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EFFECTS OF TIDAL DISTORTION ON BINARY-STAR VELOCITY CURVES AND ELLIPSOIDAL VARIATION

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ABSTRACT

Radial velocity curves for the more massive components of binaries with extreme mass ratios can show a large distortion due to tides, as first recognized by Sterne. Binaries in which the effect is large should be rare because nearly all such binaries would be in the rapid phase of mass transfer. However, the optical counterparts of some X-ray binaries may show the effect, which would then serve as a new means of extracting considerable information from the observations. The essential parts of the computational procedure are given. Light curves for ellipsoidal variables with extreme mass ratios were also computed, and were found to be less sinusoidal than those with normal mass ratios.

Subject headings: binaries — X-ray sources — radial velocities

I. INTRODUCTION

In principle, tidal distortion should modify the radial velocity curves of binary components, although in most cases the effects will be minor or entirely negligible. We wish to point out that examples may occur in which such effects are important, even to the extent that they exceed the ordinary orbital variation, although probably only for rare and unusual binaries. However, some X-ray binaries may correspond to these rare cases, so it seems important to have an acquaintance with the expected phase behavior. In the interest of simplicity we consider only synchronously rotating components, in circular orbit, although it may indeed be important to treat nonsynchronous cases. In fact, the present lack of X-ray binary velocity curves which closely resemble our computed curves may be due to nonsynchronous rotation in the real binaries. Both orbital eccentricity and nonsynchronous rotation may lead to irregular, essentially unpredictable variation in addition to systematic effects. Of course, tides will act strongly to establish synchronism, at least for the outer, most distorted layers of such stars as we consider here, and subsequently to damp orbital eccentricity. Since the time intervals over which tides have operated in particular cases can only be estimated roughly, and the number of X-ray–optical binaries is small, one cannot say at present how many suitable examples are likely to be found. However, Sk 160 (SMC X-1) and HD 77581 (2U 0900–40) may show the effect to a small degree.

Consider a situation in which the more massive component of a binary system is as large or nearly as large as its Roche lobe. In synchronous rotation, the typical velocities of its surface elements will be larger than the component's orbital velocity and, if the mass ratio is fairly extreme, will be many times larger. In fact, this statement is true even if the star is only some reasonable fraction (say one-third) of the size of its lobe. However, the disk of such a small star would have nearly perfect symmetry so that the large positive rotational radial velocities from one side would almost exactly cancel (i.e., will cancel in the sense that the effective spectral line center will not be shifted) the large negative velocities from the other side. The spectral lines of the star, although rotationally broadened, would then show only the orbital velocity. However, tidal distortion in a component which is about as large as its lobe will produce a net asymmetry of the disk (except at the conjunctions) so that rotational velocities will not cancel exactly. It is this effect which we have investigated. We find, of course, that it is most important for very small mass ratios, q, but not completely negligible even for modest q values of unity or greater. We define q as M_2/M_1, where star 1 is the component whose radial velocity is under study. Crampton and Hutchings (1974) and Hutchings (1973) have investigated effects somewhat related to those of the present paper. However, their emphasis was quite different from ours, in that they were interested in line profiles for HZ Her and for contact binaries, rather than radial velocities for cases of extreme q.

Recognition of the existence of this effect is by no means new, although references to it have been in-

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frequent. As far as we know, the earliest reference is one by Sterne (1941), who pointed out that it should occur and presented a simple formula in terms of a double sine wave (i.e., $\sin 2\theta$) for the nonorbital variation. Three separate formulations by Kopal (1945), Kopal (1959), and Kopal and Kitamura (1968) are based on higher order analyses of the problem. These references do not include graphs or tables of computed velocity corrections, but only formulae. We programmed all four formulations and found that they are not in very good agreement with our detailed integrations or, in fact, with each other. Our integrations were done in two quite different ways (viz., § II) and these agree, for all practical purposes.

We have also computed light curves for ellipsoidal variables of extreme q, and find some interesting features which seem worth mentioning.

