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STELLAR EVOLUTION AT HIGH MASS WITH SEMICONVECTIVE MIXING ACCORDING TO THE SCHWARZSCHILD CRITERION

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ABSTRACT

New evolutionary sequences of models for stars of 5–60 $M_\odot$ with four different initial chemical compositions have been constructed with the use of the Schwarzschild criterion for convection. This criterion leads to semiconvection outside the convective core (1) just after the zero-age main-sequence stage for masses greater than $\sim 12\ M_\odot$ and (2) just before the stage of central hydrogen exhaustion for masses greater than $\sim 6\ M_\odot$. The base of the semiconvective zone thereafter becomes fully convective, with the result that the hydrogen profile in the intermediate zone acquires a local plateau. Competition between the effects of the hydrogen plateau, of the envelope opacity, and of the luminosity-to-mass ratio provides the reason why the post–main-sequence evolutionary tracks on the H-R diagram are found to vary widely as a function of initial chemical composition for masses less than $\sim 17\ M_\odot$ and why, for higher masses, the tracks invariably reach the red region of the H-R diagram only toward the end of core helium burning. The tracks for the more massive stars are found to be relatively insensitive to the uncertainties in the nuclear reaction rates. Small differences among the sequences computed by different authors are probably due mostly to slight differences in the opacities adopted. It is found that the rather cool range of effective temperatures predicted for stable blue supergiants with masses above $\sim 20\ M_\odot$ is not in agreement with the observations. An observational test utilizing the surface hydrogen abundance of stellar remnants in binary systems is at present inconclusive in deciding whether the Schwarzschild criterion or the Ledoux criterion is the correct criterion for convection to use. It is strongly suggested that neither criterion with any reasonable adopted initial chemical composition can at present lead to model predictions that satisfy all the observational requirements.

Subject headings: convection — stars: evolution — stars: interiors — stars: massive — stars: supergiants

I. INTRODUCTION

Convective instability often occurs in deep-lying layers of massive stars that contain a gradient of mean molecular weight. How to mix these layers realistically has been a problem of long standing. Iben (1966*a*, *b*) was the first author to consider the problem for post–main-sequence stars, in the case in which the Schwarzschild criterion for convection is adopted. He chose to mix the unstable layers homogeneously, and found that a star of 15 $M_\odot$ begins and completes core helium burning as a blue supergiant, while a star of 9 $M_\odot$, which has a much smaller homogeneous intermediate zone, undergoes a more conventional evolution (typical of stars of low and intermediate masses) whereby core helium burning begins in the red-supergiant configuration and ends in the blue-supergiant configuration. It is now recognized that the fact that the hydrogen-burning shell promptly encountered the large hydrogen discontinuity at the base of the homogeneous intermediate zone shortly after central hydrogen exhaustion is the reason why Iben's model of 15 $M_\odot$ stabilized as a blue supergiant (Stothers and Chin 1968 [Paper I]).

Iben's solution, however, is not wholly physical, since a proper treatment of the convective instability should lead, at least initially, to only a minor adjustment of the gradient of mean molecular weight, by means of semiconvection (Schwarzschild and Härm 1958; Sakashita and Hayashi 1961). Therefore, it seemed to us (Paper I) that the conventional picture of evolution during core helium burning would probably apply at all masses. While this has been largely verified whenever the Ledoux criterion has been adopted as the criterion both for convective instability and for semiconvective mixing, such is not the case when the Schwarzschild criterion has been adopted, as Chiosi and Summa (1970) were the first to demonstrate. In fact, a large fully convective zone is found to develop at the bottom of the semiconvective zone, so that the final hydrogen profile based on the Schwarzschild criterion resembles the one obtained by Iben, with similar consequences for the H-R diagram.

The evolutionary sequences that have been based on the Schwarzschild criterion are summarized in Table 1. Not enough data have been published to include in the table the sequences at 14 $M_\odot$ and 50 $M_\odot$ derived by Eggleton (1972) and by Morris (1970), respectively. Eggleton's sequence and a sequence at 20 $M_\odot$ computed Chiosi and Nasi (1974*b*) were derived for a Population II chemical composition, while a sequence at 12 $M_\odot$ due to Ferrari d'Occhieppo *et al.* (1969) was

TABLE 1

Previous Evolutionary Sequences of Models for Massive Stars Based on the Schwarzschild Criterion

