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FUNDAMENTAL DATA FOR MASSIVE STARS COMPARED WITH THEORETICAL MODELS

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ABSTRACT

A new grid of nonrotating stellar models for the upper main sequence has been calculated for a variety of initial chemical compositions and assumed modes of evolution. Theoretical isochrones in the H-R diagram have then been constructed (with the help of published models for the post–main-sequence phases). A comparison of the theoretical data with observational data for massive members of eclipsing binary systems, clusters, and subgroups of associations in the (mass, spectral type)- and (mass, luminosity)-planes suggests that (1) currently adopted initial chemical compositions are probably correct ($X_e \sim 0.70$, $Z_e \sim 0.03$), although larger variations, particularly in Z_e, possibly occur; (2) chemical evolution, even in fast rotators, proceeds inhomogeneously; (3) the rotation of normal upper-main-sequence stars is approximately uniform; (4) mass exhange in a close binary system may lead to nonuniform rotation in the new primary component; and (5) mass loss seems to be rather small for normal massive stars, including supergiants. Two independent ages for each cluster or association containing supergiants are derived, by using (1) the spectral type of the main-sequence turnup in the H-R diagram and (2) the average luminosity of the *evolved* supergiants; the two ages generally agree well with each other, although they differ slightly for several stellar groups, probably as a result of a somewhat larger metals abundance than normal or a fast uniform rotation of the upper-main-sequence stars. The few available kinematic ages are usually much shorter than the nuclear ages.

I. INTRODUCTION

Recent interest in the evolution of massive stars has focused on a variety of perturbations of the simple scheme of evolution usually adopted in theoretical calculations. Accordingly, it has become important to investigate the available observational material and to compare it critically with theoretical stellar models built under standard assumptions. This is now possible in view of recently enlarged observational data, revised ideas about the internal structure of massive stars, and improved information on their initial chemical composition.

To provide a homogeneous theoretical framework, new models for stars evolving on the upper main sequence have been constructed with a variety of masses, chemical compositions, and assumed modes of evolution (§ II). Reliable observational data for upper-main-sequence stars and supergiants have been compiled, including masses, luminosities, and spectral types—as determined from the members of double-line eclipsing binary systems, young clusters, and subgroups of associations (§ III). Comparison of the theoretical models with the observational data (in a variety of ways) then permits the drawing of a number of conclusions regarding the initial chemical composition, axial rotation, loss of mass, mode of evolution, and age of a massive star (§§ IV and V).

II. THEORETICAL MODELS

Evolution on the upper main sequence has been computed by standard techniques for stars with masses of 5, 7, 10, 15, 30, 60, and 120 $\mathfrak{M}_\odot$. Three different sets of initial chemical composition have been chosen:

$$X_e = 0.739\,, \quad Z_e = 0.021\;;$$

$$X_e = 0.739\,, \quad Z_e = 0.044\;;$$

$$X_e = 0.602\,, \quad Z_e = 0.044\,.$$

The carbon-nitrogen abundance has been taken, in all cases, to be $X_{CN} = \frac{1}{3}Z_e$. Basic input physics is the same as that used by Stothers and Simon (1970) for their study of the zero-age upper main sequence, except that, in the present work, the detailed temperature dependence of the rate of nuclear-energy generation for the CN cycle (Clayton 1968) has been used. Rotation and magnetic fields have been ignored. Evolutionary tracks have been computed from an initially homogeneous state on the zero-age main sequence (ZAMS) to a terminal state defined by a central hydrogen content of $X_c = 0.05$, which is close to the stage at which the effective temperature attains a minimum during core hydrogen burning (TAMS).

a) Mode of Evolution

Stars heavier than $\sim 10\ \mathfrak{M}_\odot$ usually develop a convectively unstable region outside the convective core at some stage before the end of core hydrogen burning. At present, there is no general agreement concerning the redistribution of chemical composition that is set up by the convective motions. Of many possible schemes, the following schemes have been found to be among those most likely (notation of Stothers 1970).

1. A semiconvective zone connects smoothly the core and envelope (scheme S2).
2. A small semiconvective zone overlies a deep radiative zone (scheme M1).
3. A small semiconvective zone merges into a fully convective zone, which in turn overlies a deeper radiative zone (scheme N2).
4. A radiative zone connects smoothly the core and envelope (scheme R). This scheme is unrealistic if the Schwarzschild criterion for convective instability is adopted, because convective instability is found to occur in the intermediate zone of varying chemical composition (Stothers 1970). The scheme is realistic, however (as shown by calculations of this paper), if the Ledoux criterion is adopted, because convective instability then occurs, when it occurs at all, *just outside* the intermediate zone. The resulting mixing is taken to be fully convective since it occurs only in a region of homogeneous (zero-age) composition; thus the problem of semiconvection does not arise (unless convective overshooting is effective). This scheme was originally suggested by Tayler (1969), on the basis of certain calculations done in 1954, but he does not seem to have distinguished critically between the use of the Schwarzschild criterion and the Ledoux criterion, which is of prime importance here.
5. The convective core maintains a steady outward growth into the chemically homogeneous envelope (scheme C1). This scheme may be rejected on formal physical grounds (Stothers 1970), but if efficient convective overshooting occurs at the core boundary, something approaching this scheme is remotely possible. It is clearly a limiting case of the true situation, for which the opposite limiting case is the radiative scheme R.

Schemes S2, M1, and R yield very similar evolutionary tracks and ages because the hydrogen profiles in the models are nearly identical (Stothers 1970). Scheme N2 also yields a comparable track and age despite the difference in hydrogen profile, but only for masses up to $\sim 20\ \mathfrak{M}_\odot$ (Chiosi and Summa 1970); at $30\ \mathfrak{M}_\odot$ a small deviation in the evolutionary track becomes apparent (Simpson 1971). Scheme C1 produces a significantly longer age and steeper evolutionary track on the H-R diagram (Stothers 1970), and must be considered independently.

