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THE SECONDARY COMPONENT OF BETA LYRAE

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ABSTRACT

Quantitative analysis of β Lyr light curves shows that thin-disk and "no-disk" models for the secondary can be ruled out, leaving (presumably) only a thick-disk model. Some common assertions about the asymmetry and repetition of the light curves are not supported by examination of the existing observations. There is no evidence that the secondary component is underluminous for its mass, provided the observations are interpreted in terms of a thick secondary disk, most of whose luminosity emerges at the poles. The polar effective temperature of the disk is several thousand degrees K higher than that of the primary component, and is therefore the obvious candidate for the elusive source of excitation for the B2–B5 shell spectrum. Because the secondary is not underluminous, there is, at present, no reason to postulate the presence of a collapsed star at its center. Reasons are given to explain why the secondary spectrum has not been seen, despite the fact that the observed continuum flux from the secondary is of the order of one-half that from the primary. Estimates of this flux in previous work were much lower. The mass ratio, $\mathfrak{M}_2/\mathfrak{M}_1$, is estimated to be 4.2 (+3.0, −1.8) if the chemical abundances found by Boyarchuk are adopted, or about 6, with similar uncertainty, if abundances by Hack and Job are used. It is argued that the disk must be in strongly differential rotation. Future work on modeling the system might well assume the basic semitoroidal geometry of the Bodenheimer-Ostriker differentially rotating model stars. Most of the secondary mass probably lies in an embedded main-sequence star, rather than in the disk, which is probably of the order of one, or perhaps a few, solar masses.

Subject headings: eclipsing binaries — stars, individual

I. INTRODUCTION

Our present understanding of the unusual binary β Lyrae is greatly improved over that of 10 to 20 years ago, due largely to intensive work by Struve, Sahade, Huang, and many others on β Lyr itself, as well as to general advances in the theory of binary star evolution made by Kippenhahn, Paczyński, Plavec, and their co-workers. Whereas a mass ratio,[1] $\mathfrak{M}_2/\mathfrak{M}_1$, of 0.5–0.7 was given serious consideration 15 years ago (Struve 1958), thus dictating individual masses of, say, 75 and 50 $\mathfrak{M}_\odot$, virtually all recent work agrees that $\mathfrak{M}_2 > \mathfrak{M}_1$, which gives much smaller masses. Undoubtedly the major turning point for understanding the system was Huang's (1963) disk model for the secondary component, which overcame serious inconsistencies in previous interpretations. These interpretations had pictured the secondary as a relatively ordinary star, typically smaller than the B8.5 II primary and of about spectral class F. It is now generally agreed that β Lyr is nearing the end of the rapid phase of mass transfer and that the peculiar appearance of the secondary is due to its departure from equilibrium as it assimilates the large mass flux (presently $\sim 3 \times 10^{-5}\,\mathfrak{M}_\odot$ per year) from the primary. This interpretation is reinforced by the hydrogen depletion and helium overabundance (e.g., Boyarchuk 1959; Hack and Job 1965) of the primary, which indicates an example of case B mass exchange (primary reaches its Roche lobe during the shell hydrogen burning stage). That is, present computations of conservative mass transfer predict that the helium-enriched core of the mass-giving component is uncovered in case B mass exchange, but not in case A (Roche lobe reached during the first slow evolution from the zero-age main sequence [ZAMS] during core hydrogen burning).

Nevertheless, basic questions remain. Because of the importance of β Lyr as an example of a very short-lived evolutionary phase, we should like to know the present mass ratio, and thus the absolute masses, with some confidence. Because of the secondary's importance as an example of a nonequilibrium object assimilating matter on a rapid time scale, we should like to know as much as possible about its structure. Finally, we can expect such structure information to relate to the recent suggestions (Devinney 1971; Wilson 1971; Kondo, McCluskey, and Houck 1971) that a black hole may be found at the center of the secondary disk. The central observational question which concerns the black-hole possibility is that of the underluminosity of the secondary component. This question is related indirectly to two others: (*a*) Why is there so little evidence of the absorption spectrum of the secondary component despite the appreciable secondary eclipse revealed by the photometry? (*b*) What source excites the "B5" absorption spectrum (e.g., Böhm-Vitense 1954) in view of the fact that the primary is no earlier than class B8? In this paper we offer answers to these questions based on an attempt to model the system. Although the model is still preliminary, it is considerably more quantitative in certain respects than previous investigations.

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[1] Throughout this paper we define component 1 to be the B8.5 II star which is prominent in the optical observations.

II. OBSERVATIONAL BACKGROUND

Beta Lyrae is certainly the most thoroughly observed binary; and, with the exception of infrared light curves, there exist good observations of almost every conceivable kind. In particular, a number of good optical light curves have been published (e.g., Larsson-Leander 1968; Wood and Walker 1960; Landis, Lovell, and Hall 1973), and these obviously have much to tell us about the system. However, interpretation of this photometry has been hampered by two problems: (1) The system is so far outside the framework of classical models for light-curve analysis that any attempt at direct use of such models for quantitative results is simply a waste of effort. Some of the simple relations developed for ellipsoidal components have yielded limited information, but it is clearly dangerous to press this approach very far. (2) Two misleading "facts" about the light variation seem to have gained fairly widespread acceptance, and have created the impression that the light curves are more complicated than is actually the case. These "facts" are that the primary eclipse is seriously asymmetric and that the depth differs as much as 0.3 mag from one series of observations to another. Scrutiny of the published light curves shows that these statements about the depth and asymmetry are true only if one focuses attention on observations within 0.03 of zero phase. That is, from phase 0.97 to 0.03 the light variation is erratic, and the mentioned "facts" have some degree of truth. However, it is quite obvious from the spectrograms published by Sahade *et al.* (1959) that this irregular behavior is caused by increased absorption in the Balmer lines (and, presumably, the Paschen continuum). Many absorption lines, including those of He I and Ca II in addition to hydrogen, increase enormously in strength at these phases, so without question the fluctuations near phase zero are caused by attenuation in circumstellar gas. Now in developing a model for the massive constituents of the binary, we certainly do not wish to be misled by these effects, which are produced by neither the primary nor the secondary components. Therefore, we must ask whether the light curve repeats well and is sensibly symmetric at phases away from mid-primary eclipse. Apart from a significant asymmetry which *sometimes* appears in secondary eclipse, it seems that the light variation is reasonably consistent and symmetric, with only infrequent departures greater than a few hundredths of a magnitude. In fact, Stebbins's (1916) light curve, which is often cited to support the asymmetry claim, is practically devoid of points on both the ascending and descending branches of primary eclipse, so it is difficult to see how this idea first arose. Until recently, a conspicuous counterexample to light-curve consistency was provided by the photometry by Wood and Walker (1960), which showed B-magnitudes and $B - V$ colors about 0.05 mag more negative than those of all other recent observers. However, Wood (1973) has again reduced these observations and now finds magnitudes and colors within 0.01–0.02 mag of the means found by other observers. In view of the unusual properties of β Lyr in many other respects, the minor irregularities of the light curves seem noteworthy for their relative normality rather than abnormality. This, in turn, suggests that it may be possible to develop a model which will be useful for a quantitative analysis of the light variation.