$M/M_{\odot}$	X_e	Z_e	θ_α^2	$\epsilon_{3\alpha}$	log T_e (tip)	log T_e (b/y)	τ_b/τ_{He}	Author
9*............	0.708	0.020	0.780	Old	4.15	~4.0	0.9	Iben 1966*a*
11............	0.602	0.044	0.060	Old	3.96	~3.7	0.6	Barbaro *et al.* 1972
12............	0.800	0.020	0.100	Old	...	...	0.0	Robertson 1972
12............	0.708	0.020	0.100	Old	...	...	0.0†	Robertson 1972
15............	0.750	0.020	0.085	Old	4.13	~3.8	0.6	Simpson 1971
15............	0.708	0.020	0.780	Old	4.24	~4.0	1.0	Iben 1966*b*
15............	0.708	0.020	0.100	Old	4.18	~3.8	0.8	Robertson 1972
15............	0.700	0.020	0.060	Old	4.17	~4.0	1.0	Chiosi and Nasi 1974*a*
15............	0.617	0.020	0.100	Old	4.19	~3.8	0.7	Robertson 1972
15............	0.602	0.044	0.100	Old	4.15	~3.8	0.6	Robertson 1972
15............	0.602	0.044	0.060	Old	4.10	~3.8	0.6	Barbaro *et al.* 1972
16............	0.602	0.044	~0.1	New	4.18	~3.9	1.0	Varshavsky and Tutukov 1973
20............	0.700	0.020	0.060	Old	4.23	~3.9	1.0	Barbaro *et al.* 1973
20............	0.700	0.020	0.100	Old	4.21	~3.9	0.9	Sreenivasan and Ziebarth 1974
20............	0.700	0.044	0.060	Old	4.17	~3.9	0.9	Barbaro *et al.* 1973
20............	0.602	0.020	0.060	Old	4.24	~3.9	0.9	Barbaro *et al.* 1973
20‡............	0.602	0.044	0.060	Old	4.16	~3.9	0.8	Barbaro *et al.* 1973
20............	0.500	0.020	0.060	Old	4.24	~3.9	0.8	Barbaro *et al.* 1973
20............	0.500	0.044	0.060	Old	4.14	~3.9	0.6	Barbaro *et al.* 1973
30............	0.750	0.020	0.085	Old	4.17	~3.8	0.8	Simpson 1971
30............	0.602	0.044	0.060	Old	4.08	~3.8	0.7	Chiosi and Summa 1970
32............	0.602	0.044	~0.1	New	4.06	~3.8	0.6	Varshavsky and Tutukov 1973
40............	0.602	0.044	0.060	Old	3.86	(§)	(§)	Barbaro *et al.* 1971
64............	0.602	0.044	~0.1	New	3.77	~3.7	0.7	Varshavsky and Tutukov 1973

* Recalculated, with somewhat different initial parameters, by Iben 1972.

† This sequence was terminated when $Y_c = 0.459$, but no blue loop had yet developed.

‡ Also calculated by Chiosi and Summa 1970.

§ This sequence was terminated when $Y_c = 0.920$, but the subsequent evolution probably takes place at effective temperatures cooler than log $T_e = 3.86$.

discontinued shortly after the onset of core helium burning.

Our motivation for calculating additional sequences is severalfold. First of all, it is very advantageous for comparative purposes to have a grid of sequences computed with the same initial chemical compositions and physical input parameters as were adopted in our previous work based on the assumptions of (*a*) no mixing in the intermediate zone (Stothers and Chin 1973 [Paper II]) and (*b*) Ledoux-type semiconvective mixing (Stothers and Chin 1975 [Paper III]). Second, the behavior of the evolutionary tracks as a function of initial chemical composition requires much more clarification at relatively low masses (~10 $M_{\odot}$) and at very high masses (30–60 $M_{\odot}$). Third, the sensitivity of the tracks to the adopted nuclear reaction rates has not yet been studied consistently, although Iben (1966*b*, 1972) has considered some aspects of this problem at the lower masses. Fourth, a comprehensive picture of evolution based on the Schwarzschild criterion, as determined from the present large number of evolutionary sequences, is now possible, with the result that the main factors responsible for the behavior of the various sequences computed both by us and by other authors can be identified, and that a basic comparison with observational data can be reliably made.

II. SEMICONVECTION

Adaptation of the details of Schwarzschild-type semiconvective mixing to automatic numerical-relaxation programs has varied considerably among different authors (Chiosi and Summa 1970; Simpson 1971; Eggleton 1972; Robertson 1972; Varshavsky and Tutukov 1972; Sreenivasan and Ziebarth 1974; Schlesinger 1975). Our present approach resembles the procedure that we used earlier for Ledoux-type semiconvective mixing (Paper III), and is very similar to the procedure of Robertson. The various semiconvective and fully convective possibilities that can arise have already been enumerated, as follows (Stothers 1970): S2, the semiconvective zone joins smoothly the convective core; M2, the semiconvective zone is detached from all convective regions; and N2, the semiconvective zone runs smoothly down into a fully convective intermediate zone.

III. EVOLUTIONARY SEQUENCES WITH SEMICONVECTIVE MIXING ACCORDING TO THE SCHWARZSCHILD CRITERION

Evolutionary sequences for stars of 10, 15, 30, and 60 $M_{\odot}$ have been calculated to the end of core helium burning, by employing an automatic computer program described in Paper II, but modified here to handle mixing in the convectively unstable intermediate zones, in accordance with the Schwarzschild criterion. As in our earlier work, we have adopted the Cox-Stewart opacities as represented by a fitted formula, and the standard mixing-length theory of convection in the outer envelope, with the low-temperature opacities

TABLE 2

NEW EVOLUTIONARY SEQUENCES OF MODELS FOR STARS OF 10, 15, 30, AND 60 $M_\odot$ BASED ON THE SCHWARZSCHILD CRITERION*

Sequence	X_e	Z_e	θ_α^2	$\epsilon_{3\alpha}$	log T_e (tip)	log T_e (b/y)	τ_H (10^6 yr)	τ_{He}/τ_H	τ_b/τ_{He}	τ_y/τ_{He}
10-A.	0.739	0.021	0.1	New	4.00	~3.8	21.260	0.146	0.592	0.041
10-B.	0.739	0.044	0.1	New	. . .	. . .	24.011	0.137	0.042	0.007
10-C.	0.602	0.021	0.1	New	4.18	~3.9	12.291	0.171	0.937	0.040
10-D.	0.602	0.044	0.1	New	4.12	~4.0	13.584	0.175	0.664	0.061
15-A.	0.739	0.021	0.1	New	4.20	~3.8	11.992	0.115	0.848	0.036
15-B.	0.739	0.044	0.1	New	. . .	. . .	12.825	0.108	0.036	0.007
15-C.	0.602	0.021	0.1	New	4.29	~4.1	7.911	0.130	0.984	0.016
15-D.	0.602	0.044	0.1	New	4.19	~3.9	7.914	0.134	0.886	0.053
30-A.	0.739	0.021	0.1	New	4.24	~4.1	6.101	0.086	0.968	0.032
30-B.	0.739	0.044	0.0	New	4.09	~3.8	6.403	0.084	0.902	0.070
30-C.	0.739	0.044	1.0	New	4.09	~3.7	6.403	0.130	0.825	0.096
30-D.	0.739	0.044	0.1	Old	4.10	~3.8	6.403	0.103	0.850	0.044
30-E.	0.602	0.021	0.1	New	4.26	~4.1	4.110	0.116	0.920	0.080
30-F.	0.602	0.044	0.1	New	4.14	~3.8	4.097	0.111	0.993	0.007
60-A.	0.739	0.021	0.1	New	3.94	~3.6	3.830	0.086	1.000	. . .
60-B.	0.739	0.044	0.1	New	3.89	~3.6	3.840	0.086	1.000	. . .
60-C.	0.602	0.021	0.1	New	3.93	~3.6	2.697	0.115	1.000	. . .
60-D.	0.602	0.044	0.1	New	3.91	~3.6	2.712	0.117	1.000	. . .

