry of the universe.

II. GRAVITATIONAL EQUATIONS

The major problem encountered when trying to allow for possible variations of G is that Einstein's equations (with constant G) have been very successful in describing gravitation and any attempt to abandon them must be viewed as very risky.

Einstein's equations were derived as a geometrical generalization of Newton's equations, and their goal is that of describing gravitation alone. No successful attempt has yet been made which incorporates atomic and gravitational phenomena. Atomic phenomena were successfully accounted for when Bohr and Schrödinger devised the necessary machinery to give us a complete theory. In order to do atomic physics one need not know gravitation, and vice versa. When one studies an atom, the spacetime structure (say Minkowski space) is given, and the atom is then projected in such a space. No distortion of the spacetime structure due to the presence of the atom is ever considered. The theory is complete in the restricted sense that, once the spacetime structure is given, the theory itself provides the system of units to be employed. The clocks are not postulated from the outside: they are determined from within the theory itself. The introduction of the Planck constant was the ingredient needed for the theory to provide a complete system of atomic units: mass (m), time (h/mc^2), length (h/mc). Analogously, when gravity was first successfully described by Newton, a new constant appeared, G, which then allowed the introduction of a complete system of gravitational units, mass (M), time $(G\rho)^{-1/2}$, length $(c^2/G\rho)^{1/2}$, where M and R ($\rho = M/R^3$) refer to astronomical masses and radii. As we said before, we have not yet been able to construct a theory capable of unifying atomic and gravitational phenomena. If such a theory existed, it would bring with it a new system of units, which *a priori* need not be either atomic nor gravitational. In the absence of such a complete theory, it is legitimate to assume that either (a) the two systems of units have been a *constant* multiple of each other throughout the entire history of the Universe or else (b) their relation is a function of the age of the Universe. Case b is what we shall investigate in this paper. In order to do so, we must construct a theory that contains one more degree of freedom, a gauge function $\beta(x)$, which is the result of our professed ignorance of the proper coupling between atomic and gravitational physics. If the coupling is stipulated *a priori*, as in case a, one no longer has a gauge freedom. In fact, β must be a constant. Once this choice is made, there is no possibility for G to vary, *an issue which ought to be resolved by accurate experimental verification, rather than by an a priori conjecture.*

The two systems of units, atomic and gravitational, are assumed to be connected by the function $\beta(x)$. When $\beta = \text{const.}$, the gravitational equations must reduce to the ordinary Einstein equations, whereas when atomic units are used, then $\beta(x)$ is not constant. This in turn implies that G can at the same time be constant and variable, depending on the units employed. This is perfectly understandable, since G has dimensions and its being constant or not has meaning only if we specify the units we are referring to.

From the point of view of, say, a galaxy, whose dynamics is solely governed by gravity, there is no need to go to atomic units: Einstein's equations are perfectly valid as they are, and G is a true constant. However, when making observations of gravitational phenomena using atomic apparatus (as we ordinarily do), we must employ a system of gravitational equations in which $\beta(x)$ is not constant and G can in principle vary: the function $\beta(x)$ is the passport to go from one system to another. In this theory, contrary to others that entirely modify their structure, we retain Einstein's equations in their total integrity, except that we make the further specification that they are valid only when gravitational units are employed.

The extension to any other system of units is easily obtained by a conformal transformation of the form $\bar{g}_{\mu\nu} \rightarrow \beta^2(x)g_{\mu\nu}$, where, from now on, barred quantities refer to Einstein units. One then obtains the following generalized gravitational equations (Canuto, Hsieh, and Adams 1977; Canuto *et al.* 1977*b*):

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R + f_{\mu\nu}(\beta) = -8\pi G(\beta)T_{\mu\nu}(\beta)\,, \tag{2.1}$$

where, with $\beta_\mu \equiv \beta_{;\mu}$,

$$\beta^2 f_{\mu\nu}(\beta) = 2\beta\beta_{\mu;\nu} - 4\beta_\mu\beta_\nu - g_{\mu\nu}(\beta\beta^{\lambda}{}_{;\lambda} - \beta^\lambda\beta_\lambda) . \tag{2.2}$$

Equation (2.1) is the scale-invariant form of Einstein's equations, if we assume that the product $GT_{\mu\nu}$ remains unchanged under scale transformations, i.e., if $GT_{\mu\nu} = \bar{G}\bar{T}_{\mu\nu}$. The left-hand side of (2.2) is scale invariant by construction. It is interesting to note that (2.2) coincides with the so-called improved energy momentum tensor for scalar particles (Callan, Coleman, and Jackiw 1970), even though it was here derived in a purely geometrical form and is not intended to represent scalar particles.