We emphasize the consistency of the light curve not only because the counterexamples are infrequent and fairly minor, but also because they are probably caused by absorption in material which is not part of the structure of either component. This view is basically different from that of Huang (1963), who used the lack of repetition, as referenced in a paper by Guthnick (1945–46), to establish that the secondary disk is semistable. Perhaps the disk is semistable, but that does not seem to be indicated by the observations we have examined. Our first premise, therefore, is that the secondary component (whether a star, disk, or combination thereof) is *not* so variable in form as to render a study of its properties at one particular epoch a matter of little interest.

III. THE MASS RATIO

It is important to know the mass ratio not only for the evolutionary reason mentioned earlier, but also to eliminate one major uncertainty in developing models for the binary. Unfortunately q $(=\mathfrak{M}_2/\mathfrak{M}_1)$ is still poorly known, although all quantitative methods (except a naïve application of the mass-luminosity law) yield $q > 1$. Several sets of spectral lines vary approximately 180° out of phase with the lines of component 1 (Sahade *et al.* 1959; Skulskii 1971), and it is not clear which, if any, of these lines should be used to infer the velocity amplitude of component 2. However, none of these systems indicate $\mathfrak{M}_2 < \mathfrak{M}_1$. If one assumes that the bright component just fills its Roche lobe and that it rotates in synchronism with the orbital motion, the rotational broadening of its lines (Mitchell 1954) indicates a very large q (Huang 1963), and one must assume quite a narrow intrinsic line profile to avoid mass ratios greater than 6, which were considered unrealistic a few years ago.

From the amplitude of the photometric ellipticity effect, Devinney (1971) has shown that $q > 2$. His method is essentially contained within that of § V below, in which the entire light curve is fitted, including the parts between eclipses, except that here a least-squares criterion is used and a best estimate of q (rather than a limiting value) is found. This gives a value of q between 4 and 5.

In this section we reapply a basic method used before by Huang (1962) and by Woolf (1965). The reason for resurrecting this approach is that it is now 10 years since the earlier applications, and several types of corrections can now be made, based on published model-atmosphere calculations as well as on the results of § V of this paper. Indeed, even with these corrections the resulting q is uncertain by more than 50 percent of its value, so it is not surprising that Huang and Woolf found different answers. Despite the uncertainty of this method, however, it is probably the most reliable one presently available for β Lyr, and might be further refined by computing a model atmosphere specifically for the B8.5 II component. Briefly the method is as follows. Our knowledge that component 1 fills its Roche lobe couples the geometry of the lobe to that of the orbit. The relative Roche geometry depends only on q and specifies the size of the lobe relative to the distance (a_1) from component 1 to the orbital center of mass. Since we know a_1 in kilometers from the spectroscopy, we know the absolute size of the lobe, and thus of component 1, for any chosen value of q. We therefore need only to enter the energy emitted per unit area of surface (determined essentially by the surface temperature) to calculate the absolute magnitude of the star. We therefore simply choose q such that the brightness so calculated agrees with the star's observed absolute magnitude. Of course, we must know the distance modulus to find the observed absolute magnitude.

While the method appears simple in principle, it requires in practice quite accurate observational parameters to yield a useful value of q. Huang applied the method bolometrically so that it contained an uncertainty due to the bolometric correction. That is, he worked with an estimated *effective* temperature, T_e, and converted the observed M_v to $M_{\rm bol}$. He found the mass ratio to lie between 2.5 and 3.4. Woolf applied the method essentially monochromatically so that he eliminated the bolometric correction, but naturally he had to convert T_e to T_b, the brightness temperature, so that this step contained a source of error. Of course, there now exist far better published data on stellar atmospheres, by which to make this conversion, than were available to Woolf. His value for q was 5.7 ± 0.7.

The specific steps followed here are different from those by Huang and by Woolf; but the method is, in essence, the same. The binary-star light-curve computer program described by Wilson and Devinney (1971) was used as an aid in these computations because it accounts automatically for geometric and photometric ellipticity and other such effects. A fictitious binary system was generated in which one component was the B8.5 II star in β Lyr, while the other component was the Sun. One can easily show that the relative radius of the Sun, imagined to be in the β Lyr system, is given by

$$r_\odot/a = 0.0211q/(1 + q)\,, \qquad (1)$$

where the numerical coefficient is $r_\odot/a_1$. When r/a is small, as in this case, the Roche modified potential needed to produce this value of $r_\odot/a$ is given by

$$\Omega_\odot = 1 + qa/r - (q - 1)/2\,,$$

which is a special case of the general equation given by Kopal (1959). The procedure, stated simply, was to compute the V-system flux ratio of these components (β Lyr primary and the Sun) at orbital phase 0.25 for values of q ranging from 2.0 to 6.0. For each q, Ω was chosen such that the B8.5 star just filled its Roche lobe, and for all cases the following parameter

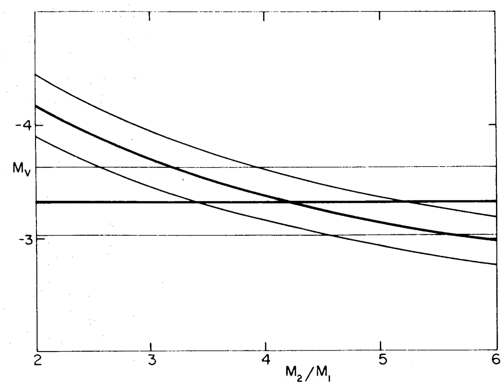

FIG. 1.—Graph for finding the mass ratio from the Roche-lobe geometry (Boyarchuk abundances). The (*heavy*) horizontal line shows the observed M_v, with estimated uncertainty (*lighter parallel lines*). The (*heavy*) curve is the theoretical relation, with estimated uncertainty (*lighter parallel curves*). See text for effect of using Hack and Job abundances.

values were inserted: $i = 85^\circ.00$, T_1(pole) = 11,250° K, $T_\odot = 5920°$ K, $x_1 = 0.50$, $x_2 = 0.60$. This provided values of $M_v(\beta$ Lyr) $- M_v$(Sun) and therefore $M_v(\beta$ Lyr) since M_v(Sun) is known. The adopted value of M_v(Sun) is +4.84 mag (Morton and Adams 1968). In figure 1 these M_v's are compared graphically with the observed value of M_v, which comes mainly from the work of Abt *et al.* (1962), who determined the H-R diagram of the visual companions of β Lyr. We assume, as did Abt *et al.*, that companions HD 174664 and BD+33°3225 are on the ZAMS (cf. discussion below). Since the magnitude difference between these two stars is quite in agreement with this assumption, we can work with either one to find the M_v of the eclipsing pair. The V-magnitude of HD 174664 is 7.22 mag (Abt *et al.* 1962), and we estimate M_v to be +0.10 mag from a consensus of the ZAMS of Sandage (1957), Blaauw (1963), and Lloyd Evans (1972). The spectrum has been classified as B6 (Slettebak 1963) and B7 (Abt *et al.* 1962), so we adopt B6.5. The value of $M_v - V$ is therefore -7.12 mag; and M_v for the eclipsing pair, whose V-magnitude at maximum light is close to +3.40 mag (Larsson-Leander 1968; Landis *et al.* 1973; Wood 1973), becomes $M_v = -3.72$ mag. We correct this by +0.40 mag (see table 3 and point 6 below) for the light of the secondary to find $M_v = -3.32$ mag for the B8.5 II primary star.

We next list some of the main corrections to the curved (theoretical) and the horizontal (observed) lines of figure 1, whose intersection determines q.