In the present paper, detailed hydrogen-burning models for schemes S2, M1, R, and C1 have been computed. The results for schemes S2, M1, and R are virtually identical and will be designated "radiative/semiconvective"; results for scheme C1 will be called "convective." Evolution beyond the phase of core hydrogen burning has not been calculated in this paper, but the effective temperature of supergiants is known to be very sensitive to the differences among the schemes (Stothers 1970).

b) ZAMS and TAMS

Luminosities, effective temperatures, and ages of our models for the ZAMS and TAMS are presented in tables 1 and 2, respectively. In agreement with previous studies

TABLE 1

THEORETICAL MODELS FOR THE ZERO-AGE MAIN SEQUENCE (ZAMS)

Composition		$\mathfrak{M}/\mathfrak{M}_\odot$ 5	7	10	15	30	60	120
X_e=0.739....	log $(L/L_\odot)$	2.719	3.230	3.736	4.261	5.047	5.687	6.213
Z_e=0.021....	log T_e	4.250	4.333	4.412	4.484	4.600	4.680	4.731
X_e=0.739....	log $(L/L_\odot)$	2.562	3.109	3.647	4.200	5.017	5.666	6.198
Z_e=0.044....	log T_e	4.192	4.282	4.369	4.446	4.570	4.656	4.709
X_e=0.602....	log $(L/L_\odot)$	2.840	3.368	3.882	4.407	5.170	5.782	6.288
Z_e=0.044....	log T_e	4.254	4.340	4.423	4.497	4.610	4.684	4.728

TABLE 2

THEORETICAL MODELS FOR THE TAMS

Composition and Mode		$\mathfrak{M}/\mathfrak{M}_\odot$ 5	7	10	15	30	60	120
X_e=0.739.................	log $(L/L_\odot)$	2.943	3.482	4.015	4.554	5.360	5.938	6.391
Z_e=0.021.................	log T_e	4.180	4.260	4.338	4.413	4.490	4.531	4.548
Radiative/semiconvective...	log τ (yr)	7.904	7.599	7.322	7.046	6.761	6.561	6.424
X_e=0.739.................	log $(L/L_\odot)$	2.765	3.340	3.909	4.474	5.313	5.907	6.367
Z_e=0.044.................	log T_e	4.122	4.209	4.291	4.371	4.453	4.493	4.513
Radiative/semiconvective...	log τ (yr)	8.019	7.681	7.377	7.072	6.770	6.566	6.427
X_e=0.602.................	log $(L/L_\odot)$	3.054	3.611	4.148	4.687	5.440	5.987	6.427
Z_e=0.044.................	log T_e	4.200	4.285	4.361	4.431	4.509	4.551	4.573
Radiative/semiconvective...	log τ (yr)	7.703	7.389	7.123	6.866	6.595	6.410	6.284
X_e=0.739.................	log $(L/L_\odot)$	...	...	...	4.602	5.453	6.003	6.431
Z_e=0.044.................	log T_e	...	...	...	4.341	4.423	4.497	4.551
Convective................	log τ (yr)	...	...	...	7.183	6.893	6.685	6.543

and simple similarity arguments (e.g., Schwarzschild 1958), we find that, at a fixed mass, an increase in X_e (lighter mean molecular weight) or Z_e (higher opacity) reduces the luminosity and effective temperature and increases the lifetime. A number of previous studies have used masses and initial chemical compositions in common with ours (Hofmeister, Kippenhahn, and Weigert 1964; Kippenhahn, Thomas, and Weigert 1965; Ezer and Cameron 1967; Hofmeister 1967; Horn, Kříž, and Plavec 1969; Kippenhahn 1969; Chiosi and Summa 1970). While agreement among the different models is satisfactory, precise agreement is never obtained for reasons already discussed in detail elsewhere (e.g., Ruben and Masevich 1966; Cesarsky 1969; Horn *et al.* 1969).

c) Locus of Constant Age

In order to compare our models with the H-R diagrams of young star clusters, the evolutionary tracks have been transformed into loci of constant age. Since a locus of constant age for the main sequence usually ends in a nearly vertical portion, we follow normal procedure in calling this portion the *turnup*. The *turnoff* from the main sequence is poorly defined for massive stars and will not be considered here. To facilitate discussion, certain parameters of the theoretical turnup can be defined, following the schematic time-line shown in figure 1 These parameters, and other useful quantities, are listed in table 3, in the following format: (1) spectral type; (2) intrinsic $B - V$ color; (3) maximum effective temperature (log T_e) of the turnup (to which the listed spectral type and

TABLE 3

THEORETICAL MAIN-SEQUENCE TURNUPS

Composition and Mode	Sp	$(B-V)_0$	log T_e	Δlog T_e (width)	M_{bol} (tip)	M_v (tip)	$\Delta_1 M$ (height)	$\Delta_2 M$ (length)	$\mathfrak{M}/\mathfrak{M}_\odot$ (evolved)	log τ (years)
X_e=0.739, Z_e=0.021, Radiative/Semiconvective.....	O7	−0.32	4.544	0.035	− 9.3	−6.0	3.0	2.4	45	6.68
	O8	−0.31	4.525	0.028	− 8.9	−5.8	2.8	2.1	38	6.73
	O9	−0.31	4.505	0.022	− 8.5	−5.4	2.6	1.8	31	6.79
	O9.5	−0.30	4.491	0.019	− 8.2	−5.2	2.5	1.7	28	6.83
	B0	−0.30	4.477	0.016	− 7.9	−5.0	2.4	1.5	25	6.87
	B0.5	−0.28	4.418	0.008	− 6.6	−4.0	2.0	1.2	16	7.06
	B1	−0.26	4.354	0.008	− 5.5	−3.5	1.9	1.1	11	7.29
	B1.5	−0.25	4.333	0.007	− 5.1	−3.2	1.9	1.0	10	7.37
	B2	−0.24	4.312	0.007	− 4.8	−3.0	1.8	1.0	9.1	7.44
	B2.5	−0.22	4.283	0.006	− 4.3	−2.7	1.8	1.0	8.0	7.54
	B3	−0.20	4.253	0.006	− 3.8	−2.3	1.7	0.9	6.9	7.65
X_e=0.739, Z_e=0.044, Radiative/Semiconvective.....	O7	−0.32	4.544	0.054	− 9.9	−6.7	3.4	3.0	68	6.58
	O8	−0.31	4.525	0.047	− 9.5	−6.4	3.2	2.7	54	6.64
	O9	−0.31	4.505	0.038	− 9.1	−6.0	3.0	2.4	44	6.70
	O9.5	−0.30	4.491	0.032	− 8.8	−5.8	2.8	2.2	38	6.74
	B0	−0.30	4.477	0.026	− 8.5	−5.6	2.6	2.0	33	6.78
	B0.5	−0.28	4.418	0.013	− 7.3	−4.7	2.3	1.5	22	6.95
	B1	−0.26	4.354	0.011	− 6.0	−4.0	2.0	1.3	14	7.18
	B1.5	−0.25	4.333	0.011	− 5.6	−3.7	1.9	1.2	12	7.26
	B2	−0.24	4.312	0.009	− 5.3	−3.5	1.9	1.1	11	7.33
	B2.5	−0.22	4.283	0.008	− 4.8	−3.2	1.8	1.0	9.8	7.44
	B3	−0.20	4.253	0.006	− 4.3	−2.8	1.8	1.0	8.7	7.54