* α = 0.4 and "unmodified" opacities in the outer convective envelope.

left "unmodified" and the ratio of mixing length to density scale height taken to be equal to $\alpha = 0.4$.

The main results of our calculations are contained in Table 2, which lists: X_e, the initial hydrogen abundance by mass; Z_e, the initial metals abundance by mass; θ_α^2, the reduced α-particle width of the 7.12 MeV level in ^{16}O; $\epsilon_{3\alpha}$, the rate of nuclear energy generation from the 3α process ("old" rate, Clayton 1968; "new" rate, Austin *et al.* 1971); log T_e (tip), logarithm of the maximum effective temperature attained during the stable phase of core helium burning (as long as any part of this phase takes place in the blue-supergiant configuration); log $T_e(b/y)$, logarithm of the effective temperature dividing the stable "blue" supergiants from the unstable "yellow" supergiants with rapidly expanding envelopes; τ_H, the lifetime of core hydrogen burning; τ_{He}/τ_H, the ratio of the lifetimes of core helium burning and core hydrogen burning; τ_b/τ_{He},

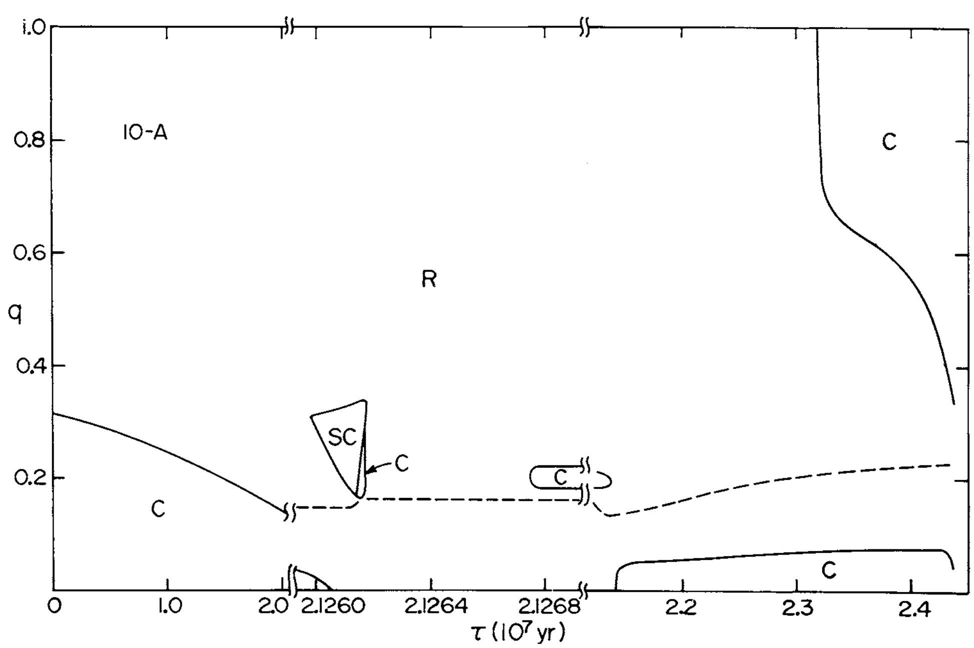

FIG. 1.—Evolution of the structural zones of a star of 10 $M_\odot$ with (X_e, Z_e) = (0.739, 0.021). Coding is as follows: R, radiative; C, convective, SC, semiconvective. The dashed line represents the peak of the hydrogen-burning shell. The ordinate is mass fraction q. The abscissa, time, is divided into three segments, representing the following three phases of evolution: (1) core hydrogen burning, (2) transitional stages, and (3) core helium burning.