1. Since the Wilson-Devinney program treats the components as blackbodies, the temperatures required are *brightness* temperatures, but published calibrations are given in terms of *effective* temperature. Corrections were made by interpolating among the Carbon-Gingerich (1969) atmospheres. It was found that $T_b - T_e \approx -380°$ K for β Lyr and +120° K for the Sun.

2. The program uses polar temperatures as input, but the observational data (spectral type, color index)

provide a *representative* temperature over the stellar disk. This was corrected approximately by computing the temperature distribution over the surface of the B8.5 star. The polar temperature is of the order of 500° K higher than the mean disk temperature.

3. $(B - V)_0$ is uncertain by perhaps ± 0.03 mag, mostly because of the small but significant interstellar reddening, which is important when finding effective temperatures for hot stars. $(U - B)_0$ is modified too strongly by abundance effects to provide much useful information on the temperature. We adopt the color excess given by Abt *et al.* (1962), $E_y = 0.065$ mag.

4. The star is a giant, and calibrations of T_e versus $B - V$ or spectral type exist only for main-sequence stars. Fortunately, graphs by Böhm-Vitense (1967) enable one to make this correction, which is approximately $+0.03$ mag in $B - V$, relative to a main-sequence star of the same $T_e(\log g = 2.6)$.

5. The chemical composition is abnormal. According to Boyarchuk (1959), hydrogen is about 5 times underabundant. Again the paper by Böhm-Vitense permits a correction. According to her graph, a star in this temperature range, with hydrogen 5 times underabundant, should be 0.03 mag bluer than one with normal composition. Thus corrections (4) and (5) virtually cancel for β Lyr. The computations for figure 1 were made for the Boyarchuk composition. However, Hack and Job (1965) find a much smaller hydrogen depletion and a somewhat smaller helium overabundance. Had we used the Hack and Job results, the correction to the $(B - V, T_e)$-relation (for abnormal abundances) would have been negligible.

6. Abt *et al.* applied a correction of 0.1 mag to convert from the observed M_v of the binary pair to that of component 1 alone. Here we must draw on a result which depends on the present model. In § V we find that the secondary contributes considerably more to the continuum flux of the binary than has generally been believed. Accordingly, the correction to M_v should be approximately 0.4 mag rather than 0.1 mag.

In finding M_v, Abt *et al.* argued that HD 174664 (B6,7 V) is not sensibly evolved from the ZAMS because it is 4 mag fainter than β Lyr, which could not have left the termination of the main sequence very long ago. However, because of the large mass transfer, it is not immediately clear on which part of the main sequence the β Lyr primary originated. Since the main sequence is of the order of 1 mag wide at type B, it is not sufficient for our purposes merely to know that HD 174664 is of luminosity class V. We must establish that it is on or very close to the ZAMS. Otherwise the resulting uncertainty in its absolute visual magnitude will render q hopelessly indeterminate by this method. However, unpublished tables of evolutionary model sequences by Stothers provide also the relation between the total masses of evolved stars and the masses of their helium cores, which is just the type object we are dealing with in the β Lyr primary (i.e., we know that the original helium core, before mass transfer, was at least as massive as the object we see now). For $q \leq 6$, the absolute mass of component 1 is at least 2 $\mathfrak{M}_\odot$ (core mass). Thus the total mass, before mass transfer, for component 1 must have been at least 13 $\mathfrak{M}_\odot$, which corresponds to a main-sequence lifetime (Stothers 1972*a*) of about 1×10^7 years or less. In this time, HD 174664 ($\sim 4.5\,\mathfrak{M}_\odot$) could not have evolved more than 0.1 mag from the ZAMS.

The visual companion BD + 33°3225 is probably a physical member of the system according to Abt *et al.* (1962). The pre-main-sequence contraction lifetime for this star (A8–9 V, $\sim 1.7\,\mathfrak{M}_\odot$, $\sim 6.5\,L_\odot$) sets a lower limit to the age of the system of roughly

$$\tau = 3 \times 10^7 (\mathfrak{M}/\mathfrak{M}_\odot)(L_\odot/L) \approx 1 \times 10^7 \text{ years} .$$

This lower limit sets an upper limit to the original mass of component 1 of about 13 $\mathfrak{M}_\odot$ and thus also an upper limit to its present (helium core) mass of roughly 2 $\mathfrak{M}_\odot$, which suggests that $q \geq 6$.

To summarize, the polar brightness temperature of 11,250° K was found by increasing the mean disk temperature of 10,750° K by 500° K to allow for gravity darkening. The mean disk brightness temperature was found by correcting the mean disk *effective* temperature of 11,130° K by $-380°$ K for departures from Planckian emissivity (i.e., by comparison of blackbodies with stellar atmospheres). The mean disk effective temperature was taken from the Morton-Adams (1968) calibration for a $(B - V)_0$ of -0.07 mag. The $(B - V)_0$ was found by correcting the mean (-0.005 mag) of several observed $B - V$ values (Larsson-Leander 1968; Landis *et al.* 1973; Iriarte *et al.* 1965; Johnson *et al.* 1966) for a color excess, E_y, of -0.065 mag, as estimated by Abt *et al.* (1962). The adopted value of $(B - V)_0$ includes canceling corrections of $+0.03$ mag, which accounts for the star's luminosity class, and -0.03 mag, which accounts for its abnormal chemical composition. If one adopts the Hack and Job (1965) abundances, the last correction for $B - V$ is eliminated and all temperatures are about 1400° K higher.

The intersection shown in figure 1 determines a q-value of 4.2 (+3.0, −1.8) if the Boyarchuk (1959) abundances are assumed; or about 6, with similar uncertainty, if the Hack and Job (1965) abundances are assumed. Each relation is shown as a band whose width has been estimated somewhat arbitrarily. However, the reader can easily reassign these widths if he so desires and thus find his own estimates of the

TABLE 1

MASSES COMPATIBLE WITH THE MASS FUNCTION

$\mathfrak{M}_2/\mathfrak{M}_1$	$\mathfrak{M}_1/\mathfrak{M}_\odot$	$\mathfrak{M}_2/\mathfrak{M}_\odot$
2.0	9.7	19.4
3.0	5.1	15.3
4.0	3.4	13.4
5.0	2.5	12.4
6.0	1.9	11.7
7.0	1.6	11.2

TABLE 2

SOME MASS-RATIO DETERMINATIONS

$\mathfrak{M}_2/\mathfrak{M}_1$	Source	Reference
~6	V_r of H and He I emission peaks	Sahade *et al.* 1959
2.9 ± 0.5	Roche-lobe geometry	Huang 1962
5.7 ± 0.7	Roche-lobe geometry	Woolf 1965
≥2	Ellipticity effect	Devinney 1971
1.7	V_r of Ca II absorption line	Skulskii 1971
4.2 (+3.0, −1.8)	Roche-lobe geometry, Boyarchuk abundances	Wilson, this paper
~6	Roche-lobe geometry, Hack and Job abundances	Wilson, this paper
4–5	Least-squares fits to light curve	Wilson, this paper
≥6	Evolutionary lifetimes	Stothers, this paper
>6	Rotational line broadening	Huang 1963; Mitchell 1954

uncertainty in q. The widths allowed here were ± 0.3 mag in the observed M_v and, for the theoretical relation, that width which corresponds to $\pm 1000°$ K in T_b. Absolute masses can be extracted from table 1. In table 2 are listed values of q found by several methods.