From equation (2.1) we can derive the energy conservation law

$$d\rho/dt + (\rho + p)u^\mu{}_{;\mu} = -\rho\frac{dG\beta/dt}{G\beta} - 3p\frac{d\beta/dt}{\beta} , \tag{2.3}$$

which, for an equation of state of the form

$$p = c_s{}^2\rho , \tag{2.4}$$

can be integrated to yield

$$\rho(R) \sim R^{-3(1+c_s{}^2)}[G(\beta)\beta^{1+3c_s{}^2}]^{-1} . \tag{2.5}$$

These equations must still be complemented by the baryon number conservation law, which in Einstein units reads

$$(\bar{\mathfrak{M}}\bar{u}_\mu)_{;\mu} = 0 , \qquad \bar{\mathfrak{M}} \equiv \bar{m}\bar{n} . \tag{2.6}$$

As explained in a previous paper (Canuto *et al.* 1977*b*), the generalization of (2.6) to arbitrary units is

$$(nu^\mu)_{;\mu} - [\pi(G) - 1]nu^\mu(\beta_\mu/\beta) = 0 . \tag{2.7}$$

Within a homogeneous comoving volume, having total mass M, equation (2.7) can be integrated to yield

$$GM\beta = \text{const} . \tag{2.8}$$

Finally, we shall need the cosmological equations derivable from (2.1) and (2.2) upon using a Robertson-Walker metric. The result is

$$\frac{(dR\beta/dt)^2}{(R\beta)^2} + \frac{k}{R^2} = \frac{8\pi}{3}G(\beta)\rho(\beta) , \tag{2.9a}$$

$$\frac{d^2R/dt^2}{R} + \frac{d\beta/dt}{\beta}\frac{dR/dt}{R} + d\left(\frac{d\beta/dt}{\beta}\right)\Big/dt = -\frac{4\pi}{3}G(\beta)[3p(\beta) + p(\beta)] . \tag{2.9b}$$

III. THE GAUGE FUNCTION $\beta(t)$

As we have discussed previously (Canuto, Hsieh, and Adams 1977; Canuto *et al.* 1977*b*), the function $\beta(x)$ cannot be determined from within the theory. External conditions must be provided based on physical considerations.

We shall show in what follows that Dirac's proposal of 1937 and the newest one of 1973 can be incorporated in our formalism. Moreover, we shall present a new gauge based on consolidation of the 3 K blackbody radiation.

a) *The Dirac Gauges*

In 1937 Dirac proposed that the existence of large dimensionless numbers cannot be accepted as a built-in feature in any physical theory, and he therefore proposed the so-called Large Numbers Hypothesis. So far, no successful mathematical scheme has been proposed which could incorporate Dirac's ideas in a consistent way. We have already shown that our formalism is indeed capable of incorporating Dirac's proposal, thus removing one of the major objectives, namely the lack of a rigorous mathematical scheme.

In 1937 Dirac proposed that

$$G \sim 1/t ; \qquad M \sim t^0 , \tag{3.1}$$

two conditions known under the name of No Matter Creation. From the exact relation (2.8), it then follows that

$$\beta(t) \sim t , \qquad G\beta = \text{const.} , \tag{3.2}$$

which fixes the gauge function. From (2.5) we then have

$$\rho_m \propto \frac{1}{R^3} , \qquad \rho_\gamma \propto \frac{1}{R^4}\frac{1}{\beta} . \tag{3.3}$$

Can we further explain the second large number, namely $N \sim t^2$? The answer is yes. We proceed as follows. We shall write for N today

$$N_0 = \frac{4\pi}{3} \frac{\rho_{m0}}{m_p} \left(\frac{c}{H_0}\right)^3 \sim 10^{80} \sim t^2 , \tag{3.4}$$

which is the only acceptable definition of what is meant by the number of nucleons in the universe; in fact, (3.4) is written in terms of observational quantities alone.

Following Dirac, we shall write (3.4) as being valid at any time t, i.e.,

$$N(t) \sim \rho_m(t) H^{-3}(t) \sim t^2 . \tag{3.5}$$

Using (3.3), we obtain

$$N(t) \sim \frac{1}{R^3} \frac{R^3}{(dR/dt)^3} \sim \frac{1}{(dR/dt)^3} \sim t^2 , \tag{3.6}$$

which implies

$$R(t) \sim t^{1/3} , \qquad H_0 t_0 = \tfrac{1}{3} , \qquad q_0 = 2 . \tag{3.7}$$

Equation (3.7) is indeed the exact solution of (2.9a) if $k = 0$ and $\beta \sim t$.