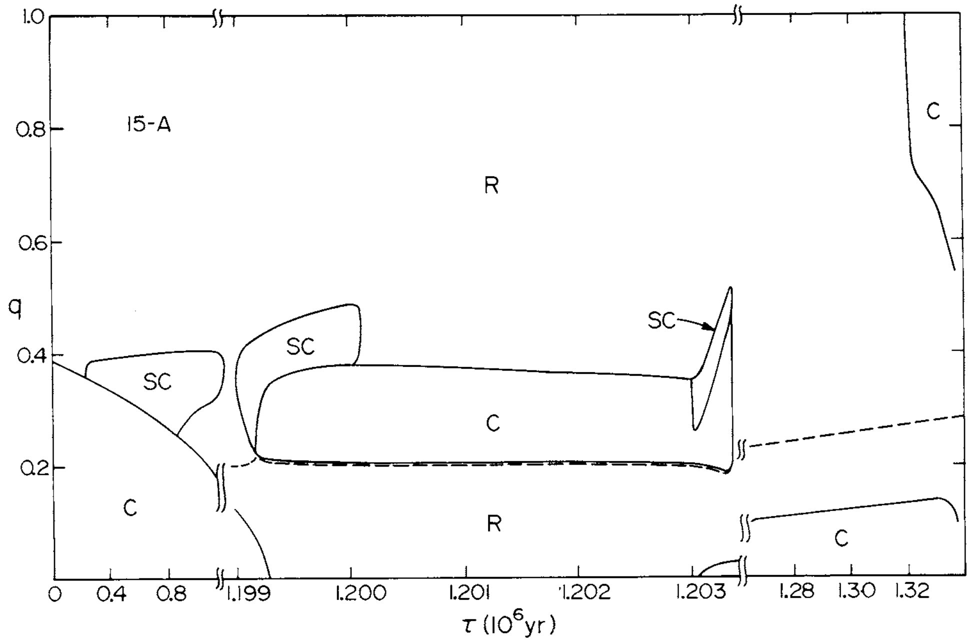

FIG. 2.—Evolution of the structural zones of a star of 15 $M_{\odot}$ with (X_e, Z_e) = (0.739, 0.021). Coding, same as in Fig. 1.

the fraction of time spent in the blue-supergiant configuration during core helium burning; and τ_y/τ_{He}, the fraction of time spent in the yellow-supergiant configuration during core helium burning.

Because of the many available descriptions of evolution in massive stars, we shall concentrate here on a summary of the modifications of the interior structure and surface parameters induced by semiconvective mixing as a result of the adoption of the Schwarzschild criterion. The interior evolution for each of the four masses is shown in Figures 1, 2, 3, and 4. The main differences from the results based on the Ledoux criterion are the great extent of the convectively unstable layers in the intermediate zone (due to the absence in the Schwarzschild criterion of the choking effect of a gradient of mean molecular weight) and the eventual development of a fully convective zone at the base of the semiconvective zone. Hydrogen profiles

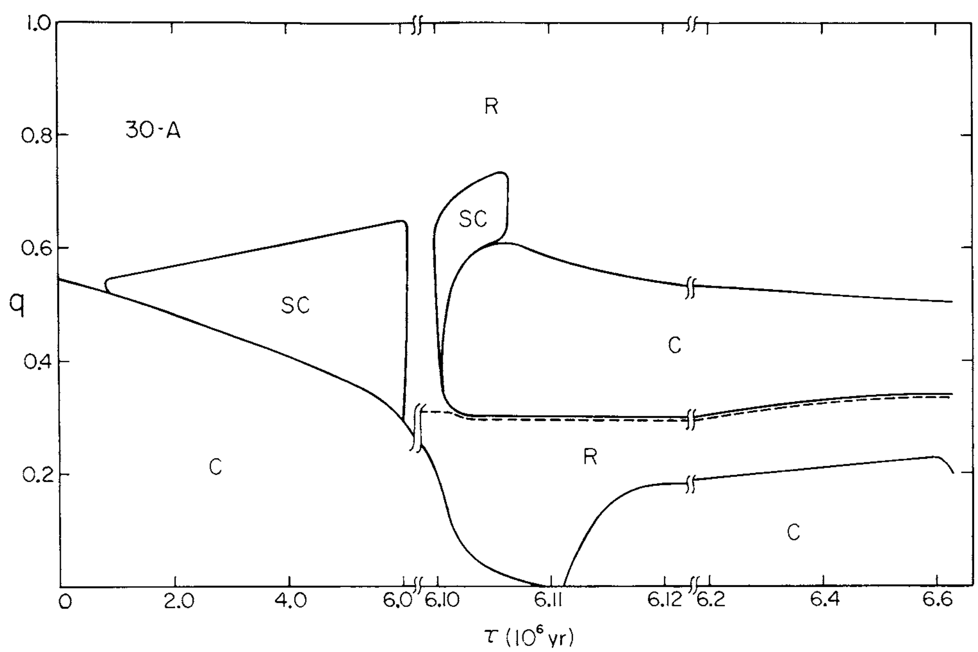

FIG. 3.—Evolution of the structural zones of a star of 30 $M_{\odot}$ with (X_e, Z_e) = (0.739, 0.021). Coding, same as in Fig. 1.

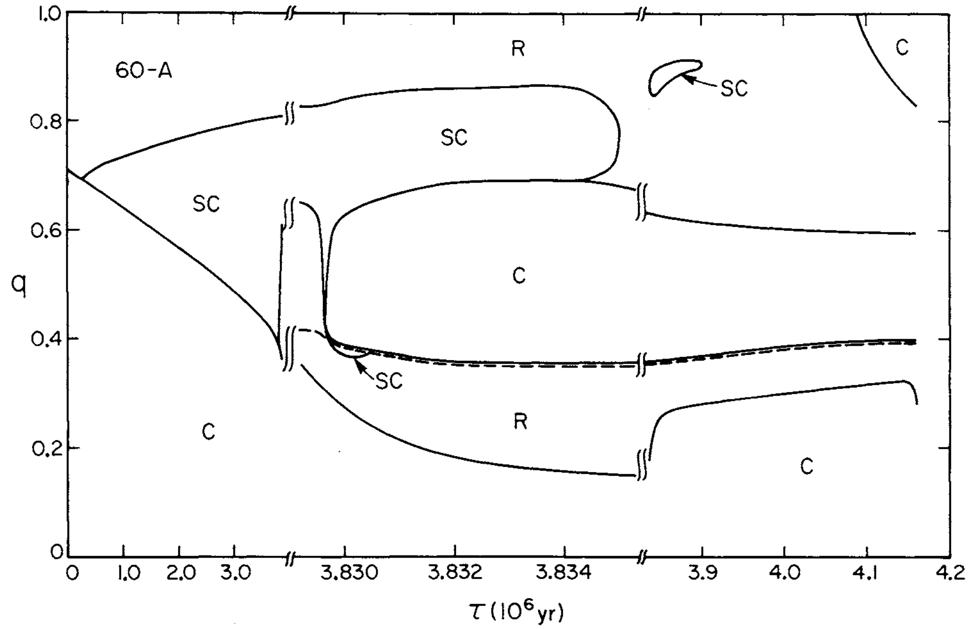

FIG. 4.—Evolution of the structural zones of a star of 60 $M_{\odot}$ with (X_e, Z_e) = (0.739, 0.021). Coding, same as in Fig. 1.

