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INFLUENCE OF ROTATION ON THE MAXIMUM MASS OF PULSATIONALLY STABLE STARS

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ABSTRACT

The critical mass for stability against radial pulsations in rotating, homogeneous main-sequence stars is found to be greater than in the case of no rotation. Analytic and detailed numerical models show that the critical mass rises steeply with increasing concentration of angular momentum to the center of the star. For uniform rotation near breakup velocity at the star's equator the critical mass is $\sim 850\ M_\odot$ if an electron-scattering opacity is used, or $\sim 5000\ M_\odot$ if the opacities of Cox and Stewart are used. For nonuniform rotation with a constant ratio of centrifugal force to gravity in the star, the critical mass becomes "infinite" long before breakup velocity is attained. The relevance of the present results to several observational problems is noted.

Subject headings: interiors, stellar — massive stars — pulsation — rotation, stellar

I. INTRODUCTION

Theoretical models of nonrotating stars are known to be violently unstable to radial oscillations energized by nuclear reactions in the core if the stellar mass is sufficiently high. Most observed stars of high mass are fast rotators, however; and some of them appear to be rotating at equatorial breakup velocity. Therefore, a realistic attempt to determine the critical mass for pulsational stability must take into account the interaction of rotation with pulsation. How rotation affects the critical mass of homogeneous main-sequence stars is the subject of inquiry in this paper.

II. BASIC EQUATIONS

Three simplifying approximations in the treatment of rotation are made here. First, rotational distortion of the star's shape is ignored. Second, the radial heat flow is assumed to be unmodified by the neglected distortion or by circulation currents. Third, the mean centrifugal force acting on each layer of the star is computed by taking an appropriate average over angles. The resulting equations agree with those of Faulkner, Roxburgh, and Strittmatter (1968), who carefully reduced the exact equations for a uniformly rotating star to the same form as for a spherical star, except that those authors obtained an additional factor (very close to unity) which multiplies the flux in the radiative transfer equation. We retain our present approximations, however, since we wish to consider other rotation laws in addition to uniform rotation. In general, the neglected terms are of second or higher order in the rotation parameter λ (defined below).

At each mass point in the star, the centrifugal force is

$$f_{\rm rot}(r, \theta) = \Omega^2 r \sin\theta\,, \tag{1}$$

where r is the distance from the stellar center and Ω is the angular velocity about the rotation axis, with which the r-vector makes an angle θ. The appropriate mean centrifugal force on a spherical layer at r is

$$f_{\rm rot}(r) = \lambda G M(r)/r^2 \tag{2}$$

where

$$\lambda = \tfrac{2}{3}\Omega^2 r^3/GM(r)\,. \tag{3}$$

Thus the gravitational force is effectively weakened by the centrifugal force, by an amount $(1 - \lambda)$. Formally, we may replace Newton's constant G by $(1 - \lambda)G$ in the ordinary equilibrium equations of stellar structure.

For the pulsation equations, we shall employ a linearization of the perturbed equations of motion of a spherical star, and shall consider radial motions only. The equations for the nonrotating case have already been given by Schwarzschild and Härm (1959). In the rotating case, we assume that each mass element conserves its angular momentum during the pulsation (Ledoux 1945), so that, at each mass point,

$$\Omega r^2 \sin^2\theta = \text{constant}\,. \tag{4}$$

Accordingly, the perturbed rotational quantities are

$$\frac{\delta\Omega}{\Omega} = -2\frac{\delta r}{r} \quad \text{and} \quad \frac{\delta\lambda}{\lambda} = -\frac{\delta r}{r}\,. \tag{5}$$

The inclusion of rotational terms changes only equation (2) of Schwarzschild and Härm's paper.[1] This equation then reads

$$x\frac{d}{dx}\left(\frac{\delta P}{P}\right) = V\frac{\delta P}{P} + V\left[4 + \left(\frac{\omega^2}{1-\lambda}\right)\frac{x^3}{q} - \left(\frac{\lambda}{1-\lambda}\right)\right]\frac{\delta r}{r}\,, \tag{6}$$

where the usual nondimensional Schwarzschild (1958) variables have been adopted. The nondimensional

[1] Note the misprint, a minus sign, in their equation (2). Their paper contains a number of other misprints and minor errors in the equations.