II. RADIAL VELOCITY COMPUTATIONS

We have computed the tidal effect on radial velocity in two nearly independent ways, as follows:

a) Direct Method

The radial velocity of an arbitrary point in a rotating coordinate frame centered on component 1 is given by

$$v = r\omega(\lambda m - \mu l) \tag{1}$$

where λ, μ are direction cosines of the star's surface coordinates, i.e.,

$$\lambda = \cos\phi \sin\theta\,,$$
$$\mu = \sin\phi \sin\theta\,; \tag{2}$$

and m, l are direction cosines of the line of sight, i.e.,

$$l = \cos\Theta \sin i\,,$$
$$m = \sin\Theta \sin i\,. \tag{3}$$

Here ω is angular velocity, Θ means orbital phase, reckoned from superior conjunction, i is inclination, r is the distance from the origin, and the surface coordinates ϕ, θ are the longitude and colatitude, respectively. In terms of these definitions, the center-of-mass velocity of the star is

$$V_c = -a_1\omega m\,. \tag{4}$$

In order to plot results which are independent of the period P and separation a, we divide equations (1) and (4) by $a\omega$, yielding

$$\frac{v}{a\omega} = r'(\lambda m - \mu l) \tag{5}$$

and

$$\frac{V_c}{a\omega} = \frac{-qm}{1+q}\,, \tag{6}$$

where $r' = r_1/a$ and $a = a_1 + a_2$. In the computations

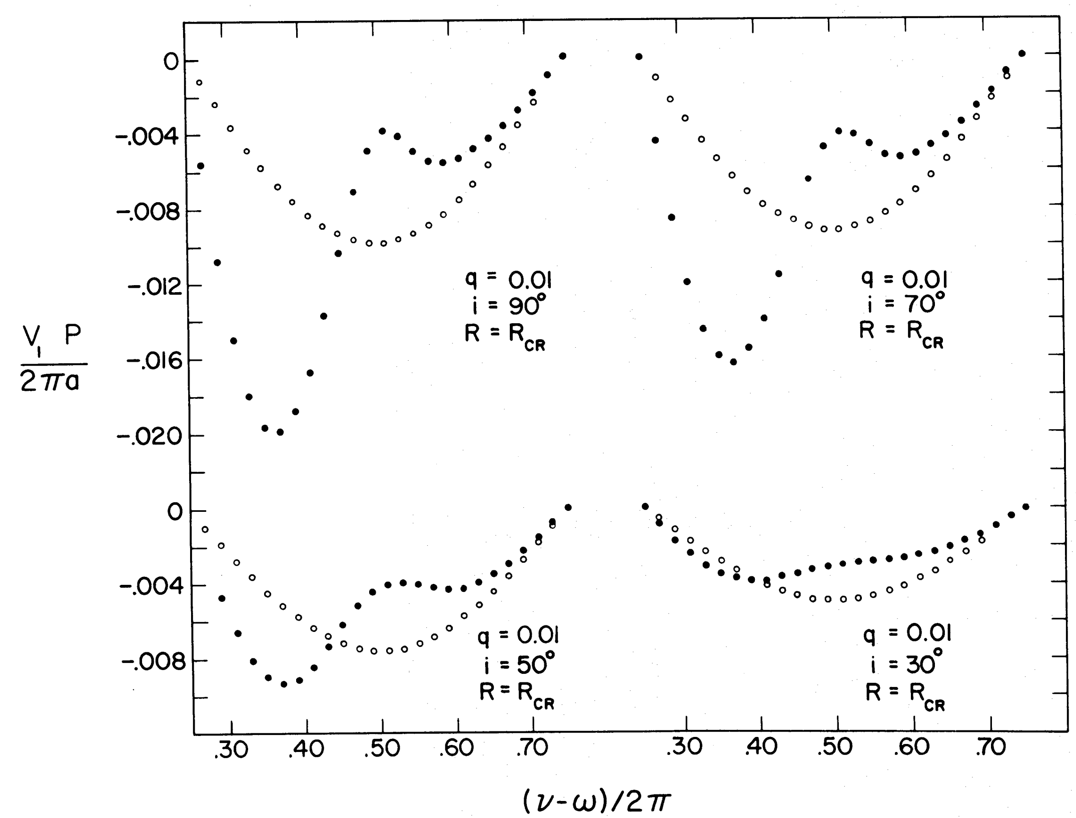

FIG. 1.—Dimensionless radial velocity curves for mass ratio 0.01 at four inclinations. Filled circles show the total (orbital plus tidal correction) velocity while open circles show the purely orbital velocity.