TABLE 3—*Continued*

THEORETICAL MAIN-SEQUENCE TURNUPS

Composition and Mode	Sp	$(B-V)_0$	$\log T_e$	$\Delta\log T_e$ (width)	M_{bol} (tip)	M_v (tip)	$\Delta_1 M$ (height)	$\Delta_2 M$ (length)	$\mathfrak{M}/\mathfrak{M}_\odot$ (evolved)	$\log \tau$ (years)
$X_e=0.602$, $Z_e=0.044$, Radiative/Semiconvective.....	O7	−0.32	4.544	0.021	− 9.3	−6.1	2.6	1.8	44	6.53
	O8	−0.31	4.525	0.014	− 8.9	−5.8	2.4	1.5	36	6.59
	O9	−0.31	4.505	0.010	− 8.5	−5.5	2.3	1.3	29	6.65
	O9.5	−0.30	4.491	0.008	− 8.2	−5.3	2.2	1.2	26	6.69
	B0	−0.30	4.477	0.006	− 8.0	−5.1	2.1	1.0	24	6.73
	B0.5	−0.28	4.418	0.002	− 6.7	−4.1	1.8	0.7	15	6.92
	B1	−0.26	4.354	0.000	− 5.5	−3.5	1.6	0.0	10	7.15
	B1.5	−0.25	4.333	0.000	− 5.2	−3.3	1.6	0.0	9.3	7.22
	B2	−0.24	4.312	0.000	− 4.8	−3.0	1.5	0.0	8.4	7.29
	B2.5	−0.22	4.283	0.000	− 4.3	−2.7	1.5	0.0	7.3	7.40
	B3	−0.20	4.253	0.000	− 3.8	−2.3	1.4	0.0	6.4	7.51
$X_e=0.739$, $Z_e=0.044$, Convective (only above $10\,\mathfrak{M}_\odot$).................	O7	−0.32	4.544	0.047	−10.3	−7.0	3.7	3.0	73	6.69
	O8	−0.31	4.525	0.048	− 9.9	−6.8	3.6	3.0	57	6.74
	O9	−0.31	4.505	0.047	− 9.6	−6.5	3.6	3.0	48	6.80
	O9.5	−0.30	4.491	0.047	− 9.3	−6.3	3.6	2.9	42	6.83
	B0	−0.30	4.477	0.047	− 9.1	−6.2	3.6	2.9	37	6.87
	B0.5	−0.28	4.418	0.034	− 7.9	−5.3	3.2	2.5	24	7.03
	B1	−0.26	4.354	0.020	− 6.6	−4.6	2.7	2.0	16	7.21
	B1.5	−0.25	4.333	0.014	− 6.0	−4.1	2.4	1.7	14	7.27
	B2	−0.24	4.312	0.010	− 5.4	−3.7	2.1	1.3	12	7.33
	B2.5	−0.22	4.283	0.008	− 4.8	−3.2	1.8	1.0	9.8	7.44
	B3	−0.20	4.253	0.006	− 4.3	−2.8	1.8	1.0	8.7	7.54

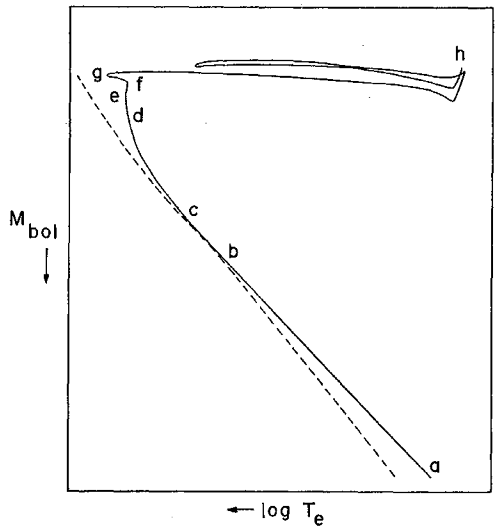

FIG. 1.—Locus of constant time for a young cluster in the theoretical H-R diagram. Critical segments are: (*a*)–(*b*) pre–main-sequence contraction; (*b*)–(*c*) unevolved main sequence; (*c*)–(*d*) main-sequence turnoff; (*d*)–(*f*) main-sequence turnup, where *e* is the point of maximum effective temperature and *f* is the tip of the turnup; (*f*)–(*g*) secondary contraction; (*g*)–(*h*) post–main sequence. The dashed line represents the unevolved main sequence (ZAMS) for stars of all masses.

color refer); (4) "width" of the turnup, defined by the difference between the maximum value of log T_e and the value of log T_e at the tip of the turnup; (5) absolute bolometric magnitude of the tip of the turnup; (6) absolute visual magnitude of the tip of the turnup; (7) "height" of the tip of the turnup, defined as the difference in absolute magnitude between the tip of the turnup and the point on the ZAMS having the same log T_e; (8) "length" of the turnup, defined as the difference in absolute magnitude between the tip of the turnup and the point on the less evolved portion of the turnup having the same log T_e; (9) the "expected" mass of a star in the helium-burning (or later) stages of evolution, defined as the mass of a star whose TAMS age is equal to 0.9 times the age of the turnup; and (10) the age of the turnup. Conversions from theoretical to observational quantities have been effected by tables prepared by Morton (1969) for O5–B0.5 main-sequence stars and by Morton and Adams (1968) for B1–B4 main-sequence stars. In the event that these calibrations become significantly revised, table 3 can be altered most simply by revising the columns for spectral type, color, and M_v.

In nearly all cases the theoretical turnup is well defined by its maximum effective temperature. However, both the length and width of the turnup are larger for a younger age. The characteristics of the turnup are affected further by the choice of initial chemical composition, mode of evolution, and (not calculated here) rotation and magnetic fields. A larger value of X_e or a larger convective-core size increases the age and length of the turnup and the expected supergiant mass, while a larger value of Z_e increases the length of the turnup and the expected supergiant mass, but decreases the age (because of a shift of higher masses to the effective temperature of the turnup). Uniform axial rotation produces the same qualitative effects on the turnup as does a larger value of Z_e (e.g., Maeder 1971), but the effects of highly nonuniform rotation may mimic the effects of a larger X_e (Bodenheimer 1971). Interior magnetic fields are still largely an unknown factor, but their strength is probably nowhere near being comparable to gas pressure very deep in the interior, as we may infer from the absence of severe splitting or broadening of the spectral lines in known remnants of binary mass exchange (Stothers 1972*c*). Although moderate fields are possible and may be important for the rotation (e.g., Mestel 1965), we shall ignore their direct effects on the luminosity and effective temperature.