pulsational eigenfrequency ω is defined by $\omega^2 = \sigma^2 R^3/GM$, where $\sigma = 2\pi/\text{Period}$. For opacities other than Thomson scattering by free electrons, equation (5) of Schwarzschild and Härm's paper must also be changed to

$$\frac{\delta L(r)}{L(r)} = 4\frac{\delta r}{r} - \alpha\frac{\delta\rho}{\rho} + (4+\eta)\frac{\delta T}{T} + T\frac{dr}{dT}\frac{d}{dr}\left(\frac{\delta T}{T}\right), \quad (7)$$

where the thermodynamic derivatives of opacity are defined by $\alpha = \partial \ln \kappa/\partial \ln \rho$ and $\eta = -\partial \ln \kappa/\partial \ln T$. We shall ignore energy losses at the surface of the star due to mass ejection and to running waves in the atmosphere.

III. NUMERICAL RESULTS

Equilibrium models for homogeneous main-sequence stars have been calculated with the input physics described in Stothers and Simon (1970) but modified by the rotational term given in § II. Two opacity representations have been adopted: (1) Thomson scattering by free electrons, and (2) Cox-Stewart opacities as represented by the interpolation formula in the paper just cited. Two rotation laws have been considered: (1) uniform rotation with Ω = constant, and (2) nonuniform rotation with λ = constant. These two laws can be expressed in terms of the central condensation parameter $\rho_c/\langle\rho\rangle$ as $\lambda_R/\lambda_c = \rho_c/\langle\rho\rangle$ and $\Omega_c^2/\Omega_R^2 = \rho_c/\langle\rho\rangle$, respectively. Here R refers to the surface and c to the center of the star.

In order to check the accuracy of our results based on the simplifying approximations of § II, the uniformly rotating models computed with the more exact equations of Faulkner *et al.* (1968) by Sackmann and Anand (1970) for 15 $M_\odot$ with $\lambda_R = 0$ and $\lambda_R = 0.3007$ (in our notation) have been recomputed with our program for the same chemical composition, opacity representation, and rate of nuclear energy generation as were used by those authors. Our results for the basic quantities $\log L/L_\odot$, $\log R/R_\odot$ (mean radius), $\log T_c$, and $\log \rho_c$ are virtually identical to theirs, to three significant figures, even for the extreme case of $\lambda_R = 0.3007$, at which centrifugal force just balances gravity at the star's equator.

Pulsational stability or instability of the rotating models with respect to the fundamental radial mode has next been determined in the quasi-adiabatic approximation by using the prescription of Schwarzschild and Härm (1959) with the modifications mentioned in § II. The critical masses separating stable from unstable models are listed in table 1, along with several other quantities of interest (the relative radiation pressure is $1-\beta$). The (hydrogen, metals) content is taken to be $(X, Z) = (0.70, 0.03)$.

The critical masses for the zero-rotation models based on Thomson scattering and on Cox-Stewart opacities are those derived originally by Schwarzschild and Härm (1959) and by Stothers and Simon (1970), respectively. It is seen that rotation tends to stabilize the models pulsationally, since the critical mass rises steeply as λ_R is increased. For a fixed value of λ_R, nonuniform rotation (with the angular velocity increasing toward the rotation axis) is clearly more effective than uniform rotation in stabilizing the star. For nonuniform rotation with λ = constant, the critical mass becomes "infinite" when $\lambda \sim 0.2$ (see § IV).

A curious phenomenon occurs in the case of uniform rotation with $\lambda_R = 0.3$ and with Thomson scattering opacity. From 850 $M_\odot$ up to roughly 1200 $M_\odot$ the sequence of models alternates between marginal stability and instability. The destabilizing effect of higher radiation pressure at the larger masses is delicately balanced in this case by the stabilizing effect of the centrifugal force, which increases by just the right amount the central condensation of the models. Above $\sim$1200 $M_\odot$ the models are unstable in the usual way.