IV. MODELS FOR THE SECONDARY COMPONENT

The Huang (1963) disk model provides the basic point of departure for quantitative study of the β Lyr secondary. Should any question remain that the secondary might be an underluminous star (without disk), it can be stated that fairly thorough attempts with the Wilson-Devinney light-curve program to satisfy the photometry were completely unsuccessful. A flattened geometry is the only one permitted by the observed light curves. In fact another point, not noted previously, favors a flattened geometry for the secondary. Since the primary eclipse is much deeper than secondary, and the components are close, one should

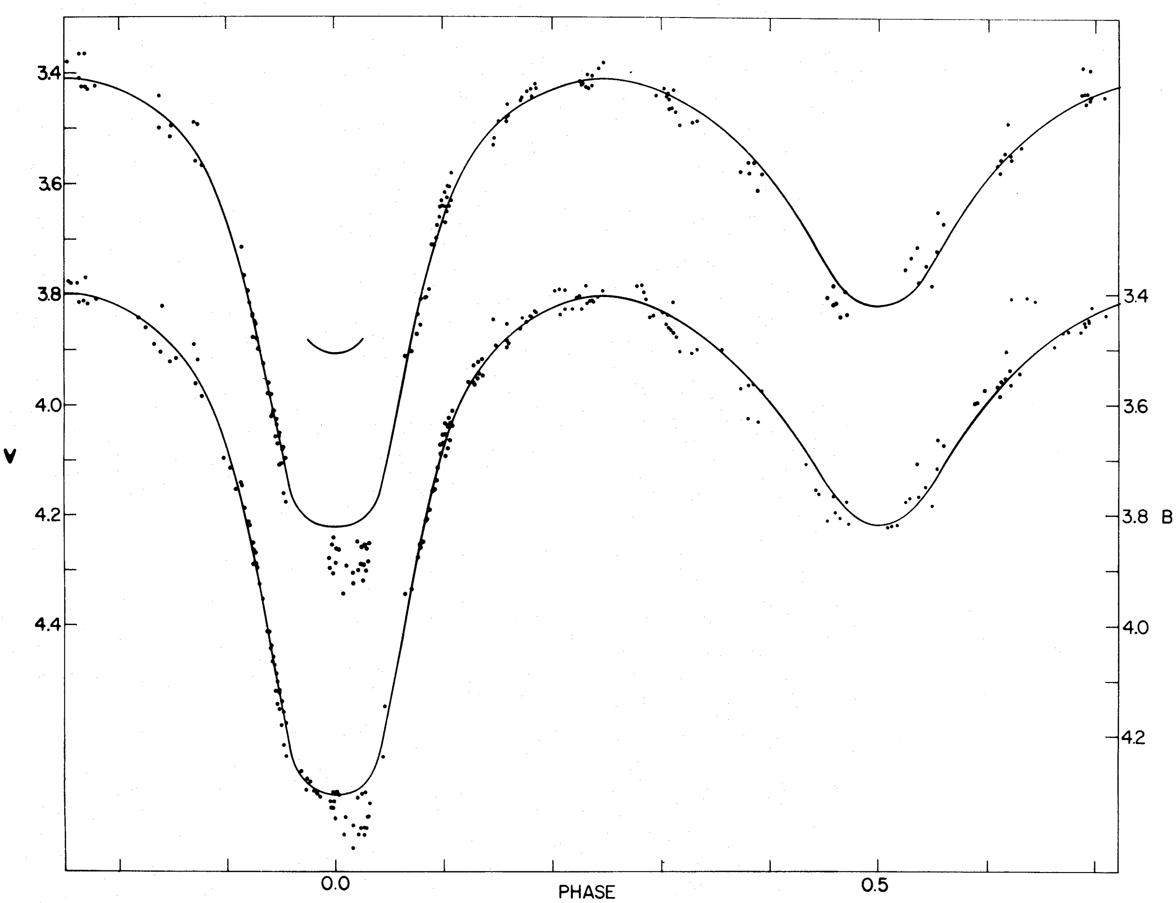

FIG. 2.—V (*above*) and B (*below*) light curves from the 1959 International β Lyr Campaign, and best-fitting model curves for a mass ratio of 4. The points near zero phase were excluded from the fitting process for reasons given in the text. The short arc near (phase zero, $V = 3.9$) shows the greatest possible eclipse depth for a thin disk model.

expect quite a large differential reflection effect (approximately 0.10 mag semiamplitude in the usual $\cos\theta$ term). In fact, it is somewhat unclear whether *any* differential reflection effect can be detected among the several good light curves. The model of this paper requires a semiamplitude for the $\cos\theta$ differential reflection term of only 0.015 mag. Although it *should* be possible to measure an effect of this size, it is small enough to be masked by fairly minor transients in the light curve.

Figure 1 in Huang's paper showed a disk with a fairly small ($< 0.05a$) but nonnegligible thickness, with a central bulge of radius roughly $0.1a$. Naturally we must understand this figure to be schematic, but it is important to find the quantitative dimensions of the disk if we hope to understand its structure and evolution. Of course, a straightforward classical approach with a Russell model is pointless, and we mention it in passing only because it has actually been tried, even recently. One might try to modify the Russell or Kopal rectification theory to allow for the flattening of the secondary, but clearly standard rectification theory is not amenable to such major changes and, in fact, already has had difficulties in treating many ordinary close binaries. The present approach, therefore, is that of direct computation of theoretical light curves for comparison with observations.

The primary component is treated as a tidally distorted star which shows gravity- and limb-darkening, as well as the reflection effect. The model, in regard to this component, does not differ from that in the general close-binary light-curve program (Wilson and Devinney 1971; Wilson *et al.* 1972). In view of the present rudimentary understanding of the secondary component, the model for the "disk" must be simple yet potentially informative. Because of the extreme proximity of the components, the "disk" must have symmetry about the orbit plane. At this point we encounter the basic question of whether the disk is (geometrically) thin or thick. Proceeding on the first hypothesis, we wrote a program which computed light curves for the case that the disk is thin and self-luminous. Extensive trials established that such a system would have a primary eclipse much less deep than that observed, even for optimally chosen mass ratio and inclination. The V-light curve of figure 2 includes a short arc at zero phase which shows the greatest possible depth for a primary eclipse caused by a *thin* disk with radius equal to the *largest* radius of the secondary Roche lobe. In fact, the radius of the disk really could not be that large, so the depth shown by the arc is decidedly overestimated, and still it is far less than that observed. The discrepancy is so large that it could not be accounted for by any additional eclipse effect due to a main-sequence star in the center of the disk. Therefore, either the disk has a considerable thickness or it has a central bulge (presumably a star) which is larger than main-sequence dimensions. If the latter explanation were correct, however, we should expect a marked change in the slope of the light curve at the phase where the bulge begins to eclipse the B8 star. Since there is no hint of such a change in slope in the observations, we conclude that the first alternative (a generally thick disk) must be the correct one.