III. MASSES

a) Upper-Main-Sequence Stars

Several recent reviews of the empirical data for upper-main-sequence stars (Hack 1963; Harris, Strand, and Worley 1963; Cester 1965; McNamara 1966; Plavec 1967; Popper 1967; Stephenson and Sanwal 1969) have encouraged a number of new attempts to fit theoretical stellar models to the observational data, particularly in regard to masses, radii, luminosities, and effective temperatures (Morris and Demarque 1966; Ruben and Masevich 1966; Iben 1967; Barbaro and Fabris 1968; Morton and Adams 1968; Popov 1968; Kříž 1969; Hutchings and Hill 1971). Most authors concur that, while agreement between theory and observation seems to be satisfactory, a precise determination of the chemical composition of extreme Population I stars is still not possible by using models for stellar interiors. Generally speaking, eclipsing binary systems have been the major source of fundamental data, of which the greatest quantity and quality are probably spectral types and masses. In view of (1) the rather limited character of previous investigations (confined mostly to the ZAMS) and (2) our extensive network of unevolved and evolved models, it seems worthwhile to utilize more of the available observational material than is usually done, in an effort to extend earlier results.

Table 4 contains data for 39 eclipsing binary systems with completely determined masses and with at least one component having a spectral type in the range O5–B4. (This table is, incidentally, complete for all Wolf-Rayet stars and all supergiants with completely determined masses.) The data are arranged as follows.

Columns (1) and (2): HD number and name of the binary system.

Column (3): orbital period in days, from Batten (1967).

Columns (4) and (5): spectral type and luminosity class of the brighter and fainter components (MK classes when available) from Jaschek, Conde, and de Sierra (1964), Sahade (1962), Cester (1965), Batten (1967), Lesh (1968*b*), Olson (1968*a*), Hiltner, Garrison, and Schild (1969), Kříž (1969), Stephenson and Sanwal (1969), Garrison (1970), Wright (1970), or Herbison-Evans *et al.* (1971).

Columns (6) and (7): masses (in solar units) of the brighter and fainter components from Batten (1967, 1968), Sahade (1962), Cester (1965), Wright (1970), or Herbison-Evans *et al.* (1971).

Columns (8) and (9): projected rotational velocity (in km s^{-1}) of the brighter and fainter components, from Boyarchuk and Kopylov (1964), Koch, Olson, and Yoss (1965), Hill (1967), Olson (1968*b* and private communication), Slettebak (1968), or Wright (1970).

Stellar mass is plotted against spectral type for 53 early-type components in figure 2. The most reliably determined masses (Harris *et al.* 1963; Popper 1967; Batten 1968) are indicated by filled symbols. Theoretical loci for the ZAMS and TAMS are shown only for two of the initial chemical compositions. Loci for $(X_e, Z_e) = (0.602, 0.044)$ are nearly identical to those for $(X_e, Z_e) = (0.739, 0.021)$ and therefore are not plotted. Use of the "convective" models for the TAMS, instead of the "radiative/semiconvective" models that we have preferred, introduces little change into figure 2.

Examination of the figure shows that, at a given mass, (1) the earliest spectral type that is observed agrees well with the theoretical ZAMS and (2) later spectral types are associated with more evolved stars (brighter luminosity classes), as expected theoretically for chemically inhomogeneous evolution. However, there seem to be too many stars of too late spectral type, which cannot be explained by the small number of stars expected to be evolving in the rapid pre– and post–main-sequence phases. Interaction between the binary components may give rise to a spurious spectral type (Batten 1970), particularly for the highly discrepant "contact" system EO Aur; and a few of the systems are known, or suspected, to have suffered mass exchange and possible helium enrichment (e.g., Plavec 1967, 1971). Nevertheless, most of the systems appear to be normal, and Stephenson and Sanwal (1969), in their discussion of some of the evolved sys-

TABLE 4

EARLY-TYPE ECLIPSING BINARY SYSTEMS WITH COMPLETELY DETERMINED MASSES

HD	Name	P (days)	Sp₁	Sp₂	$\mathfrak{M}_1/\mathfrak{M}_\odot$	$\mathfrak{M}_2/\mathfrak{M}_\odot$	$v_1 \sin i$ (km s^{-1})	$v_2 \sin i$ (km s^{-1})
1337...	AO Cas	3.5	O9 III	O9 III	19	26	160	124
25204...	λ Tau	4.0	B3 V	A4 IV	6.1	1.6	75	...
25833...	AG Per	2.0	B4	B5	5.2	4.6	91	72:
32068...	ζ Aur	972	K4 Ib	B6: V	8.0	5.8	< 10	...
33088...	TT Aur	1.3	B3	B7	6.7	5.3	...	...
33357...	SX Aur	1.2	B3 V	B3 V	11.2	5.9	240	...
34333...	EO Aur	4.1	B3 III	B3 III	28.3	28.3	...	...
36486...	δ Ori A	5.7	O9.5 II	B	28	10	141	...
42933...	δ Pic	1.7	B0.5 III	B0.5–3	16.3	7.3	270	...
44701...	IM Mon	1.2	B3	B8	9.0	6.0	...	...
47129...	...	14.4	O8 V	O8f	~51	~64	130	110
57060...	UW CMa	4.4	O7f	O7:	24	29	141	...
65818...	V Pup	1.5	B1 V	B2:	18	10	180	...
77464...	CV Vel	6.9	B2 V	B2 V	5.6	5.4	...	...
116658...	α Vir	4.0	B1.5 IV–V	B3 V	10.9	6.8	172	...
151890...	μ¹ Sco	1.4	B1.5 V	B6	14	9	220	...
156247...	U Oph	1.7	B4 V	B5 V	5.0	4.5	107	87
156633...	u Her	2.1	B1.5: V	B5	7.9	2.8	119	90:
163181...	V453 Sco	12.0	B0.5 I:	B	27	30	...	...
168206...	CV Ser	29.7	WC7	B	8	25	...	...
173787...	V356 Sgr	8.9	B3 V	A2 II	12.1	4.7	350	90
175227...	DI Her	10.6	B5 III	B4 III	3.7	4.1	...	...
176853...	V599 Aql	1.8	B4 III	B8 III	11.8	7.3	...	...
181987...	Z Vul	2.5	B4 V	A2–3 III	5.4	2.3	155	...
185507...	σ Aql	2.0	B3 V	B3 V	6.8	5.4	123	142
187879...	V380 Cyg	12.4	B1 III	B3 V	14.1	8.0	88	...
190967...	V448 Cyg	6.5	B1 Ib–II	O9.5 V	17.5	22.4	170	...
227696...	V453 Cyg	3.9	B0.5 IV	B0.5 IV	17.8	13.7	260	...
192577...	31 Cyg	3784.3	K4 Ib	B4 V	9.2	6.2	< 10	...
192909...	32 Cyg	1140.8	K5 Iab	B4 IV–V	19	10	< 10	...
228854...	V382 Cyg	1.9	O7	O8	37.4	32.8	...	...
228911...	V470 Cyg	1.9	B2	B2	12.5	11.0	...	...
193576...	V444 Cyg	4.2	WN5	O6	10	25	...	...
193611...	V478 Cyg	2.9	B0 V	B0 V	14.6	14.8	...	...
198846...	Y Cyg	3.0	B0 IV	B0 IV	17.5	17.5	145	140
208816...	VV Cep	7450	M2 Ia:	B	~20	~20	~ 0	...
211853...	GP Cep	6.7	WN6	B0: I:	7.6	19.6	...	...
216014...	AH Cep	1.8	B0.5 IV	B0.5 IV	16.1	13.9	210	195
218066...	CW Cep	2.7	B1.5 V	B1.5 V	10.0	9.8	>215	...