IV. ANALYTIC RESULTS

a) *Uniform Rotation*

For a slow uniform rotation of the star, Ledoux (1945) showed that the square of the pulsational

TABLE 1

CRITICAL MASSES FOR THE PULSATIONAL STABILITY OF MASSIVE ROTATING STARS WITH $(X, Z) = (0.70, 0.03)$

Opacity	Rotation	λ_R	$M/M_\odot$	β_c	$\rho_c/\langle\rho\rangle$	ω^2
Thomson	Zero	0	54	0.65	21	3.0
	Uniform	0.1	82	0.57	25	2.8
		0.2	161	0.44	33	2.6
		0.3	$\sim$850	0.22	69	2.3
	Nonuniform*	0.1	119	0.52	22	2.7
Cox-Stewart	Zero	0	106	0.51	29	2.8
	Uniform	0.1	209	0.39	37	2.6
		0.2	800	0.22	71	2.3
		0.3	$\sim$5000	0.09	161	2.2
	Nonuniform*	0.1	540	0.29	36	2.4

* λ = constant.

eigenfrequency is given approximately by

$$\sigma^2 = (3\langle\Gamma_1\rangle - 4)(-W/I) + (5 - 3\langle\Gamma_1\rangle)(\Omega J/I)\,, \tag{8}$$

provided that the central condensation of the star is not too high. Here $\langle\Gamma_1\rangle$ is a weighted mean of the first adiabatic exponent, W is the total gravitational potential energy of the star, J is the total angular momentum, and I is the total moment of inertia with respect to the center. Introducing the relation $J = \frac{2}{3}I\Omega$ and the reduced expression for σ^2 given in the Appendix for the case of no rotation, we rewrite equation (8) as

$$\omega^2 = \frac{3\langle\Gamma_1\rangle - 4}{2n - 1}\frac{\rho_c}{\langle\rho\rangle} + (5 - 3\langle\Gamma_1\rangle)\lambda_R\,, \tag{9}$$

where n is the effective polytropic index. Unless $\langle\Gamma_1\rangle$ is very close to 4/3, the first (gravitational-energy) term is always much larger than the second (rotational-energy) term, which we shall henceforth neglect.

In a uniformly rotating star the parameter λ, which equals $\lambda_R\langle\rho\rangle/\langle\rho(r)\rangle$, is virtually negligible everywhere except near the surface, because $\langle\rho(r)\rangle$ increases very rapidly below the surface. The deep interior of the star is thus practically unaffected by the rotation. The loosely bound outer layers are, however, distended by centrifugal force. The change in mean stellar radius is obtainable, for example, from the first-order perturbation theory of Sweet and Roy (1953), who considered a rotating Cowling model constructed with Kramers opacity and the CN cycle of energy generation. From their results, it follows that

$$\delta \log(\rho_c/\langle\rho\rangle) = 0.47\lambda_R\,. \tag{10}$$

Eddington's (1926) standard model of a star provides a relation between $\langle\Gamma_1\rangle$ and stellar mass. The standard model is a nonrotating polytrope of index $n = 3$, for which $\rho_c/\langle\rho\rangle = 54$ and β = constant. Since Γ_1 is a unique function of β, it is necessary only to solve Eddington's quartic equation in order to find Γ_1 from the stellar mass M and mean molecular weight μ:

$$(M/M_\odot)\mu^2 = 18(1 - \beta)^{1/2}\beta^{-2}\,. \tag{11}$$

This equation may be used for a uniformly rotating standard model since its derivation depends very little on the outer layers of the star.

The critical mass for pulsational stability of homogeneous stars fueled by the CN cycle is known to be characterized by $\omega^2 \sim 3$ (§ III; Simon and Stothers 1969). This criterion and equations (9), (10), and (11) allow us to calculate the critical mass for any value of λ_R. We find, by simple inspection, that uniform rotation raises the critical mass primarily through the increase of central condensation. As is well known, a higher central condensation induces smaller pulsation amplitudes in the core and therefore less nuclear destabilization. To overcome this, the relative radiation pressure must be increased, and so a higher mass is required.

b) *Nonuniform Rotation*

Only the simplest rotation law, λ = constant, will be considered here. In a straightforward modification of Ledoux's (1945) work on uniformly rotating models, ΩJ in equation (8) can be replaced by $2E_{\rm rot}$ for any moderate distribution of angular velocity, where $E_{\rm rot}$ is the total rotational energy of the star. The virial theorem in the present instance gives $E_{\rm rot} = -(\lambda/2)W$. We then obtain in place of equation (9):

$$\omega^2 = \frac{(3\langle\Gamma_1\rangle - 4) + (5 - 3\langle\Gamma_1\rangle)\lambda}{2n - 1}\frac{\rho_c}{\langle\rho\rangle}\,. \tag{12}$$