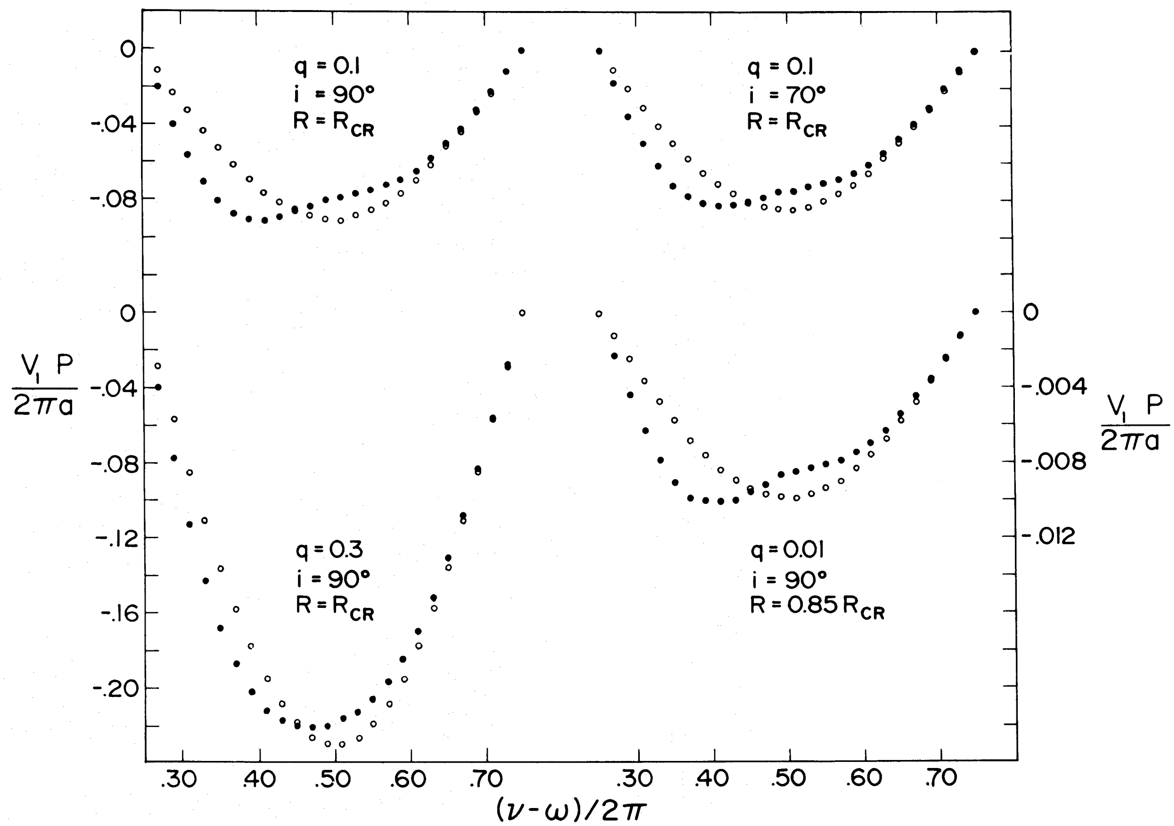

FIG. 2.—Dimensionless radial velocity curves for miscellaneous cases of interest. The symbols are the same as those of Fig. 1.

we made use of the general eclipsing-binary light-curve program by Wilson and Devinney (1971) in which all required quantities already occur. In addition, the flux F in the direction of the line of sight exists in the program, so this may be used as a weighting function to compute a mean effective radial velocity for the disk of the star, relative to the component's center of mass. That is, we compute

$$\Delta V = \frac{\int vFd\sigma}{\int Fd\sigma}, \qquad (7)$$

where $d\sigma$ is an element of surface area; and in Figures 1 and 2 we plot the dimensionless quantity

$$\frac{VP}{2\pi a} = \frac{1}{a\omega}(V_c + \Delta V) \qquad (8)$$

as a function of (spectroscopic) orbital phase for eight cases. V is the actual observable radial velocity.