tems, have inferred that mass loss has probably been unimportant for most of the stars.[1]

If the scatter of late spectral types in figure 2 is assumed to be due solely to a cosmic variation in initial chemical composition, we find that the range in hydrogen content is $X_e = 0.60$–0.80 for $Z_e = 0.044$, or that Z_e ranges for fixed X_e as follows:

$$X_e = 0.60\,, \quad Z_e = 0.044\text{–}{>}0.10\,;$$

$$X_e = 0.74\,, \quad Z_e = 0.021\text{–}\ \ 0.08\,;$$

$$X_e = 0.80, \quad Z_e = 0.015\text{–}\ \ 0.044\,.$$

[1] Schöneich (1966) has criticized strongly the inference of mass loss and evolution down the main sequence suggested by Sobolev (1961) on the basis of luminosities derived from (probably uncertain) radii of components of eclipsing binary systems.

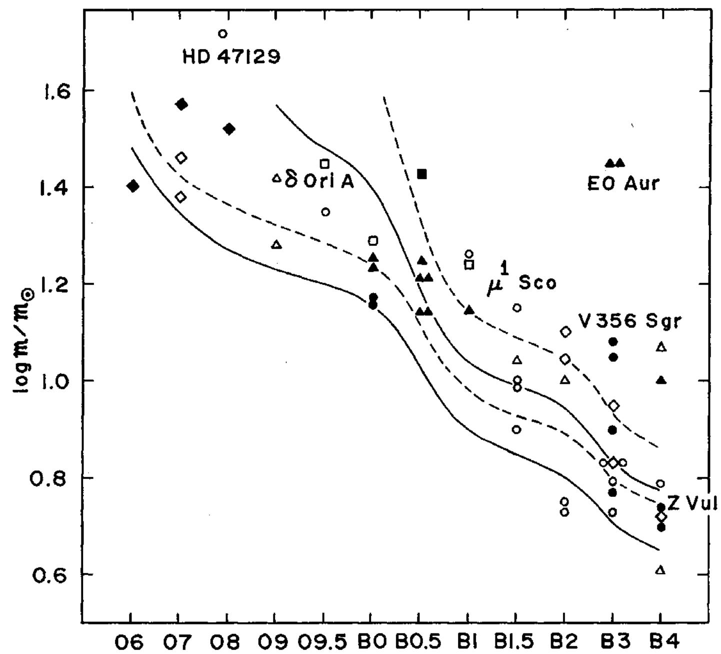

FIG. 2.—(Mass, spectral type)-diagram for observed stars (*symbols*) and theoretical stellar models (*lines*). Empirical masses have been derived from eclipsing binary orbits; the most reliable masses are represented by filled symbols. The shape of a symbol refers to the stellar luminosity class: *squares*, I–II; *triangles*, III–IV; *circles*, V; *diamonds*, unknown luminosity class. The theoretical stellar models are based on no rotation and $(X_e, Z_e) = (0.739, 0.021)$ (*solid lines*) or $(0.739, 0.044)$ (*dashed lines*). The lower and upper lines of each type refer to the ZAMS and TAMS, respectively.

The most recent spectroscopic and other evidence, however, suggests that $N(\text{He})/N(\text{H}) = 0.10 \pm 0.02$ (e.g., Norris 1971) and $Z_e = 0.03 \pm 0.01$ (e.g., Morton 1968), implying that $X_e = 0.70 \pm 0.05$. Unless the cosmic variation in initial chemical composition is larger than expected, rotation is the likeliest source of the scatter in figure 2.

The projected rotational velocities listed in table 4, taken in conjunction with Bodenheimer's (1971) models of massive rotating main-sequence stars, appear adequate to explain the late spectral types in figure 2, if the latter are due to a reduction of the effective temperature. However, there is no obvious correlation between $v \sin i$ and spectral type (or luminosity class) at a given mass. This puzzling feature, as well as lack of information concerning the angle of axial inclination and the interior distribution of angular momentum for each particular star, does not permit us to transform figure 2 into a *unique* empirical diagram for "nonrotating" stars. Furthermore, we do not know precisely how spectral type is correlated with effective temperature and gravity for a rotating star (see Ireland 1967; Hardorp and Strittmatter 1968).

To circumvent these difficulties in part, we have determined luminosities for the O5–B4 components of seven eclipsing binary systems that are known members of clusters and nearby associations with accurately determined distance moduli. (Various members of more distant associations, discussed below, have less reliably determined luminosities.) We have corrected for interstellar reddening in the usual way and have adopted estimates of the magnitude difference between the binary components as quoted by Batten (1967). Intrinsic color and bolometric correction as a function of spectral type were taken from Morton (1969) and Morton and Adams (1968). An alternative method of obtaining luminosities, from empirical radii and effective temperatures of the components, is of considerably less value here, because luminosity is very sensitive to small errors in radius and effective temperature; this method demonstrably produces a large scatter in the mass-luminosity plane (e.g., Popov 1968). But, for five systems (Y Cyg, U Oph, V356 Sgr, μ^1 Sco, and Z Vul), the radii derived from the orbits seem to be well determined (Harris *et al.* 1963), and therefore luminosities will be deduced for them. In the case of α Vir, Herbison-Evans *et al.* (1971) have combined interferometric measurements with spectroscopic and photometric data to derive the mass and luminosity of the primary component. For the secondary component, they have simply applied the difference of apparent visual magnitude between the two components. It is worth noting that α Vir is the only binary system in table 4 that has a meaningfully determined trigonometric parallax (Jenkins 1963). But unfortunately its parallax is probably less accurate than the indirect parallax determined by Herbison-Evans *et al.* Luminosities for 17 early-type components of 12 binary systems are collected in table 5, where the O8f secondary of HD 47129 has been omitted because of its uncertain bolometric correction.