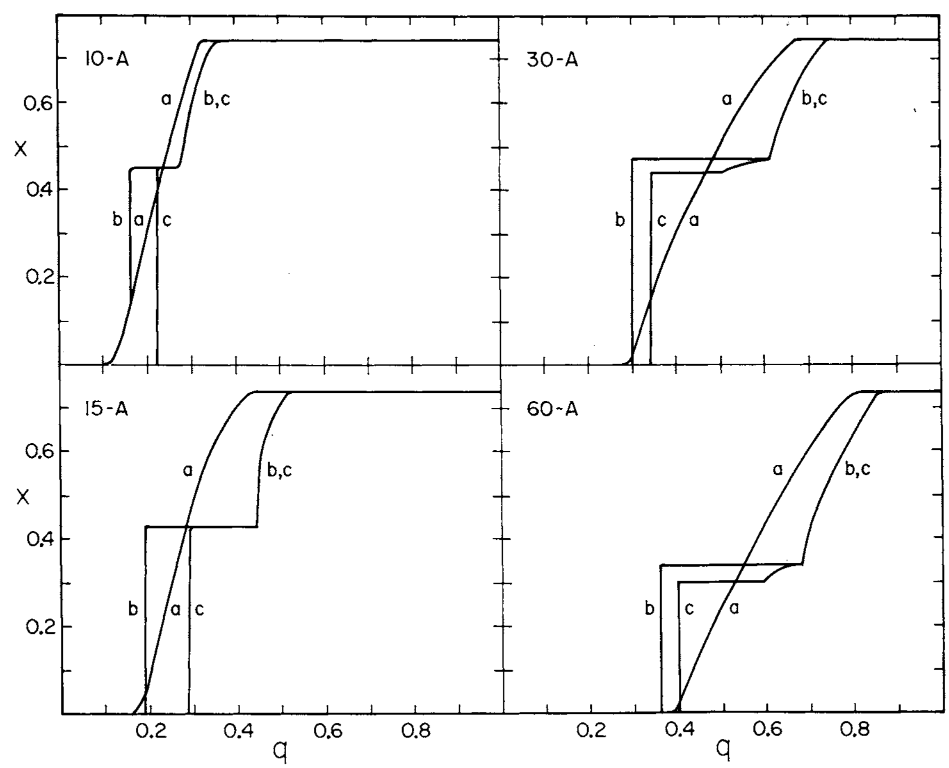

FIG. 5.—Hydrogen profiles for stellar models of 10, 15, 30, and 60 $M_{\odot}$ with Z_e = 0.021 (sequences 10-A, 15-A, 30-A, and 60-A). Lettering index: *a*, end of core hydrogen burning; *b*, maximum extent of fully convective intermediate zone; and *c*, end of core helium burning.