A very simple model for this thick disk was chosen. A second program was written in which the secondary is an ellipsoid of revolution with equatorial radius R_{eq} and polar radius R_{pole}. Horizon and eclipse effects as well as the reflection effect, limb darkening, and other projection effects are treated in the same way as in the basic Wilson-Devinney program. From the presence of a secondary eclipse we know that component 2 is self-luminous, and the lack of a noticeable reflection effect indicates that this energy originates in the secondary component. Therefore, we must postulate some law by which the emitted energy is distributed over the surface of the ellipsoid. Since we have little knowledge of the internal structure of the disk, we should select a very simple law. One such assumption is that the luminosity suffers essentially a geometrical dilution on the way from center to surface, so that the bolometric surface brightness obeys an inverse square law. Since T_e is proportional to (local flux)$^{0.25}$ and the local flux is proportional to R^{-2}, local effective temperatures on the secondary will be given by

$$T_e = T_e(\mathrm{pole})/(R/R_{\mathrm{pole}})^{1/2} .$$

It would be surprising if this law is grossly incorrect—for example, von Zeipel gravity darkening on a rotating star behaves very nearly in this way—so an inverse square law was built into the program. The local monochromatic surface flux is determined by the Planck function at a local effective temperature determined by the Stefan law, as in the case of the primary component. The local fluxes so computed are required to sum to the 4π steradian luminosity of the disk. The local fluxes, and thus also the 4π luminosity and line-of-sight flux, are coupled to the polar effective temperature, so we are now in a position to find quantitative values for all of these interesting quantities. Throughout this paper an important distinction is made between *luminosity* and *observed flux*. For many stars one can discuss the observed (monochromatic) flux as if it were the object's (monochromatic) luminosity since, at a fixed distance, these quantities are related essentially by a multiplicative constant. However, this is not the case for β Lyr (especially the secondary) where strong aspect factors also enter. In fact, as shall be shown, the long-standing puzzle of the secondary's underluminosity can be resolved if one makes this distinction clearly and consistently.

It is important to point out that the derived luminosity of the secondary, which is one of the main quantitative results of this study, depends strongly on both the assumed law of surface flux distribution and the law of limb darkening (here the linear cosine law, $I/I_0 = 1 - x + x\cos\gamma$). The same statement can be made about the polar effective temperature of the secondary, so the numerical results for both these quantities (table 3) must be viewed in the context of these uncertainties.

TABLE 3

ADJUSTED PARAMETERS OF β LYRAE MODEL

Parameter	$q = 2$, $\Omega_1 = 5.2517$	$q = 3$, $\Omega_1 = 6.6163$	$q = 4$, $\Omega_1 = 7.9116$	$q = 5$, $\Omega_1 = 9.1636$	$q = 6$, $\Omega_1 = 10.3855$
i	90°	89°	85°	85°	85°
T_2 (pole) (°K)	11,000	12,400	13,000	13,900	14,000
r_2/a (eq)	0.52	0.54	0.56	0.57	0.57
r_2/a (pole)	0.21	0.16	0.17	0.15	0.15
$L_2/(L_1 + L_2)$ [V]	0.43	0.50	0.59	0.64	0.66
$L_2/(L_1 + L_2)$ [B]	0.39	0.47	0.57	0.62	0.65
$F_2/(F_1 + F_2)_{0.25}$ [V]	0.23	0.22	0.31	0.32	0.34
$\sum wr^2$	0.0858	0.0793	0.0732	0.0746	0.0756
Radii of Primary Component					
r_1/a (pole)	0.300	0.269	0.248	0.233	0.221
r_1/a (point)	0.429	0.389	0.362	0.341	0.325
r_1/a (side)	0.313	0.280	0.258	0.242	0.230
r_1/a (back)	0.345	0.313	0.291	0.275	0.262

Fixed Parameters

$g_1 = 1.00$	$x_1[V] = 0.45$
T_1(pole) $= 11{,}300$° K	$x_1[B] = 0.50$
$A_1 = 0.20$	$x_2[V] = 0.50$
$A_2 = 0.00$	$x_2[B] = 0.60$

V. MAJOR RESULTS AND INFERENCES

The B, V observations of the 1959 International β Lyrae campaign (Larsson-Leander 1968) were used for comparison with the model of § IV. As mentioned in § II, those within phase 0.03 of mid-primary eclipse show erratic variation due to circumstellar absorption, and so were deleted for purposes of fitting the light curve. It quickly became obvious that trial-and-error fitting procedures, although capable of satisfying the observations, would never yield results free of serious personal bias. Therefore, the fitting was accomplished by the method of least squares. A differential corrections program used previously for several other binaries (e.g., Wilson and Devinney 1971) required only some minor alterations in order to be used for this purpose. Separate iterated adjustments were made for each of the five mass ratios 2.0, 3.0, 4.0, 5.0, and 6.0. For each mass ratio the Roche "potential," Ω_1, was chosen so that component 1 just filled its lobe. The bolometric albedos, A_1 and A_2, deserve some comment. Although we expect the true heat albedo of the primary to be unity, a value of 0.20 was estimated *a priori* for A_1 because the primary will "see" the flattened secondary as a heat source of low apparent efficiency. This is because most of the secondary

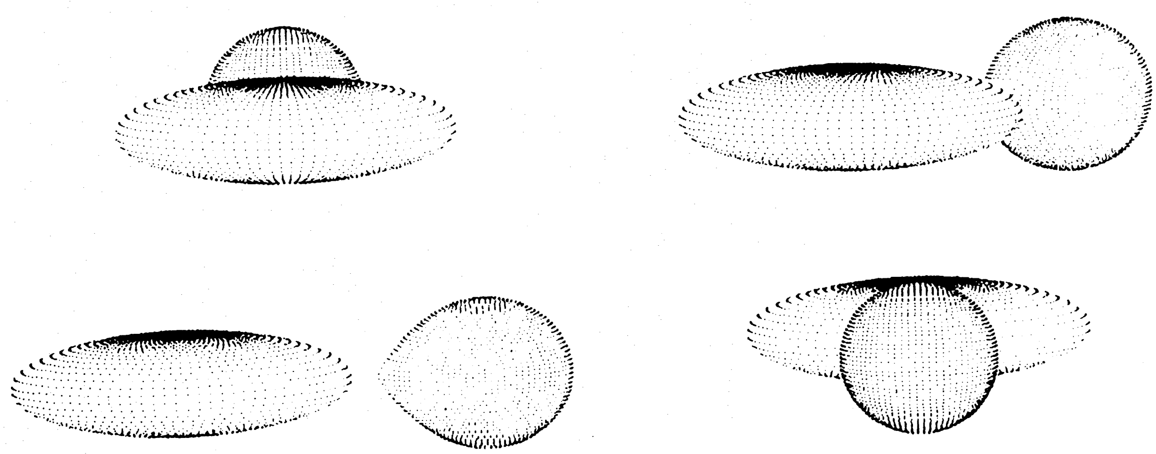

FIG. 3.—Computer-generated "pictures" of the final adjusted model for a mass ratio of 4 at phases 0.00 (*upper left*), 0.125 (*upper right*), 0.25 (*lower left*), and 0.50 (*lower right*). The projected grid points have been made equally black (i.e., there is no attempt at a gray scale). The blackening near the poles of component 2 does not, therefore, portray the high intensity of radiation near the poles, although, by coincidence, it may be used to visualize the polar brightening.

luminosity emerges at the poles. When the adjustments were completed, it became known [via the $F_2/(F_1 + F_2)$ and $L_2/(L_1 + L_2)$ entries in table 3] that the value $A_1 = 0.20$ was, in fact, a fairly good estimate. A_2 was set equal to 0.00 because the 1959 photometry shows no differential reflection effect whatever, unless it is one with the opposite sign (+) to that expected (−) in theory. It should be noted, however, that the 1958 photometry (Wood and Walker 1960) shows a very small differential reflection effect of the expected sign. The results of the differential corrections solutions are given in table 3. Figure 3 shows "pictures" of the system at several phases for the case $q = 4$. This figure was generated by the program from the surface grid points used in the numerical flux integrations.