A mass-luminosity diagram of the 17 components is shown in figure 3, where the theoretical ZAMS and TAMS are also indicated. Loci for $(X_e, Z_e) = (0.602, 0.044)$ in this figure would be brighter than the loci shown for $(X_e, Z_e) = (0.739, 0.044)$ by about 0.7 mag at 5 $\mathfrak{M}_\odot$ and 0.2 mag at 60 $\mathfrak{M}_\odot$. Thus there is some indication that the initial hydrogen abundance of the stars plotted is closer to $X_e = 0.739$ than to 0.602 (unless Z_e is very large). Furthermore, agreement with the theoretical models for nonrotating (or approximately uniformly rotating) stars is excellent because almost none of the observed objects, which show projected rotational velocities of 107–260 km s^{-1}, is significantly underluminous for its mass. A striking reduction of luminosity has been predicted theoretically by Bodenheimer (1971) for large differential interior rotation, while no significant reduction of luminosity is predicted for even very fast uniform rotation (e.g., Sackmann and Anand 1970; Bodenheimer 1971).

The two possible exceptions in figure 3 are the primaries of μ^1 Sco and V356 Sgr. Significantly, the secondary of each system is more evolved, but has a lower mass, than the primary, which suggests that the primary was once the mass-accreting component dur-

TABLE 5

Reliably Determined Luminosities for the Components of Early-Type Eclipsing Binary Systems with Completely Determined Masses

Star	Sp	$M_{\rm bol}$	Group	Photometric $(m-M)_0$
AG Per.....	B4	−2.7	Per OB2	8.0
δ Ori A......	O9.5 II	−9.1	Ori OB1b	8.2
HD 47129...	O8 V	−8.5	Cr 106	10.7
μ^1 Sco.......	B1.5 V	−4.5*	Sco OB2	6.1
V448 Cyg...	B1 Ib–II	−6.8†	NGC 6871	11.5
	O9.5 V	−7.6		
V453 Cyg...	B0.5 IV	−6.4	NGC 6871	11.5
	B0.5 IV	−5.9		
CW Cep....	B1.5 V	−5.1	Cep OB3	9.5
	B1.5 V	−4.8		
U Oph......	B4 V	−2.6	...	...
Z Vul.......	B4 V	−3.3	...	...
Y Cyg......	B0 IV	−6.3	...	...
	B0 IV	−6.3		
V356 Sgr....	B3 V	−3.6	...	...
α Vir.......	B1.5 IV–V	−5.4	...	...
	B3 V	−3.0		

* $M_{\rm bol} = -4.3$ from the kinematic distance modulus (Jones 1970) or −4.6 from binary orbital data.

† $M_{\rm bol} = -7.3$ from binary orbital data.

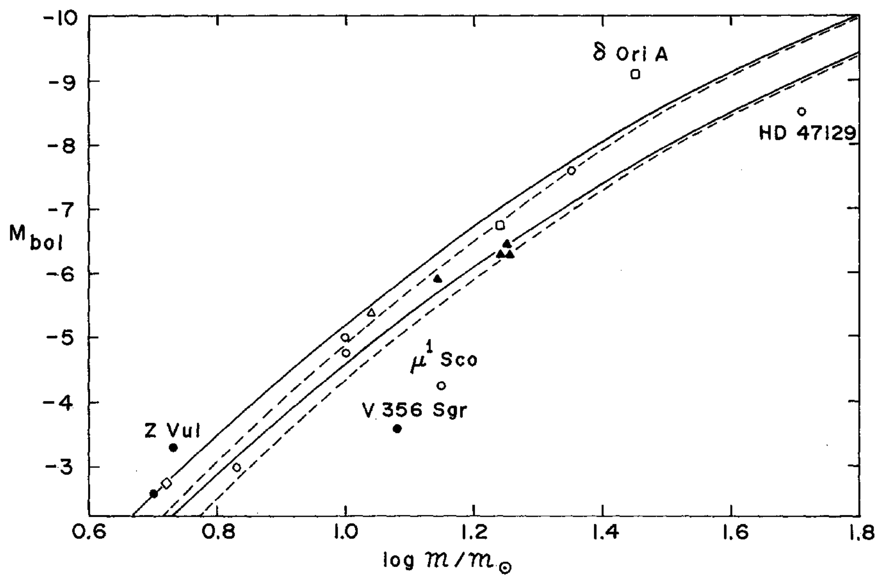

FIG. 3.—Mass-luminosity diagram for observed stars (*symbols*) and theoretical stellar models (*lines*). The meaning of the symbols and lines is the same as in fig. 2. The lower and upper lines of each type refer to the ZAMS and TAMS, respectively.

ing a previous mass exchange (Plavec 1967). On this basis, Stothers and Lucy (1972) have suggested that the underluminosity, lateness of spectral type, and approximate synchronism (or slight defect) of surface rotational period with respect to orbital period of these two primaries are due to rapid differential interior rotation, induced by the accretion of mass from their companions with a subsequent slowing of their surface layers by tidal friction. Although this explanation makes it difficult to explain the normal luminosity (or slight overluminosity) and normal spectral type of the primary of Z Vul, which has probably also experienced an accretion of mass (Plavec 1967), the masses of both components of this system are small, and rotation may never have been important in the primary, even with the acquisition of some material enriched in angular momentum. Or the system may be older than μ^1 Sco and V356 Sgr, so that tidal friction has slowed down the deep interior (but the surface rotational velocity seems to be somewhat too fast for synchronism). The "unevolved" components in other systems which may have suffered mass exchange—e.g., V448 Cyg—seem also to have normal luminosities and spectral types. In this connection, the *overluminosity*, but normal spectral type, of the primary of δ Ori A may also require explanation, although the mass for this star is rather uncertain.

In all, it appears probable from figures 2 and 3 that the rotation of *normal* main-sequence stars is approximately uniform, while departures from uniform rotation can be induced by mass accretion in a binary mass exchange as well as by normal evolutionary processes (e.g., *large* envelope expansion).

b) Supergiants

Five methods will be used here to derive masses for supergiants, since few orbital masses are available.