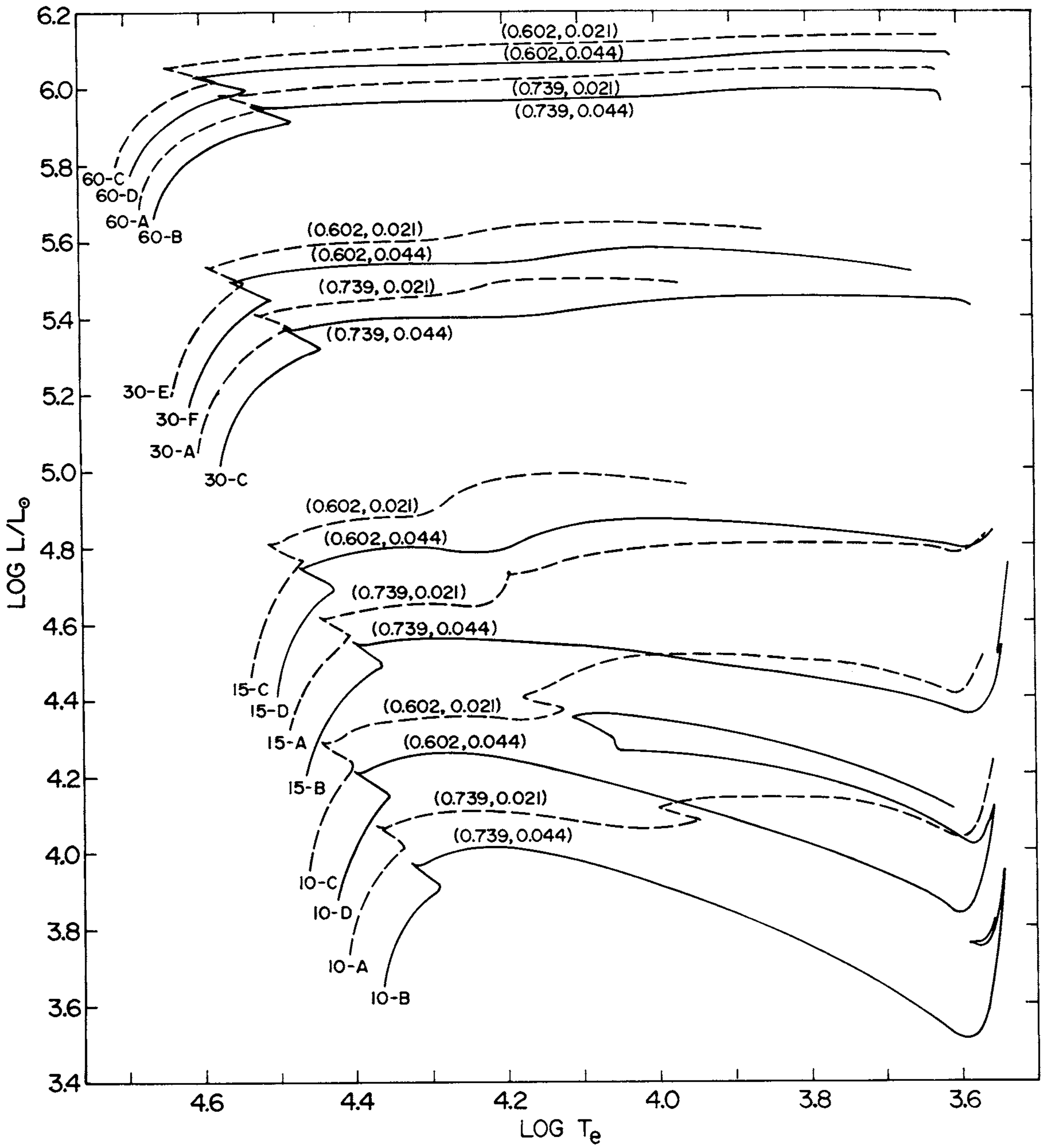

FIG. 6.—Theoretical evolutionary tracks in the H-R diagram for stars of 10, 15, 30, and 60 $M_{\odot}$. Tracks are labeled with (X_e, Z_e).

throughout the star at various critical stages of evolution are shown in Figure 5. The main factors responsible for the behavior of the evolutionary tracks on the H-R diagram, as plotted in Figure 6, will be summarized next by combining our new results (including some unpublished results for 5 and 7 $M_{\odot}$) with the available older results (see especially Barbaro *et al.* 1971, 1973).

First, however, we remark that, although semiconvection has been treated differently by different authors, the semiconvective and fully convective zones that develop are very similar for analogous published evolutionary sequences. The differences in the sequences that do occur seem to be mainly due to differences in the adopted opacities. For example, Iben (1966*a*, *b*) employed the Keller-Meyerott opacities, which are somewhat smaller than the Cox-Stewart opacities adopted by us (in the form of a fitted formula) and by Barbaro *et al.* (in the form of tables, which yield slightly larger opacities than does the formula). Robertson (1972) used the largest opacities of all, being modifications of the Cox-Stewart opacities.

Recognition of these opacity differences can lead to a ready explanation of the differences in the derived evolutionary tracks, as may be inferred from the discussion below. In most cases, however, the derived differences are rather small. For example, for a fixed initial chemical composition at 15 $M_\odot$, the scatter of the derived values of $\log T_e$ (tip) around their mean value never exceeds ± 0.05.

With this assurance, we can turn to our brief description of stellar evolution based on the Schwarzschild criterion. If the stellar mass exceeds $\sim 12\ M_\odot$, semiconvection is found to develop in layers just above the boundary of the convective core shortly after the zero-age main-sequence stage. Transport of fresh hydrogen into the core prolongs this phase of evolution. Eventually the semiconvective zone breaks away from the core, shrinks (or even dies out), and reappears later with a fully convective zone attached to its base. This late appearance of semiconvection occurs at all masses above $\sim 6\ M_\odot$ around the time of central hydrogen exhaustion. At the higher masses, the convectively unstable zones are both larger in mass fraction and more permanent in duration. This is also true if the initial hydrogen or metals abundance is lower, at a fixed stellar mass. Although, for a short time at the highest masses, less than 1 density scale height is found to separate the convective core from the overlying convective intermediate zone, the estimated amount of hydrogen flux into the core by semiconvective overshooting is so small as to be negligible.

The hydrogen abundance in the fully convective zone is eventually stabilized at a value of $X \approx 0.46$, 0.45, 0.43, and 0.30 for stellar masses of 10, 15, 30, and 60 $M_\odot$, respectively, with the single exception of sequence 15-B (where $X = 0.28$). Merger of the hydrogen-burning shell with the inner edge of the hydrogen plateau that is formed by the fully convective zone occurs much earlier in the evolution if the stellar mass is very high. This merger tends to halt the rapid envelope expansion that follows the cessation of central hydrogen burning. It therefore tends to make the star "blue." However, it is opposed by the effects of a variable envelope opacity and of a high luminosity-to-mass ratio, which tend to make the star "red" and which grow in importance as the stellar mass is raised.[1] Consequently, a maximum of $\log T_e$ (tip) is reached at an intermediate mass of $\sim 20\ M_\odot$. This maximum effective temperature, and the entire blue edge of the domain of stable blue supergiants on the H-R diagram, is much more sensitive to the initial metals abundance adopted than to the initial hydrogen abundance; thus, the blue edge can become considerably hotter for a smaller value of Z_e.

For masses less than $\sim 17\ M_\odot$, the competition between the hydrogen plateau in the intermediate zone and the high envelope opacity in the overlying layers is so delicately balanced that a small change of initial chemical composition can give rise to a large difference in the evolutionary tracks. However, as long as the star does not have a preliminary red-supergiant phase during core helium burning, the present uncertainty in the nuclear reaction rates has little influence on the tracks. This preliminary red phase never occurs for masses greater than $\sim 17\ M_\odot$, but if the initial hydrogen or metals abundance is very low (i.e., if the envelope opacity is small), the actual switchover mass may be as low as 10 $M_\odot$.