Although the program computes formal probable errors, these are not listed, as they would be misleading for the following reason. Since the preliminary nature of the model justifies only coarse surface grids (for numerical integration of the light of the binary), the iterative differential corrections process cannot be expected to converge to extremely small corrections. In fact, parameter differences from one iteration to the next were always at least several times larger than the probable errors of the parameters. Therefore, uncertainties in the numbers in table 3 can best be judged by their roughness as functions of q.

The thin-disk experiments (§ IV) showed that the secondary object, although flattened, must have quite an appreciable thickness. Given that the disk is somewhat thick, table 3 provides an impersonal answer to the question "how thick?" Of course, the ellipsoid model, with inverse-square surface-brightness distribution, is quite simplified; but one can scarcely doubt that it provides the correct approximate dimensions of the secondary. That is, it seems safe to say that the secondary is a flattened object with the ratio of dimensions being roughly 3 to 1. The careful reader will notice that the equatorial radius of the disk tends to be about 10 percent larger than the "side" radius of the secondary Roche lobe. It will later be shown that this formal result (which should not be accepted uncritically) has a strong bearing on the true physical nature of the secondary.

The next interesting inference to be drawn from table 3 is that, contrary to all past ideas, the secondary probably equals or exceeds the primary in luminosity! In fact, we see that for $q = 5$ and $q = 6$, L_2 is about twice L_1. If this seems incompatible with the secondary's lack of optical prominence, we must remember that the program integrates the surface flux to find the 4π *steradian* luminosity of each component, and therefore takes due account of the fact that most of the secondary's energy is emitted in directions away from the orbit plane (and thus away from the observer, who is near the orbit plane). This point helps explain the much discussed underluminosity of component 2. In earlier work, it has been a tacit assumption that the ratio of *observed flux* to *intrinsic luminosity* is the same for both components; however, the computations show that this ratio is only about 0.25–0.30 as large for component 2 as for component 1 (both observed at phase 0.25). This corresponds to an *apparent* underluminosity of about 1.4 mag which is not real but only a consequence of our position near the orbit plane. The present computer results *further* show that even the directly observed flux from the secondary has probably been underestimated in most earlier work. The entry $[F_2/(F_1 + F_2)]_{0.25}$ in table 3 gives the ratio of the monochromatic fluxes from the components as observed at phase 0.25 (or 0.75). This ratio is not highly sensitive to the assumed brightness distribution law, as is the luminosity ratio, and requires an explanation as to why so little spectroscopic evidence of the disk is seen if it provides so much light. Such an explanation will be offered later in the section. For now the key matter is that actually two points have contributed to the supposed underluminosity. They are: (*a*) most of the luminosity of component 2 is emitted in directions other than that of the observer; and (*b*) the flux actually received from component 2 has been underestimated.

Stothers (1972*b*) has pointed out that a good part of the underluminosity problem can be accounted for simply by realizing that component 1 should have nearly the luminosity of a pure helium star of the same mass, since it is certainly the helium core of an evolved star which has lost most of its hydrogen envelope. The small remnant hydrogen envelope results in a lower T_e than one expects for such a helium star. Thus part of the supposed underluminosity of the secondary is due to the "overluminosity" of the primary. Stothers's table 2 lists the remaining underluminosity, δM_{bol}, of the secondary which is not explained by this idea. Two lines from Stothers's table are shown in table 4 of this paper. We see that, within the range of q considered here, the secondary is underluminous by about 1–4 mag, even after accounting for the "overluminosity" of the primary (in Stothers's complete table, δM_{bol} is a monotonic function of $\mathfrak{M}_2/\mathfrak{M}_1$, although this is not obvious from the abstracted two lines). Four additional columns have been added to table 4, which are the corrections

TABLE 4

UNDERLUMINOSITY VERSUS MASS RATIO

$\mathfrak{M}_2/\mathfrak{M}_1$	Stothers's $\delta(M_{\mathrm{bol}})_2$ (mag)	Correction for Flux Underestimate (mag)	Correction for Luminosity Underestimate (mag)	B.C. (secondary) (mag)	Corrected $\delta(M_{\mathrm{bol}})_2$ (mag)
2.5..........	+1.4	−0.7	−1.2	−0.7	−1.2
6.0..........	+3.8	−1.3	−1.4	−0.7	+0.4

for points (*a*) and (*b*) above, a bolometric correction for the secondary (which Stothers neglected since at that time T_e for the secondary was thought to be much lower than our result) and the corrected $\delta M_{\rm bol}$. For the sake of brevity, let us call point (*a*) the luminosity underestimate and point (*b*) the flux underestimate. After correcting for the luminosity and flux underestimates; and for $(\mathrm{B.C.})_2$ which are all in the same sense, we find that $\delta M_{\rm bol}$ is within 1 mag of zero at all points within the range of reasonable values of q. Now, of course, this is not a very precise result, but the main point is the following: within our ability to measure it, the underluminosity of the secondary is *zero*.[2] In finding this result, it was not even necessary to invoke magnetic fields, which Stothers (1972*b*) considered and dismissed, or differential rotation of the underlying star, which Stothers and Lucy (1972) considered a likely explanation. Having considered only the basic observational material and noting that component 1 should show the luminosity of a star with its expected helium content, we find the secondary to have about the normal energy output for its mass. This finding eliminates the basic argument (underluminosity) advanced by Devinney (1971), who postulated that a black hole may be located in the center of the disk. If it becomes firmly established that a structure approximately of the type here proposed is, in fact, present (the only alternative is a torus, which would have a very short Kelvin contraction time before becoming thin), one could then actually rule out a black hole. A "thick disk" of the present type, whether approximately ellipsoidal or not, must be supported by a pressure force which must extend to its center—that is, to the hypothetical black hole. There is then no way for the black hole to avoid accreting material at the maximum rate set by the Eddington limit and generating the Eddington luminosity of $10^{38}\,\mathfrak{M}/\mathfrak{M}_\odot$ ergs s^{-1}, or $2.5 \times 10^5\,L_\odot$ for a $10\,\mathfrak{M}_\odot$ black hole. Since the luminosity of the secondary certainly cannot exceed roughly $3 \times 10^4\,L_\odot$, no black hole can be present unless its mass is of the order of $1\,\mathfrak{M}_\odot$ or less.

It has been a major puzzle to identify the source of excitation for the B2 to B5 absorption line spectrum, which has come to be known as the shell spectrum, following Struve's (1941) proposal of a shell around the entire system. Now we see, consulting table 3, that there does indeed exist in the system a source of continuum radiation which is considerably hotter than the B8.5 component. The source is the bright polar regions of the secondary, whose temperature is estimated to be 2000°–3000° K higher than that of the primary star. In fact, if our guess as to the proper form for the distribution law of surface flux (analogous to gravity darkening in a normal star) is inadequate, we may have underestimated the polar temperature of the secondary. Of course, it is just as possible to have overestimated it, but the high excitation of the shell spectrum suggests the first possibility.