In 1973, Dirac extended his original proposal to include matter creation, i.e., (3.1) should be changed to

$$G(t) \sim 1/t ; \qquad M \sim t^2 . \tag{3.8}$$

If so, equation (2.8) yields

$$\beta(t) \sim 1/t , \qquad G \sim \beta . \tag{3.9}$$

From (2.5), we now have

$$\rho_m \propto \frac{1}{R^3} \frac{1}{\beta^2} ; \qquad \rho_\gamma \propto \frac{1}{R^4} \frac{1}{\beta^3} . \tag{3.10}$$

Equations (3.8)–(3.10) are compatible with the dynamic equations (2.9) if

$$R \sim t , \qquad H_0 t_0 = 1 , \qquad q_0 = 0 \tag{3.11}$$

(up to multiplications by slowly varying functions of t, such as ln t).

b) *The New Gauge: The* 3 K *Blackbody Problem*

A major stumbling block in the acceptance of Dirac's ideas has been not only the lack of a dynamical scheme, which we have now provided, but also the explanation of the 3 K radiation. Dirac (1975) himself has been concerned with this problem.

We know that today the 3 K radiation has a blackbody spectrum; what can we say about the spectrum in the past? The only fact we are sure of is that in the past matter and radiation were in equilibrium: in standard cosmology this can be demonstrated to be equivalent to stating that the radiation had a blackbody spectrum, i.e., the blackness of the spectrum existed from the very beginning. Since we also observe it today, we must find a mechanism to preserve it during the expansion. This can be achieved in standard cosmology. However, it must be stressed that the blackbody nature in the past is not an observational fact but rather an unavoidable inference from the physical description given by kinetic theory and Einstein's gravitation theory. In the present framework the mode of description must be changed, and the above mentioned inference need not follow. Consequently, many criticisms of Dirac's Large Numbers Hypothesis (LNH) based on the difficulties with the blackbody radiation, such as the one given by Steigman (1978 and references quoted therein), do not apply to our formulation. We shall have occasion to elaborate on this in a subsequent publication. But here, even assuming that radiation at equilibrium did indeed have a blackbody spectrum, i.e., that no β factors were attached to ρ_γ at decoupling, we can show that a blackbody spectrum can still be preserved to the present if we impose in (2.5) with $c_s^2 = \frac{1}{3}$ as gauge condition

$$G\beta^2 = \text{const.} \tag{3.12}$$

We shall therefore have for ρ_γ

$$\rho_\gamma \sim R^{-4} , \tag{3.13}$$

exactly as in ordinary cosmology. From the geodesic equations for photons (Canuto *et al.* 1977*b*, eq. [2.28]) it can be shown, using standard methods, that for a Robertson-Walker metric, the redshift relation is

$$\nu R = \text{const.} \tag{3.14}$$

Equation (3.14) is independent of any specific choice of the scale factor β and is valid in any system of units, Einstein or atomic. Dirac, using more intuitive arguments, arrived at the same relation.

Now, since $\rho_\gamma \sim \nu n_\gamma$, it follows that

$$n_\gamma \sim R^{-3}, \tag{3.15}$$

and consequently that

$$n_\gamma \sim \rho_\gamma^{3/4}, \tag{3.16}$$

a well-known relation. We shall now derive the previous results in a different way, which will stress their generality.

For radiation, characterized by an equation of state of the form $3p = \rho_\gamma$, the right-hand side of Einstein's equations is (Canuto *et al.* 1977*b*, eq. [2.10] and following)

$$G\rho_\gamma u_\mu u_\nu . \tag{3.17}$$

Since the left-hand side is scale invariant by construction, (3.17) must also be scale invariant, i.e., it must have power zero under a scale transformation. The four-velocity u_μ has power $+1$; and since β has power -1, the product $u_\mu u_\nu$ has the same power as β^{-2}. It therefore follows that the invariance of (3.17) can be expressed as

$$\pi(G\rho_\gamma \beta^{-2}) = 0 ; \tag{3.18}$$

i.e., the power of the quantity $G\rho_\gamma\beta^{-2}$ must be zero. Equation (3.18) can further be written as

$$\pi(G\rho_\gamma R^4 R^{-4}\beta^{-2}) = \pi(\rho_\gamma R^4) + \pi(G\beta^2) + \pi(R^{-4}\beta^{-4}) = 0 . \tag{3.19}$$

Since, by definition, the scale factor has power $+1$, the quantity $R\beta$ has power zero, so that equation (3.19) becomes

$$\pi(\rho_\gamma R^4) + \pi(G\beta^2) = 0 . \tag{3.20}$$

Equation (3.20) admits several solutions since the only requirement is that the sum of the powers of ρ_γ, R^4, G, and β^2 add up to zero. If we want to recover the dependence that ρ_γ has in standard cosmology, namely $\rho_\gamma \sim R^{-4}$, we have to impose the condition

$$G\beta^2 = 1 , \tag{3.21}$$

which is the same as (3.12). We must choose the so far unknown power of G so as to compensate the power of β^2. We have therefore shown that, as far as radiation is concerned, it is possible to choose a gauge such that the present theory becomes indistinguishable from ordinary cosmology.