Since the ordinary equilibrium equations of stellar structure remain unaltered by the present rotation law except for the replacement of G by another constant $(1 - \lambda)G$, the central condensation of the star, i.e., $\rho_c/\langle\rho\rangle$, is also unchanged. Adopting Eddington's standard model as before, we must rewrite the quartic equation for an altered value of mean effective gravity:

$$(M/M_\odot)\mu^2 = 18(1 - \beta)^{1/2}\beta^{-2}(1 - \lambda)^{-3/2}\,. \tag{13}$$

With the aforementioned criterion for vanishing pulsational stability $\omega^2 \sim 3$, the critical mass may be derived from equations (12) and (13) for any value of λ. In the present case, rotation is found to increase the critical mass directly through the total rotational energy (or, equivalently, the total angular momentum) of the star and through the reduction of the relative radiation pressure.

In the limit of very high masses, our analytic approximation predicts that $\omega^2 \approx 11\lambda$. The detailed results of § III predict that an "infinite" critical mass would be characterized by $\omega^2 < 2.3$. Since realistic stellar models are close to Eddington's standard model as the mass becomes very large, it follows that pulsational instability will disappear at all masses when $\lambda \sim 0.2$ (or possibly a somewhat lower value).

V. CONCLUSION

The critical mass for stability against radial pulsations in rotating, homogeneous main-sequence stars has been determined. Simple analytic models show that rotation raises the critical mass over the nonrotating value. Part of this increase is due to the change in the equilibrium model alone, and part to the interaction between the pulsation and the rotation. In the case of uniform rotation (Ω = constant) the increase is brought about primarily by the change in central condensation of the star. In the case of nonuniform rotation with a constant rotation parameter λ [i.e., $\Omega \propto M(r)^{1/2}r^{-3/2}$] the increase arises primarily from the high angular-momentum content of the star and the drop in the relative radiation pressure. For a fixed value of λ at the stellar surface, nonuniform rotation produces a higher critical mass than does uniform rotation. It is likely that nonuniform rotation with a constant angular momentum per unit mass ($\Omega \propto r^{-2}$) would produce an even higher critical mass than does rotation with constant λ.

Detailed numerical models computed with a purely electron-scattering opacity show that the critical mass rises from 54 $M_\odot$ for no rotation to ~ 850 $M_\odot$ for uniform rotation near breakup velocity at the star's equator. If the "hydrogenic" opacities of Cox and Stewart are adopted, the critical masses are 106 $M_\odot$ and ~ 5000 $M_\odot$, respectively. For nonuniform rotation with λ constant throughout the star, the critical mass becomes "infinite" when $\lambda \sim 0.2$, i.e., before breakup velocity is reached.

Nonrotating stars composed of pure hydrogen are known to have higher critical masses than do nonrotating stars of normal composition (Boury 1963). Therefore, our present results for rotating stars cast some doubt on (but do not rule out) the often quoted idea that an early generation of massive pure-hydrogen stars could have evolved by ejecting mass as a result of pulsational instability, and thereby could have produced most of the helium observed in the Galaxy. Our results suggest, further, that rotation is capable of stabilizing even supermassive stars against radial pulsation, and this may affect various galaxy and quasar models that invoke a pulsating central supermassive star (or stars).

No stars are definitely known to have masses exceeding ~ 60 $M_\odot$ (Stothers and Simon 1968), although such stars could be disguised in various ways (see the review by Talbot 1971). This fact raises the question of whether very massive stars are prevented from reaching a main-sequence state by high radiation pressure during their formative stages, which could impose a limit on how much mass is eventually accreted (Eddington 1926). Larson and Starrfield (1971) have examined additional mechanisms for limiting the mass, viz., radiative heating of the protostellar material, formation of an H II region, and the constraint imposed by the collapse time scale itself. We suggest here that these mechanisms are aided, perhaps crucially, by fast rotation. Conservation of angular momentum during the collapse phase of the original gas cloud could prevent the formation of very massive individual stars if fission of the cloud and equatorial spin-off of material from the fragments (Jeans 1928) are efficient. Not enough is yet known, however, about these early stages to predict a meaningful mass limit for pre-main-sequence stars.