The question now arises as to why the direct absorption spectrum of the bright polar cap has not been seen, considering that its continuum radiation plays such a prominent role both in exciting circumstellar gas and in providing the secondary eclipse (see entry $F_2/[F_1 + F_2]$ in table 3). However, it is well known that lines formed in true absorption processes weaken greatly toward the limb of a star, at least under conditions which approximate local thermodynamic equilibrium (LTE). Examples of center-to-limb variation in the profiles of lines have been published by Mihalas and Auer (1970) for LTE and non-LTE theory. In the LTE case, the lines virtually disappear at the limb. In the non-LTE case a deep, narrow core remains, while the wings are greatly weakened. Despite this sharp, black core, rotational broadening should cause an observed line at the limb to be much weaker, even in the line center, than at the center of the disk. Figure 2 of Mihalas and Auer shows this effect for several rotational velocities. It is difficult to estimate the effective rotational velocity for the polar regions of the secondary, since we are not certain whether or not it is in differential rotation. However, under the reasonable assumption of differential rotation (cf. discussion below) with angular velocity increasing toward the center, linear velocities of several hundred km s^{-1} over the polar regions would seem conservative. In summary, one can list three reasons why the line spectrum of the secondary's bright polar cap should be very weak even though its continuum flux may be as much as about half that of the primary. The first of these is obvious and was not mentioned above.

a) Because of the very large variation in effective temperature over its surface the secondary is, in effect, a composite radiation source and should show a composite spectrum. Each region (e.g., polar and equatorial) therefore contributes only partly to the total received flux and its characteristic lines will be less prominent in the overall spectrum than if that region contributed all of the secondary's light.

b) Since we are near the equatorial plane of the flattened secondary, we see the polar cap (which provides the majority of the light) at near grazing incidence. Absorption lines should "wash out" to a considerable extent, as discussed above, especially in conjunction with the expected rotational broadening.

c) If the object is in extreme differential rotation, the rotational broadening of lines formed near the poles will cause them to appear diffuse, and thus less easily observed, even without process (*b*).

We can now understand why a correction of 0.4 mag rather than 0.1 mag is appropriate in converting from

[2] In a recent preprint, Kříž (1974) also argues that the secondary is not underluminous, based partly on spectroscopic and partly on photometric evidence. A condensed version (Kříž 1973) of this paper has also appeared. Kříž concludes that component 1 is significantly more luminous than a pure helium star of the same mass; in fact, this is supported by evolutionary computations made recently by Ziółkowski (1973) specifically for β Lyr. The arguments bearing on the plausible values for L_1 are too extensive to be given here, but Stothers's conservative estimate would seem to lie at the lower end of the permissible range. Of course, this is a fine point when viewed in the context of prior thinking, since in no case does it any longer seem likely that component 2 is underluminous.

the M_v of the binary pair to that of component 1 alone (cf. § III). The above discussion shows that it is entirely reasonable that the continuum radiation of the secondary is fairly large, as shown by values of $F_2/[F_1 + F_2]$ in table 3, while spectroscopically the secondary is essentially invisible.

Incidentally, the author's (Wilson 1971) argument (based on the Orbiting Astronomical Observatory far-ultraviolet photometry) to the effect that the secondary must be a composite radiation source, is given independent support by the results of this paper. However, with this new information at hand, one would identify the "central blue component" as the hot polar region of the disk rather than with a thermal-bremsstrahlung accretion source surrounding a black hole. In fact, this possibility was suggested to the author in 1972 by J. P. Ostriker in private discussions, and it now appears to be borne out.

We now come to a point whose significance was briefly alluded to earlier. Table 3 shows that, for all q, the equatorial radius of the secondary is about 10 percent larger than the side radius of the secondary Roche lobe. Since a solution of the type given here (which mainly satisfies the eclipse geometry) will respond mostly to the "side" radii of the components, we cannot doubt that it yields dimensions for component 2 which are at least as large, and probably somewhat larger than, its Roche lobe. Even in the case of complete corotation this situation is physically impossible, for then the outer material of component 2 would form a common envelope extending around component 1. That is, we would have a contact binary, which is not the case. In fact, the situation is even more contradictory than this, because we know that the secondary object rotates faster than synchronously because of its flattening, and in this case the effective critical lobe is even somewhat smaller than the Roche lobe. Therefore, we cannot believe that the numbers given in table 3 for r_2(eq) can be accepted literally. The true values must be at least 10 percent smaller, and probably 15–20 percent smaller. However, we also cannot believe that this result is due to some transient perturbation of the light curve, since the light curve is fairly stable as discussed in § II, nor that it can be due to some defect in the differential corrections procedure, since independent solutions for five q-values yielded reasonably consistent results (also the DC program has been tested thoroughly for several years). The obvious source of the problem would seem to be in the naïvely simple model adopted for component 2 which, of course, was deliberately kept simple in this first attempt at quantitative analysis of the light variation. Consider now the problem faced by the automatic adjustment procedure in fitting the light curve while using a rotational ellipsoid model for the secondary. Basically it must choose the dimensions of the ellipsoid so as to produce sufficient area to give a primary eclipse of the observed depth, and also choose the longitudinal dimension so as to produce the observed eclipse durations. Now suppose the secondary in fact is not an ellipsoid but has the shape of the differentially rotating masses studied theoretically by Bodenheimer (1971) and by Bodenheimer and Ostriker (1970). Viewed from the side, the more extreme of these objects have profiles which differ considerably from those of ellipsoids, being similar to those of toroids. One can see from figure 5 of Bodenheimer and Ostriker that such objects have smaller equatorial radii, by about 25 percent, than the closest matching ellipsoids; and if $i \neq 90°$, they also have smaller equatorial radii than ellipsoids which produce the same eclipse durations. Therefore, the next step in modeling the system might involve a secondary component similar to the Bodenheimer-Ostriker models. Of course, it is not clear that one can accept these models literally since the time scale for reaching a stable angular-velocity distribution may be of the same order as the Kelvin contraction time scale (cf. Kippenhahn 1969), but a better alternative is lacking at present. Notice that to decide between the two kinds of figures (ellipsoidal or pseudo-toroidal) would be very difficult on the basis of the shapes of the eclipse curves alone, but inclusion of the constraint that the object not exceed its Roche lobe discriminates against the ellipsoid.

We propose the following working hypothesis, parts of which are not new, for future studies of the system. The secondary consists of a main-sequence star which is in the process of assimilating the large mass flux from the B8.5 star. The mass in the surrounding thick disk is at least a good fraction of a solar mass, and perhaps several solar masses, as shown by the stability of the disk which, in turn, is inferred from the reasonable constancy of the light curve. That is, the stability of the disk implies that it is a structure in at least quasi-hydrostatic equilibrium and must presently be in Kelvin contraction. The disk should be in differential rotation and has relative dimension, $R_{\text{eq}}/R_{\text{pole}}$, somewhat less than 3:1, so that it corresponds to the more extreme cases of rapid differential rotation treated by Bodenheimer and Ostriker. Whether the main-sequence star itself is in strongly differential rotation, as proposed by Stothers and Lucy (1972) to account for the "underluminosity problem" cannot be answered at present. However, it should be noted that the observational reason for postulating differential rotation for the star itself no longer exists since L_2 is normal within the uncertainty of measurement.