1. In a double-line eclipsing binary system, an "orbital" mass may be determined completely empirically (table 4).

2. For a pulsating variable, the pulsation-constant method yields a "pulsational" mass if the period, effective temperature, and luminosity (from the distance) are known (e.g., Stothers and Leung 1971; Parsons and Bouw 1971).

3. The surface gravity and the effective temperature, obtained from photometry or spectroscopy in conjunction with theoretical model atmospheres, determine the ratio of "atmospheric" mass to luminosity (e.g., Parsons and Bouw 1971).

4. Knowledge of the luminosity and adoption of the theoretical mass-luminosity relation for supergiants yields an "evolutionary" mass (Stothers 1969).

5. If the supergiant belongs to a cluster or association, a theoretically "expected" mass can be inferred from the spectral type of the main-sequence turnup (table 3).

A sixth method, yielding a "general-relativistic" mass, is based on the Einstein gravitational redshift of spectral lines and a knowledge of the stellar radius (Trumpler 1935). The accuracy of this approach for massive stars, however, is very doubtful (Stothers and Simon 1968 and references therein).

Since one (or more) of the foregoing methods depends on a theoretical mass-luminosity relation for supergiants, such a relation is given here, based on stellar models quoted in § IV and adjusted to fit an initial hydrogen content of $X_e = 0.70$.

(1) For helium-burning "blue" supergiants ($-11.3 \leq M_{\rm bol} \leq -5.5$),

$$\log (\mathfrak{M}/\mathfrak{M}_\odot) = 0.86 + 0.068\, M_{\rm bol} + 0.015\, M^2_{\rm bol}\,.$$

(2) For *young* helium-burning "red" supergiants ($-9.0 \leq M_{\rm bol} \leq -5.5$),

$$\log (\mathfrak{M}/\mathfrak{M}_\odot) = 0.62 - 0.023\, M_{\rm bol} + 0.008\, M^2_{\rm bol}\,.$$

(3) For carbon-burning "red" supergiants (with or without neutrino emission, $-9.0 \leq M_{\rm bol} \leq -5.5$),

$$\log (\mathfrak{M}/\mathfrak{M}_\odot) = 2.74 + 0.61\, M_{\rm bol} + 0.052\, M^2_{\rm bol}\,.$$

For *old* helium-burning "red" supergiants, the expression for the "blue" supergiants can be used.

Table 6 contains all the available data on masses of luminous supergiants for which three or more mass determinations have been made, except for the variable M-type supergiants in distant associations (for which see Stothers and Leung 1971). In addition, two mass determinations can be made for the blue supergiants in clusters and associations (§ IV). Evolutionary masses and expected masses in table 6 have been derived on the basis of an initial chemical composition of $(X_e, Z_e) = (0.70, 0.03)$; the B-type supergiants have been referred to the end of core hydrogen burning, however, rather than to the middle of core helium burning as for the other supergiants (see § IV).

With the exception of the poorly determined "atmospheric" masses and of the "expected" mass of ξ Per, the various methods of mass determination yield acceptable agreement with each other. ("Atmospheric" masses for supergiants can easily be in error by a factor of 2.) The large "evolutionary" mass of ζ Per suggests that this star is younger than the observed B1 turnup in Per OB2, an association which probably also contains the O7 star ξ Per. Apparently, none of the listed supergiants has suffered much mass loss (see also Stothers and Leung 1971), although some exchange of mass is known to be occurring, or to have occurred, in the five binary systems, as deduced from the spectroscopic appearance or, in some cases, the mass ratio of the components. This important conclusion about mass loss extends the results of Stephenson and Sanwal (1969) to stages of evolution beyond the main sequence.

IV. YOUNG CLUSTERS: STELLAR POPULATIONS AND AGES

Four characteristics can be used to date a young stellar group: (*a*) the distribution of pre-main-sequence stars on the H-R diagram, (*b*) the location of the main-sequence turnup on the H-R diagram, (*c*) the luminosity of the supergiants, and (*d*) the rate of expansion of the group. A comparison of the first three characteristics with the results of theoretical stellar-model calculations yields a *gravitational* or *nuclear* age of the group.

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TABLE 6

MASS DETERMINATIONS FOR SUPERGIANTS

Star	Sp	M_v*	Group*	Turnup Sp*	$(m-M)_0$*	Orbital $\mathfrak{M}/\mathfrak{M}_\odot$	Pulsa-tional $\mathfrak{M}/\mathfrak{M}_\odot$	Atmo-spheric $\mathfrak{M}/\mathfrak{M}_\odot$†	Evolu-tionary $\mathfrak{M}/\mathfrak{M}_\odot$	Expected $\mathfrak{M}/\mathfrak{M}_\odot$
GP Cep.......	B0: I:	−4.5	Cep OB1	B0.5:	12.4	19.6	...	...	14	16
V453 Sco......	B0.5 I:	−6.9	Sgr OB5	B0:	11.8	27	...	...	32	23
V448 Cyg.....	B1 Ib–II	−4.8	NGC 6871	B0	11.5	17.5	...	...	16	23
ζ Per.........	B1 Ib	−6.2	Per OB2	B1:	8.0	...	...	37	23	11
α Cyg.........	A2 Ia	−8.2	Cyg OB7	B0.5:	9.3	...	...	14	22	17
HD 33579.....	A5: Ia-0	−9.7	LMC	...	18.7	...	...	27	45	...
δ CMa........	F8 Ia	−7.6	Cr 121	B0.5:	9.0	...	...	34	18	17
32 Cyg........	K5 Iab	−5.4	Cyg OB7	B0.5:	9.3	19	...	...	13	17
VV Cep.......	M2 Ia:	−6.0	Cep OB2	O9.5:	9.5	∼20	18	...	18	29

* Group membership.

† Determined by combining M_v with the atmospheric analysis of ζ Per (Cayrel 1958), α Cygni (Groth 1961), HD 33579 (Przybylski 1968), and δ CMa (Parsons and Bouw 1971).

A combination of the fourth characteristic (deduced from the motion of either the group as a whole or an individual "runaway" star) with measurements of stellar position yields a *kinematic* age; this is a strictly empirical age. The principles involved in the application of these methods of dating have been discussed in detail elsewhere (e.g., von Hoerner 1957).

a) Pre–Main-Sequence Stars

Although none of the four methods is free from uncertainty, the greatest uncertainty is usually associated with the location of pre–main-sequence stars on the H-R diagram (or some equivalent diagram). The reasons are the following: (1) the pre–main-sequence stars, being faint, are often inadequately observed (when they are observed at all); (2) the points representing them on the H-R diagram show a wide scatter, much of which is probably due to the influence of circumstellar shells, to the fact that star formation is still going on, and to the presence of some stars that are not physical members of the group; and (3) the transformation from observational to theoretical quantities on the H-R diagram is still uncertain for these stars (Strom, Strom, and Yost 1971). For the foregoing reasons we shall not discuss this method of dating further.

b) The Main-Sequence Turnup

The location of the main-sequence turnup on the color-magnitude diagram is the basis of the "classical" method of dating. Despite the continued use of this method, a number of improvements can and will be made here, after pointing out what we believe to be inadequacies in earlier attempts.