The computer program used in the present work to determine the pulsational properties of the models is a modified version of one kindly provided by Dr. Norman R. Simon for nonrotating models.

APPENDIX

The fundamental eigenfrequency of radial pulsation in a nonrotating star, σ, obeys the inequality (Ledoux and Pekeris 1941)

$$\sigma^2 \leq (3\langle\Gamma_1\rangle - 4)(-W/I)\,,$$

where, in the usual notation,

$$\langle\Gamma_1\rangle = \frac{\int \Gamma_1 P dV}{\int P dV}\,, \qquad -W = \int \frac{GM(r)}{r}\, dM(r)\,,$$

$$\mathrm{I} = \int r^2\, dM(r)\,.$$

A convenient reduction of the ratio of integrals W/I begins with the introduction of the variable $\langle\rho(r)\rangle = 3M(r)/4\pi r^3$ into the definition of W, and then follows with the replacement of $\langle\rho(r)\rangle$ by its maximum value ρ_c. Thus

$$-W \leq (4\pi/3)G\rho_c I\,.$$

By adopting $\omega^2 = \sigma^2 R^3/GM$, we then obtain

$$\omega^2 \leq (3\langle\Gamma_1\rangle - 4)\rho_c/\langle\rho\rangle\,.$$

A better approximation to ω^2 results from fitting an expression of this type to detailed solutions for polytropes of index n (see table 12 of Ledoux and Walraven 1958). We find, for $n \leq 3$,

$$\omega^2 \approx (3\langle\Gamma_1\rangle - 4)(2n - 1)^{-1}\rho_c/\langle\rho\rangle\,.$$

Extension of these results to rotating stars is given in the main text.

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STARSPOTS ON FLARE STARS

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ABSTRACT

Sizes of starspots on flare stars can be derived from the author's convection-cell hypothesis. The sizes are in fair agreement with those observed on YY Gem, CC Eri, and BY Dra by Bopp and Evans. The hypothesis predicts that periodic brightness variations due to starspots are restricted to stars brighter than a critical absolute visual magnitude. A convective model of a starspot on YY Gem has been computed, assuming that the missing flux is in the form of Alfvén waves. It is found that the surface field must exceed 10^4 gauss, and is probably less than about 3×10^4 gauss. With a surface field of 2×10^4 gauss, the effective temperature of the spot is in the range $T_e = 1590°$–$1890°$ K, depending on the field gradient. These figures are to be compared with $T_e = 2000°$ K estimated from observations by Bopp and Evans. Efficient dynamo action is shown to be a possible mechanism for generating such large surface fields. There is a possibility that tidal effects may influence starspot formation.

Subject headings: flare stars — hydromagnetics — late-type stars — magnetic stars

I. INTRODUCTION

Dark starspots have been proposed as an explanation for periodic brightness changes in certain red dwarfs (Kron 1952; Chugainov 1966; Krzeminski 1969; Evans 1971; Torres, Ferraz Mello, and Quast 1972; Chugainov 1973; Bopp and Evans 1973; Bopp 1973). The stars involved are all now known to be flare stars. Estimates of the sizes of the spots are deducible in those cases where the stars happen to be in binary systems, such as BY Dra, CC Eri, and YY Gem. (It is not clear whether or not there is a relationship between membership in a binary and flare activity; see Gershberg 1970, the English translation of which is hereinafter referred to as GT.) Thus, in YY Gem, the spot cannot extend more than 60° in longitude (Bopp 1973); in CC Eri, spots at different times extend 30°–60° in longitude, and 40°–60° in latitude (Bopp and Evans 1973); and in BY Dra, spots extend 45°–105° in longitude, and 25°–50° degrees in latitude (Bopp and Evans 1973). For the latter two stars at least, the extent of the spots depends on the effective temperature assumed for the spots. If the spots are cooler than the assumed $T_e = 2000°$ K,[1] the size of the spots would decrease, although there are constraints on the range of parameters which can be used (Bopp and Evans 1973).