We can comment on the expected overluminosity due to the liberation of gravitational potential energy in the accretion process. Alternatively, this can be regarded as the Kelvin contraction energy of the disk. We have

$$L_{\text{grav}} = (G\mathfrak{M}/R)dm/dt\,, \tag{3}$$

where $\mathfrak{M}$ and R are the mass and radius of the underlying star and dm/dt is the present mass accretion rate. If L_2/L_1 is approximately 2 (table 3) and the bolometric correction for the secondary is roughly 1 mag (Morton and Adams 1968), then L_2 is about $10^4 L_\odot$. We have assumed the embedded mass to be a main-sequence star, so we substitute the approximate main-sequence mass-radius relation

$$R = R_{\odot}(\mathfrak{M}/\mathfrak{M}_{\odot})^{0.7} \quad (4)$$

into equation (3) to obtain the accretion rate needed to generate L_2,

$$dm/dt = 3.5 \times 10^{-4}(\mathfrak{M}/\mathfrak{M}_{\odot})^{-0.3} \quad (5)$$

in solar masses per year. Equation (5) shows a very weak dependence on the mass of the embedded star and says, essentially, that the accretion rate must be a few times $10^{-4}\,\mathfrak{M}_{\odot}$ per year, if accretion is to account for most of L_2. The present rate of mass *transfer* can be estimated from the relation

$$\frac{dm}{dt} = \frac{\mathfrak{M}_2}{3(q-1)}\frac{1}{P}\frac{dP}{dt}\,. \quad (6)$$

Assuming $\mathfrak{M}_2$ and q to be about $13\,\mathfrak{M}_{\odot}$ and 4.2, respectively, the first factor is of the order $1.5\,\mathfrak{M}_{\odot}$. The last factor, $P^{-1}dP/dt$, is about 2×10^{-5} per year (Struve 1958) and is fairly well determined, so dm/dt (transfer) should be $3 \times 10^{-5}\,\mathfrak{M}_{\odot}$ per year. It must be stressed that equation (6) assumes conservative mass transfer at the present epoch, which may not be the case. Also, it is quite likely that the present rate of mass *accretion* is greater than the rate of mass *transfer*, since the binary should be nearing the end of rapid mass transfer. We cannot, therefore, rule out the possibility that much of L_2 is accretion luminosity, but the rates of mass transfer and mass accretion computed above suggest that this is not the case. If we interpret L_2 as the nuclear luminosity of the embedded (presumably main-sequence) star, we can find the mass of that star and thus, by differencing, the mass of the disk. Our estimate of L_2 is $1 \times 10^4\,L_{\odot}$, which corresponds to a main-sequence mass of $12\,\mathfrak{M}_{\odot}$, or about all of the mass of the secondary. This suggests that the disk contains, at most, a few solar masses.

This paper would have appeared much later if not for the enthusiasm for studies of β Lyrae communicated, in both discussion and correspondence, by Dr. M. Plavec. The work was greatly aided by the author's having been granted a National Academy of Sciences–National Research Council Senior Research Associateship at the Goddard Institute for Space Studies. There have been numerous valuable discussions with Dr. R. Stothers and, in particular, the argument that HD 174664 cannot be sensibly evolved from the ZAMS (§ III) is due to Dr. Stothers. Interesting conversations with Dr. P. Biermann are also acknowledged.

REFERENCES

Abt, H. A., Jeffers, H. M., Gibson, J., and Sandage, A. R. 1962, *Ap. J.*, **135**, 429.
Blaauw, A. 1963, in *Basic Astronomical Data*, ed. K. Aa. Strand (Chicago: University of Chicago Press), p. 383.
Bodenheimer, P. 1971, *Ap. J.*, **167**, 153.
Bodenheimer, P., and Ostriker, J. P. 1970, *Ap. J.*, **161**, 1101.
Böhm-Vitense, E. 1954, *Ap. J.*, **120**, 271.
———. 1967, *ibid.*, **150**, 483.
Boyarchuk, A. A. 1959, *Astr. Zh.*, **36**, 766.
Carbon, D. F., and Gingerich, O. 1969, in *Theory and Observation of Normal Stellar Atmospheres*, ed. O. Gingerich (Cambridge, Mass.: MIT Press), pp. 377, 401.
Devinney, E. J. 1971, *Nature*, **233**, 110.
Guthnick, P. 1945–46, *Abh. Deutsch. Akad. Wiss. Berlin*, Mathnaturw. Kla, No. 1.
Hack, M., and Job, F. 1965, *Zs. f. Ap.*, **62**, 203.
Huang, S. 1962, *Ap. J.*, **136**, 903.
———. 1963, *ibid.*, **138**, 342.
Iriarte, B., Johnson, H. L., Mitchell, R. I., and Wisniewski, W. K. 1965, *Sky and Tel.*, **30**, 21.
Johnson, H. L., Mitchell, R. I., Iriarte, B., and Wisniewski, W. Z. 1966, *Contr. Lunar and Planet. Lab.*, No. 63, p. 99.
Kippenhahn, R. 1969, *Astr. and Ap.*, **2**, 309.
Kondo, Y., McCluskey, G. E., and Houck, T. E. 1971, *Proceedings of IAU Colloquium No. 15*.
Kopal, Z. 1959, *Close Binary Systems* (New York: Wiley), p. 127.
Kříž, S. 1973, *Nature*, **245**, 36.
Kříž, S. 1974, *Bull. Astr. Soc. Czechoslovakia* (in press).
Landis, H. J., Lovell, L. P., and Hall, D. S. 1973, *Pub. A.S.P.*, **85**, 133.
Larsson-Leander, G. 1968, *Ark. f. Astr.*, **5**, 253.
Lloyd Evans, T. 1972, *Quart. J.R.A.S.*, **13**, 177.
Mihalas, D., and Auer, L. H. 1970, *Ap. J.*, **161**, 1129.
Mitchell, R. I. 1954, *Ap. J.*, **120**, 274.
Morton, D. C., and Adams, T. F. 1968, *Ap. J.*, **151**, 611.
Sahade, J., Huang, S., Struve, O., and Zebergs, V. 1959, *Trans. Am. Phil. Soc.*, Vol. **49**, Part 1.
Sandage, A. R. 1957, *Ap. J.*, **125**, 435.
Skulskii, M. Y. 1971, *Astr. Zh.*, **48**, 766.
Slettebak, A. 1963, *Ap. J.*, **138**, 118.
Stebbins, J. 1916, *Lick Obs. Bull*, No. 277, **8**, 186.
Stothers, R. 1972*a*, *Ap. J.*, **175**, 431.
———. 1972*b*, *Nature*, **238**, 5.
Stothers, R., and Lucy, L. B. 1972, *Nature*, **236**, 218.
Struve, O. 1941, *Ap. J.*, **93**, 104.
———. 1958, *Pub. A.S.P.*, **70**, 5.
Wilson, R. E. 1971, *Nature*, **234**, 406.
Wilson, R. E., DeLuccia, M. R., Johnston, K., and Mango, S. A. 1972, *Ap. J.*, **177**, 191.
Wilson, R. E., and Devinney, E. J. 1971, *Ap. J.*, **166**, 605.
Wood, D. B. 1973, *Ap. J.*, **186**, 615.
Wood, D. B., and Walker, M. F. 1960, *Ap. J.*, **131**, 363.
Woolf, N. J. 1965, *Ap. J.*, **141**, 155.
Ziółkowski, J. 1973, paper presented at the 141st meeting of the AAS in Tucson, Arizona.