Thermal instability in the hydrogen-burning shell can occur if the shell, while still thick, makes early contact with the fully convective intermediate zone (Stothers and Chin 1972). In the present (more realistic) models, the instability is found to be just as inconsequential as we found before, chiefly as a result of the great steepness of the hydrogen gradient through the shell.

During the later stages of core helium burning, the evolving star, which has so far been a blue supergiant, begins to move rapidly to the right in the H-R diagram. This transition takes place earlier if the initial metals (and, possibly, the initial hydrogen) abundance is high. The reason for the transition in general is the steady increase in mass fraction and in mean molecular weight of the core at the expense of the envelope. In the present case, this differentiation between core and envelope is aided by the large chemical inhomogeneity in the lower part of the envelope. The range of $\log T_e$ swept out by the star as a stable blue supergiant is greater if Z_e is large or if X_e is small; it is greatest for a mass somewhere near 15 or 20 $M_\odot$.

In contrast, the evolutionary sequences based on the Ledoux criterion (Paper III) nearly always have a preliminary red-supergiant phase during the early stages of core helium burning. This is because the original slope of the hydrogen profile in the intermediate zone is ordinarily reduced very little by semiconvection. However, deep convection originating at the surface of the star when it is a red supergiant homogenizes practically the entire envelope. The subsequent merger of the hydrogen-burning shell with the inner edge of the hydrogen plateau formed by the surface convection causes a "blue loop" to develop on the H-R diagram. Such a complete chemical homogeneity of the envelope prevents the star, once it is a blue supergiant, from making any significant transition to the right in the H-R diagram until helium is exhausted at the stellar center. The range of $\log T_e$ covered by the star as a stable blue supergiant is usually smaller than in the case where the Schwarzschild criterion is adopted; but the maximum effective temperature attained is hotter, and $\log T_e$ (tip) increases monotonically with stellar mass (as long as a blue loop exists).

IV. AN OBSERVATIONAL TEST OF THE MODE OF SEMICONVECTIVE MIXING

Theoretical studies of stellar evolution in close binary systems indicate that, when the initially more massive component (the primary) evolves and fills its Roche lobe, matter flows from its surface into the

[1] The often ignored influence of the luminosity-to-mass ratio alone may be seen in earlier models constructed without convective modifications and with constant electron-scattering opacity (Stothers and Simon 1968, see Table 1). The roles of envelope opacity and of the hydrogen plateau have been described in Paper I.

vicinity of the secondary star. If the mass transfer is heavy enough, layers that were formerly inside the hydrogen-burning core are exposed at the surface of the primary, which will thereby exhibit an enhanced helium abundance.

Two cases of mass loss are usually distinguished. In case A, mass loss begins before hydrogen is exhausted at the stellar center. In that case, the hydrogen profile throughout the star will not be altered very much by semiconvection, regardless of whether the Schwarzschild or Ledoux criterion is adopted. In case B, mass loss starts after the stage of central hydrogen exhaustion. The hydrogen profile immediately above the hydrogen-burning shell will depend strongly on which criterion is adopted during this phase of evolution, since the Schwarzschild criterion leads to an extensive fully convective intermediate zone while the Ledoux criterion does not. It is a characteristic of case B models that mass loss always continues practically down to the hydrogen-burning shell. Therefore, the surface hydrogen abundance of the remnant will depend critically on the adopted criterion for convection. Detailed models of evolution in close binary systems, with an initial (but not critically important) chemical composition of $(X_e, Z_e) = (0.602, 0.044)$, indicate that the exposed hydrogen content will be $X \approx 0.45$ if the Schwarzschild criterion is adopted (Barbaro *et al.* 1969; Kippenhahn 1969) or $X \approx 0.20$ if the Ledoux criterion (or no mixing) is adopted (Kippenhahn and Weigert 1967; Tutukov *et al.* 1973).

The best observed candidate for case B evolution is probably β Lyrae (Plaveč 1968). The observed surface hydrogen abundance of the primary is low, lying in the range $X = 0.05$–0.20 (Boyarchuk 1959; Hack and Job 1965). Most likely the mass ratio of the two components is close to 0.2 (Kříž 1974; Wilson 1974), which implies that, since the mass function of this eclipsing system is 8.5, the individual masses are $M_1 + M_2 \approx 2 + 10$ (in solar units). The present orbital parameters of the system are modeled very well by the theoretical sequence due to Kippenhahn and Weigert (1967) for an initial set of masses $M_1 + M_2 = 9 + 3.1$, as has been shown elsewhere (Stothers 1972; Kříž 1974). Taken at face value, the observed surface hydrogen abundance of the primary implies that the Ledoux criterion for convection is correct. However, the initial mass of the primary ($\sim 9\ M_{\odot}$) is dangerously close to the lower end of the mass range in which one may hope to distinguish clearly between the two criteria for convection.

The only other good candidates for case B evolution are certain Wolf-Rayet stars in binary systems (Paczynski 1967, 1973). However, the observed rates of mass ejection from these stars are so high that the extreme hydrogen deficiency seen at their surfaces is probably a result of continuing violent mass ejection after the evolutionary mass exchange has ceased. We conclude that it is necessary to catch a massive binary system just at the end of the original phase of mass exchange, in order to use the surface hydrogen abundance to test for the correct criterion for convection.