Starspots are also thought to exist in two subgiant stars, AR Lac and RS CVn (Kron 1952). In AR Lac, the primary component is spotted, while in RS CVn, the secondary is spotted. The spectral types of the spotted stars are G2 and G8, with masses of 1.32 and 1.40 $M_\odot$, and radii of 1.8 and 4.00 $R_\odot$, respectively (Popper 1967). In AR Lac, up to 20 percent of the area of the surface can be covered with spots, with individual patches covering 3–5 percent of the area (Kron 1947). Although we are mainly interested in red dwarfs in this paper, it must be kept in mind that apparently subgiants may also be spotted. That the spots on AR Lac are indeed not dissimilar to sunspots, and to those on flare stars, is perhaps indicated by the recent discovery of radio emission from AR Lac (Hjellming and Blankenship 1973), perhaps associated with a radio burst similar to the emission from regions surrounding sunspots.

The giant stars HD 209813, α Aur, and λ And are also possible candidates for spotty stars. They share a certain feature with the dwarfs of BY Dra type, namely, they exhibit photometric variations which have a different period from the binary spectroscopic period (Blanco and Catalano 1970). The spectral classes of the components in these systems are G0 III, G5 III, K0 III, and F IV.

By analogy with sunspots, starspots may be supposed to be associated with locally strong magnetic fields in the star, and these fields are presumably responsible for providing the necessary energy for flare activity. In support of this idea, one may cite the example of EV Lac, where in a certain observing period, all of the observed flares occurred during the passage of a dark spot over the surface (Cristaldi *et al.* 1969). As a further example, periodicity in flare activity (Andrews 1966; Chugainov and Korovyakovskaya, quoted in GT, p. 207) is best understood in terms of the rotational modulation of visibility of an active region moving across the stellar disk. The active region is presumably a region of enhanced fields, if the analogy with the solar case is valid. The analogy between sunspots and starspots is used by Bopp and Evans (1973) to support their choice of effective temperature in the starspot, and they believe that the analogy even extends to the magnetic field strengths in starspots. Their results seem to indicate that field strengths in starspots are comparable to those in sunspots (about 2 kilogauss). This last analogy, however, breaks down when their field estimates are corrected by multiplying by a factor of 8π (Evans 1974).

In this paper, we wish to consider other aspects of

[1] See note added in proof.

the analogy between sunspots and starspots. We indicate how starspot diameters might provide useful information on the depth of convection zones in red dwarfs, since it appears that spot diameters are consistent with a recent suggestion (Mullan 1973*b*) to the effect that spots are convection cells of some sort penetrating throughout the entire depth of the convection zone. We also present a detailed model of a starspot on YY Gem, using the author's method for computing sunspot models (Mullan 1974). Effective temperatures in the spot are indeed found to be about 2000° K, as Bopp and Evans require, but only if the surface fields are as high as 20–30 kilogauss. Indirect evidence suggests that these are plausible field strengths at the surfaces of flare stars. Finally, we speculate briefly on the origin of such large fields, and on the possible effects of tides on the appearance of starspots and flare activity.

II. EXPECTED DIAMETERS OF STARSPOTS

The author has suggested (Mullan 1973*b*) that sunspots can be considered as convection cells of some type extending to the bottom of the solar convection zone. With depth H_c and diameter D_c, the ratio D_c/H_c for such cells has a minimum value of 2.04, and a maximum value which depends on the structure of the convection zone. (The ratio D_c/H_c refers to "super" convective cells penetrating to the bottom of the convection zone. These cells therefore extend over many scale heights. One must distinguish between D_c/H_c and D/H, where the latter refers to "normal" convection cells. According to our assumptions, "normal" cells extend over 1 pressure scale height, $H = H_p$. The ratio D/H is a local parameter of the convection zone, and it varies with depth (see fig. 2), whereas D_c/H_c is a global property ascribed to the entire convection zone.) As was pointed out by Mullan (1973*b*), if starspots are indeed analogous to sunspots, then along the main sequence, at later spectral types than the Sun, the diameter of the spots should increase as the depth of the convection zone, H_c, increases. What is needed now is models of the convection zones of stars at late spectral types.