The next important point concerns the compatibility of the previous argument with the Large Numbers Hypothesis. By adopting the LNH, we immediately have that $G \sim t^{-1}$. From (3.21), it then turns out that the gauge function β must vary as

$$\beta \sim t^{1/2} . \tag{3.22}$$

Can we now explain the other large number 10^{80}, corresponding to the number of nucleons within the visible universe? We shall demand that

$$N_n \propto \rho_m (ct)^3 \propto t^2 . \tag{3.23}$$

Using (2.5) with $c_s = 0$ for ρ_m, (3.23) becomes

$$\frac{1}{R^3}\frac{1}{G\beta}t^3 \propto t^2$$

or

$$R(t) \propto \left(\frac{t}{G\beta}\right)^{1/3} \propto (t\beta)^{1/3} \propto t^{1/3} , \tag{3.24}$$

making use of (3.21) and (3.22). Equation (3.24) fixes the scale factor uniquely. We therefore conclude that

$$R(t) = R_0(t/t_0)^{1/2} , \qquad H_0 t_0 = \tfrac{1}{2} , \qquad q_0 = 1 . \tag{3.25}$$

This is a very interesting result, since very few cosmological theories are able to predict a value for the deceleration parameter not in conflict with observations.

The problem is however not entirely solved, since we have not shown that (3.25) is compatible with the dynamic equations. Using (2.5) with $c_s = 0$ for ρ_m, equation (2.9a) can then be solved exactly. The result is (for $k = 0$)

$$\beta(t)R(t)/R_0 = \left\{\frac{3}{2}\left[H_0 + \frac{(d\beta/dt)_0}{\beta_0}\right]\int_0^t \beta(t)dt\right\}^{2/3} . \tag{3.26}$$

Substituting (3.22), it is easy to verify that (3.25) is indeed the solution. The problem is then fully consistent, if the curvature is zero.

IV. CONCLUSIONS

Einstein's theory is usually employed in conjunction with the unproved assumption that gravitational and atomic forces have been in the same ratio throughout the entire evolution of the Universe. Since such a fact should not be determined by an *a priori* conjecture, but rather by experiments—the alternative possibility that gravitational and atomic forces have been changing at a certain rate must also be investigated and confronted with observational facts. The work presented here was set up to precisely investigate this possibility.

As shown above, the present theory is capable of determining the geometry of the Universe uniquely. A unique solution is found without having to resort to observational facts other than the well established 3 K blackbody radiation and the Large Numbers Hypothesis. Quite surprisingly, one of the most important facts in cosmology, namely the blackbody nature of the 3 K radiation, is not used in standard cosmology to differentiate between possible geometries: one resorts to the magnitude versus z relation, which is fraught with evolutionary uncertainties.

The reason why the 3 K radiation plays an important role in our theory can be traced back to the very basis of our line of thought: If G is to be considered constant only when analyzing gravitational phenomena and can vary with respect to atomic phenomena, it is clear that in order to preserve the blackbody nature of the 3 K radiation (an atomic phenomenon by definition: just recall Einstein's derivation of the Planck formula), a compensating relation must exist between $G(\beta)$ and the scale factor β in the term

$$G(\beta)T_{\mu\nu}(\beta)\,,$$

where $T_{\mu\nu}(\beta)$ represents radiation. Such a relation turns out to be equation (3.12).

We should, however, keep in mind that within the framework of scale covariance, radiation in equilibrium with matter may possess a non-blackbody spectrum (work in progress by the authors confirms such a possibility) of such a nature that the $\beta(t)$ factors entering in (3.3) and (3.10) can just compensate the initial grayness and establish a blackbody spectrum at later times.

The cosmological ideas and results presented here have clearly indicated the fruitfulness of examining more closely the relation between atomic and gravitational phenomena.

We have so far dealt with classical equations only. These ideas must emerge naturally into a quantum-mechanical scheme once we have learned how to make the transition to low quantum numbers of the scale-covariant equations presented here.

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