V. CONCLUSION

In the present study of stellar evolution based on the Schwarzschild criterion for convection, it has been shown that the evolutionary tracks for stars more massive than $\sim 17\ M_{\odot}$ are affected surprisingly little by (1) modest changes of the initial chemical composition, (2) uncertainties in the nuclear reaction rates, (3) penetrative convection between the envelope and core, (4) thermal instability of the hydrogen-burning shell, and (5) mild differences in the atomic opacities adopted by different authors. Unfortunately, a similar invariance of the results is not found at the lower masses, where a wide variety of evolutionary tracks is possible. Nevertheless, the most probable "zone of occupation" of the blue supergiants in the H-R diagram, and the fraction of time that supergiants spend burning core helium in this zone, are listed in Table 3 for the case of the Schwarzschild criterion; they have already been listed in Table 3 of Paper III for the case of the Ledoux criterion.

The predicted "zones of occupation" are plotted in Figure 7, in which are also plotted the individual supergiants actually observed in young clusters and associations of the Galaxy (Humphreys 1970). It is clear that the models based on the Schwarzschild criterion are insufficiently hot compared with the bulk of the blue supergiants, and that they predict far too many red supergiants for masses greater than $\sim 20\ M_{\odot}$, where the predictions are most secure. On the other hand, the models based on the Ledoux criterion reproduce the general trend (though not the full width) of the distribution of blue supergiants, but they also predict an unobserved excess of red supergiants at high stellar masses. A further observational test of the proper criterion for convection, based on members of evolved binary systems and discussed in § IV, is inconclusive at present. Other tests have been proposed, and some have been carried out (e.g., Stothers and Lloyd Evans 1970; Stothers 1975), but, as long as the basic test on the H-R diagram fails for both criteria for convection, the favoring of one criterion over the other by these other tests cannot be considered very meaningful at present. We conclude that something is fundamentally wrong with the stellar models.

TABLE 3

PARAMETERS OF THE STABLE BLUE PHASE DURING CORE HELIUM BURNING

Z_e	$M/M_{\odot}$	$\log T_e$ (b/y)	$\log T_e$ (tip)	$\tau_b/\tau_{\mathrm{He}}$
0.021.....	10	$\sim$3.9 ± 0.1	4.10 ± 0.10	0.7 ± 0.2
	15	$\sim$3.9 ± 0.1	4.20 ± 0.10	0.8 ± 0.2
	30	$\sim$4.0 ± 0.1	4.20 ± 0.05	0.9 ± 0.1
	60	3.75*	3.90 ± 0.05	0.5 ± 0.1
0.044.....	10	$\sim$3.9:	4.05:	0.7:
	15	$\sim$3.85 ± 0.05	4.15 ± 0.05	0.8 ± 0.2
	30	$\sim$3.80 ± 0.05	4.10 ± 0.05	0.8 ± 0.2
	60	3.75*	3.85 ± 0.05	0.6 ± 0.1

* Adopted value, because the speed of evolution through effective temperatures cooler than $\log T_e$ (tip) is approximately uniform.

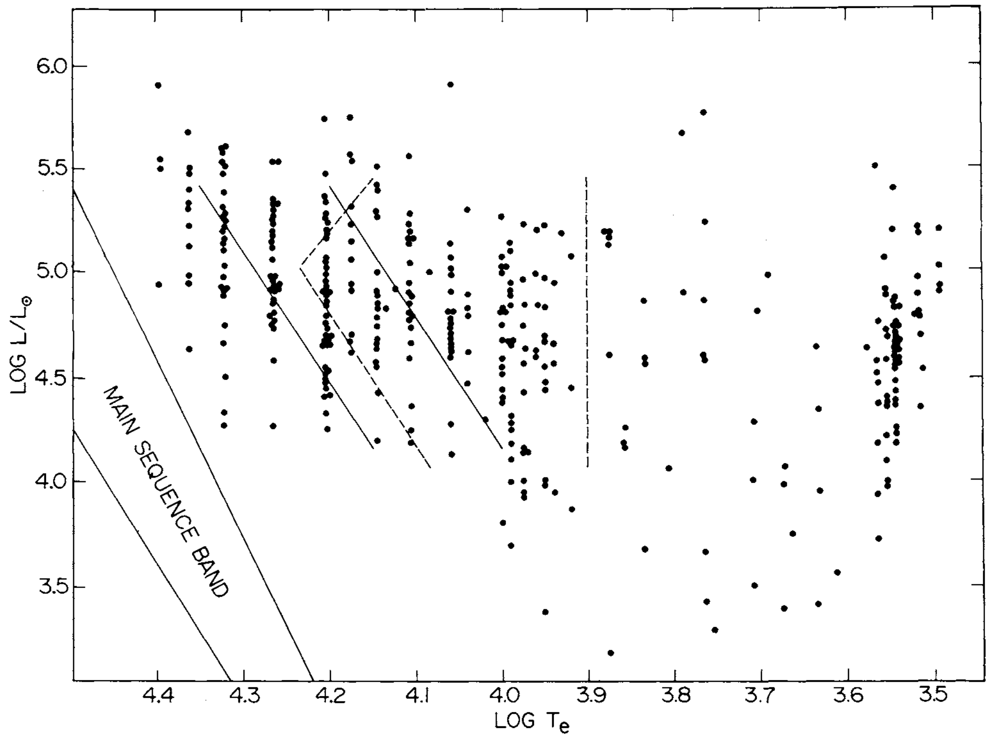

FIG. 7.—Theoretical H-R diagram showing the approximate boundaries of the "zone of occupation" for blue supergiants as predicted from the Schwarzschild criterion (*dashed lines*) and from the Ledoux criterion (*solid lines*). The observed supergiants (*dots*) are taken from the work of Humphreys. The theoretical main-sequence band is also indicated.

Perhaps the extent of convective overshooting has been underestimated, but we rather suspect that the stellar opacities and the neglect of mass loss are at fault.

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