Osterbrock (1953) computed a model for YY Gem (mass = 0.63 $\mathfrak{M}_\odot$, radius R = 0.63 $R_\odot$, effective temperature T_e = 3600° K). The convection zone was found to have a depth of 0.33 R. Using Öpik's cellular convection model (see Mullan 1971*b*), a model for this convection zone has recently been computed, assuming cell depth equals pressure scale height, and the depth of the convection zone turned out to be in good agreement with that obtained by Osterbrock. (As an aside, this incidentally indicates that in certain parts of the main sequence, the depth of the convection zone, H_c, is insensitive to the convection model used. Unfortunately, the Sun happens to lie in a region where H_c is sensitive to the convection model. Another region in which H_c is sensitive to the convection model occurs in the mass range from 0.2 to 0.3 solar masses, where the models are becoming completely convective.) From our model, the maximum superadiabaticity is found to be $\delta\beta/\beta = 0.43$. Using equation (4) of Mullan (1973*b*), the corresponding value of m can be interpreted according to the results of Vickers (1971) as setting an upper limit of 2.5 on D/H in this star. The expected spot size on YY Gem is therefore less than 0.83 R, corresponding to an upper limit of about 50° in longitude (and latitude) for the extent of the spot. This is consistent with Bopp's (1973) upper limit of 60°.

In the cases of BY Dra and CC Eri, the masses are known from the binary orbit only to within a factor of $\sin^3 i$. It is necessary to use a mass-luminosity relationship in order to obtain masses of individual components of these systems. Recent discussions have shown that there is a certain amount of scatter about a mean relationship at the lower end of the main sequence (Lippincott and Hershey 1972), and in this paper we have simplified the problems by assuming a linear relation between absolute visual magnitude M_v and $\log \mathfrak{M}/\mathfrak{M}_\odot$. The linear curve is assumed to pass through the Sun ($M_v = +4.7$), and UV Ceti ($M_v = +15.8$, Gliese 1969; $\mathfrak{M}/\mathfrak{M}_\odot = 0.108 \pm 0.008$, Harrington and Behall 1973). This leads to the following relation:

$$\log \mathfrak{M}/\mathfrak{M}_\odot = -0.087\,(M_v - 4.7)\,. \qquad (1)$$

It must be stressed that this relation is only approximate, for there is no physical justification for a linear relation between $\log \mathfrak{M}$ and M_v extending over a range of masses from 0.1 to 1 solar mass. Errors in masses estimated from equation (1) are likely to be largest at about the midpoint of the range of M_v, i.e. at $M_v \simeq 10$. The errors are such that at $M_v \simeq 10$, formula (1) leads to an underestimate of the mass of a star with a given M_v.

In BY Dra, about 60 percent of the light is contributed by the primary (Bopp and Evans 1973), so with $M_v = +7.4$ for the total light of the system, we find the individual values $M_v = 7.95$ and 8.39 mag. Equation (1) then gives masses of 0.52 and 0.48 $\mathfrak{M}_\odot$ i.e., a mass ratio of 1.08. This is not consistent with the spectroscopic mass ratio of 1.21 (Bopp and Evans 1973). However, the mass ratio can be improved if the light contributed by the primary is larger than 60 percent. If the contribution were as large as, say, 70 percent, then the mass ratio obtained from formula (1) would be 1.20. In this case, the individual absolute visual magnitudes would be 7.77 and 8.73 mag, corresponding to masses of 0.54 $\mathfrak{M}_\odot$ and 0.45 $\mathfrak{M}_\odot$, respectively.

In CC Eri, the companion is at least 2 mag fainter than the primary (Evans 1959). Hence, with a combined M_v = 8.4 mag, the primary has $M_v \leq 8.6$ mag, and a mass therefore greater than 0.46 $\mathfrak{M}_\odot$. In view of the errors involved in formula (1), these masses must be regarded as lower limits.

The depths of the convection zones in these stars can be derived from results of Copeland, Jensen, and Jörgensen (1970), plotted in figure 1. Copeland *et al.* find that stars with $\mathfrak{M}/\mathfrak{M}_\odot = 0.3$ are completely convective, although Straka (1971) suggests that the limit of completely convective stars is about 0.20 $\mathfrak{M}_\odot$.