b) Semi-intuitive Method

As outlined in § I, the effect arises due to a small asymmetry in the "visible" disk, which causes its center of light not to coincide with the position of the center of mass of the star, as projected into the plane of the sky. At a given phase we expect that, to first order, the observed radial velocity will be proportional to the projected moment arm from the system center of mass to the center of light of the distorted star. More precisely (since for $i \neq 90°$ the center of light may not lie along the projected line of centers) we must adopt for our effective projected moment arm the component of the (system center of mass to star light center) distance which lies parallel to the projected line of centers. For each surface grid element, we therefore find the quantity

$$x = \rho \cos A\,, \qquad (9)$$

where ρ is the distance from the star's center to the surface point, as projected into the plane of the sky, and A is the projected (plane of sky) angle between the line of centers and the ρ vector. We then find the mean of x weighted with the line-of-sight flux, and then adopt the effective moment arm for computing radial velocities as this mean ($\bar{x}$), which is already a projected length, plus the projection of a_1. Thus, after multiplying through by the appropriate projection factor, we have

$$a_{1_{\rm eff}} = a_1 + \bar{x}(\sin^2\Theta + \cos^2 i \cos^2\Theta)^{1/2}\,. \qquad (10)$$

This approach yields velocity curves which agree with those of the direct method (above) to within about 10 percent, and thus gives assurance that those curves are correct. However since our two computational approaches have some elements in common, we hope that someone will make independent computations.

As expected, the departure from a normal velocity curve depends on q, i, and the radius as a fraction of Roche lobe radius, $R/R_{\rm Roche}$. Figures 1 and 2 show examples of the general behavior as found from the direct-method computations. All cases correspond to a linear cosine limb darkening coefficient of 0.50 and to classical gravity darkening, with local bolometric flux proportional to local gravity. For $q = 0.01$ a large negative spike appears just after superior conjunction, while a large positive spike appears just before superior conjunction. Of course, the curves are antisymmetric about the conjunctions, so only half the phase range is shown (however, the other half was computed, as a check). The detailed behavior at intermediate phases is rather peculiar, but is almost certainly beyond detection by observations of achievable accuracy. One must remember that most of the curves shown would have quite small amplitude because we are dealing with the (considerably) more massive component.

III. RELATION TO OBSERVATIONS OF REAL BINARIES

As noted in § II, tidally induced distortion of radial velocity curves is primarily a function of q, i, and $R/R_{\rm Roche}$. Failure to observe the effect in a specific binary may therefore be due to any of these parameters having a value such that the effect is very small. If we apply this reasoning in reverse, however, we see that much information could be obtained from observations of a binary which does show a large such effect. That is, it would be certain that the star fills or nearly fills its Roche lobe and also that q is quite small. Since the curves show much more harmonic content than those of normal cases, and the effect can be very large (relative to the orbital variation), in principle one should be able to determine q, i, and $R/R_{\rm Roche}$ from high-quality observations of suitable examples.

It should be pointed out that the effect should be exceedingly rare for normal binaries because a component which fills its Roche lobe and is more massive than its companion must be in the rapid phase of mass transfer (Paczynski 1971; Plaveč 1968). We know, at most, only a few examples of optically discovered binaries which are now in this very brief phase of stellar evolution, and these are not in the very early stages of the process in which $q < 1$. Perhaps the effect could be found for a primary star which only nearly fills its lobe and has a value of q of, say, 0.3 (Fig. 2 then shows a correction of 20 percent at some phases), but these are rather special requirements. Some of the X-ray binaries may fulfill the needed conditions, although we do not know of one whose velocity curve resembles our figures in detail. However, the estimated parameter for Sk 160 = SMC X-1 (Osmer and Hiltner 1974; Avni and Bahcall 1975; Wilson and Wilson 1976) are such that the effect should be noticeable in observations which have small scatter. In Figure 3 we compare our predicted velocity curve for Sk 160 with that of the purely orbital variation.[1] The assumed (consensus) parameters are ($q = 0.17$, $i = 75°$; $R/R_{\rm Roche} = 1.0$). While the illustrated systematic effect is small compared with the scatter of the observations, the new curve seems to be marginally preferable to the orbital sine curve because it tends to mimic the long phase interval of nearly constant velocity shown by the observations, especially in the latter half of the cycle. However, the main purpose of Figure 3 is to show the size of the distortion for a concrete example, rather than to convince anyone that the effect is present in the observations.

When the effect is small, as in the illustrated case (Fig. 2) for $q = 0.3$, it would be natural to interpret it as a small eccentricity, with the longitude of periastron approximately 90°. When the effect is large, however, as in the cases with $q = 0.01$ (Fig. 1), it would not be

[1] HD 77581 (2U 0900-40) should also show the effect, but here it is complicated by an eccentric orbit (Zuiderwijk *et al.* 1974; Hutchings 1974*b*; Petro and Hiltner 1974; Wallerstein 1974).

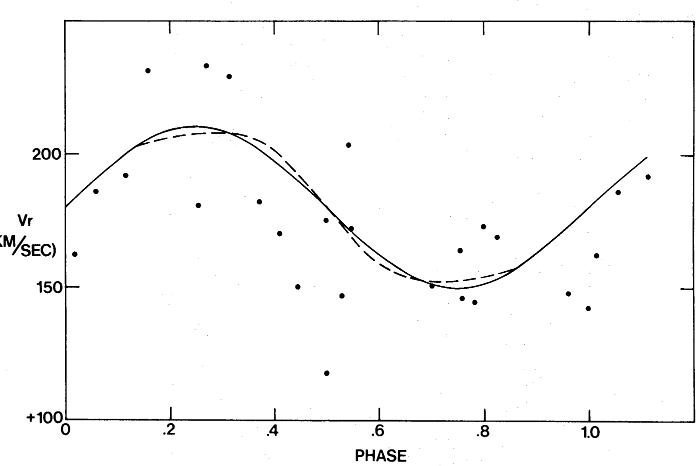

FIG. 3.—Comparison of an orbital velocity sine curve (*solid curve*) with a (*dashed*) curve including a correction for tidal distortion, for the case of Sk 160. The observations are by Osmer and Hiltner (1974), with the X-ray eclipse corresponding to zero phase.

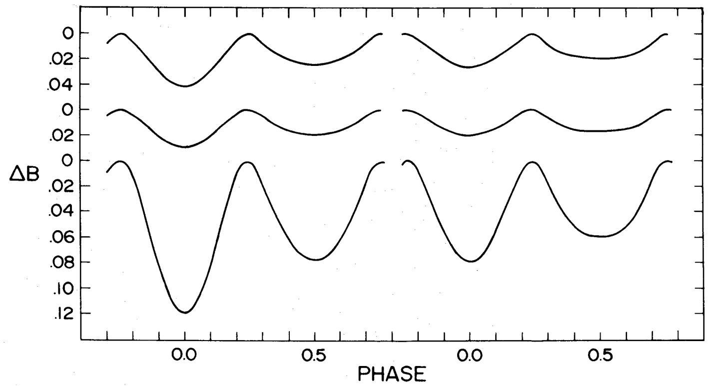

FIG. 4.—Theoretical λ4400 light curves for ellipsoidal variables with (*left*) classical gravity darkening and (*right*) no gravity darkening. The lowermost curves are for $M_2/M_1 = 0.10$, while the upper four are for $M_2/M_1 = 0.01$. The middle curves have $i = 60°$; the other four have $i = 90°$.

mistaken for orbital eccentricity unless the phase coverage were incomplete.

IV. ELLIPSOIDAL VARIATION FOR EXTREME MASS RATIO

Figure 4 shows six theoretical light curves for the more massive components of binary systems having q values of 0.01 and 0.10. The computational procedure has been described elsewhere (Wilson and Devinney 1971), and such curves for moderate values of q have been published by Hutchings (1974*a*) and others. The curves of Figure 4 all differ considerably from the cosine curves one sees for moderate q, in that they have sharply peaked maxima and relatively broad, dish-shaped minima. This is true of cases with classical gravity darkening (*left side* of figure) and of those with no gravity darkening (*right side*). Therefore, cases in which the ellipticity variation is small because the mass ratio is extreme can potentially be clearly distinguished from cases in which it is small because $i \ll 90°$ or because $R \ll R_{\mathrm{Roche}}$.

V. CONCLUSION

At present the main interest in the effects presented above would seem to lie in their possible use as indicators of extreme mass ratio. Given suitably accurate observations, the effects might decide the controversy as to whether Cen X-3, for example, has $q \approx 0.015$ to 0.05 (Wilson 1972; Osaki 1972; van den Heuvel and Heise 1972) as indicated by the X-ray eclipse duration, or whether that type of argument can partly be circumvented by various devices, such as that advanced by Davidson and Ostriker (1973).

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