The earliest modern work (Sandage 1957; Lohmann 1957; von Hoerner 1957), which is still occasionally quoted, relied on theoretical models of massive stars that were simply extrapolations of models with lower mass in which the effects of radiation pressure are unimportant; however, neglect of radiation pressure at high mass leads to stellar cores that are too small and luminosities that are too bright, and therefore to lifetimes that are too short by approximately a factor of 2 when the mass is greater than $\sim 10\,\mathfrak{M}_\odot$. This error was implicitly corrected in later work based on more realistic models of massive stars (Henyey, LeLevier, and Levée 1959; Sandage 1963; Gray 1963, 1965; Dluznevskaya 1966; Kippenhahn 1966; Kühn 1967; Hjellming 1968; Lindoff 1968; Barbaro, Dallaporta, and Nobili 1966; Barbaro, Dallaporta, and Fabris 1969; Schlesinger 1969, 1971; Simpson *et al.* 1970; though not Schmidt 1963). However, virtually no attention has so far been paid to the influence of the choice of initial chemical composition or of the mode of evolution on the inferred age of a cluster. On the observational side, the traditional use of a color-magnitude diagram involves the following risks: (1) observational errors affect mainly the unreddening of the stellar colors and the determination of the cluster distance (and therefore the stellar absolute magnitudes), and (2) color is relatively insensitive to effective temperature on the upper main sequence. In § II we have shown that the *spectral type of the main-sequence turnup* is, by itself, a good indicator of age (and is independent of distance) provided that luminosity classes as well as spectral types of the bright stars are observed.

This method will be applied to the clusters and subgroups of associations tabulated by Stothers (1969, 1972*b*) and reobserved, in a few cases, by Lesh (1968*a*) and by Schild (1970). In table 7 the groups are listed in order of spectral type of the main-sequence turnup; a colon indicates, usually, that the spectral type may be earlier than indicated—the result of a poorness in stars—and an "e" indicates the presence of one or more Be stars. Occasionally a turnup is so well populated that the formal resolution of the MK scale of spectral types can be exceeded if one carefully examines the spectral-type distribution of the turnup stars (e.g., B0.5–B1); in fact, a formal subdivision of the scale of stellar spectral types has recently been proposed by Walborn (1971). Ages based on the turnup are shown in column (3) of table 7 for an adopted chemical composition of

TABLE 7

NUCLEAR AGES OF YOUNG CLUSTERS WITH SUPERGIANT MEMBERS
(in Units of Years)

Cluster	Turnup Sp	Turnup log τ	Blue log τ	Red log τ	(1) log τ	(2) log τ	(3) log τ	(4) log τ	(5) log τ
Sco OB1....	O7	6.6	6.5	...	...	...	...	...	...
NGC 6231...	O9–O9.5	6.7	6.7	...	...	...	...	...	...
Ori OB1b....	O9–O9.5	6.7	...	...	...	...	...	...	...
NGC 6910...	O9.5::	6.8::	6.6	...	...	...	...	<7	...
NGC 6913.....	O9.5–B0	6.8	...	...	...	...	...	<7	...
IC 1805.....	B0:	6.8:	...	...	...	6.30	5.95	...	...
Pi 20.......	B0:	6.8:	6.4	...	...	...	...	...	...
NGC 6823...	B0:	6.8:	...	...	...	...	...	<7	...
NGC 6871...	B0(e)	6.8	...	...	...	...	...	<7	...
Cr 107......	B0–B0.5	6.9	...	...	...	...	...	...	...
IC 2581.....	B0.5::(e)	7.0::	6.9	...	...	...	...	...	...
h Per.......	B0.5:	7.0:	6.9	...	6.64	6.50	...	7.0:	7.08
Ori OB1a....	B0.5:(e)	7.0:	6.8	6.9:	...	...	...	...	...
NGC 4755...	B0.5–B1(e)	7.1	7.0	7.0	...	...	7.25	<7	6.85
NGC 3293...	B0.5–B1	7.1	...	7.0	...	...	...	<7	6.88
Sco OB2....	B0.5–B1	7.1	...	7.1	<6.60	...	6.40	...	...
NGC 957....	B1::	7.2::	6.9	...	...	...	...	7.19	...
NGC 457....	B1::(e)	7.2::	7.0	7.1	7.18	7.10	6.60	7.05	7.08
χ Per.......	B1:(e)	7.2:	7.2	6.9:	6.64	6.50	...	<7	7.08
Per OB2....	B1:	7.2:	...	...	6.74	...	6.55	...	...
NGC 3766...	B1.5–B2(e)	7.3	...	7.2	...	...	...	7.06	7.34
NGC 2129...	B2:(e)	7.4:	7.0	...	...	...	...	7.21	...
NGC 581....	B2:	7.4:	7.1	7.2	...	7.95	7.15	7.0:	7.16
NGC 7235...	B2:	7.4:	7.2	...	...	...	...	<7	...
NGC 6530...	B2(e)	7.4	7.2	...	...	...	6.05	<7	...

References to previously determined ages based on the turnup: (1) von Hoerner (1957); (2) Sandage (1963); (3) Gray (1965); (4) Lindoff (1968); (5) Barbaro *et al.* (1969).

$(X_e, Z_e) = (0.70, 0.03)$. For comparison, we list also, in columns (6)–(10), the ages determined previously by von Hoerner (1957), Sandage (1963), Gray (1965), Lindoff (1968), and Barbaro *et al.* (1969), respectively. It is worth mentioning that the observed and theoretical turnups agree very well in their "fine detail" for most of the groups.

Possible sources of error in our age determinations are the following: (1) cosmic variation of initial chemical composition, (2) uncertainty of the physical input data in the stellar models, (3) a "convective" rather than a "radiative/semiconvective" mode of evolution (for stars earlier than B0.5), (4) an intrinsic dispersion of the observed main-sequence turnup (half a spectral subclass), (5) uncertainty of the adopted Sp-T_e-B.C. relations, and (6) the effect of stellar rotation. Altogether, the likely maximum error in log τ is ± 0.2, although, in a poor cluster, uncertainty about the location of the turnup could result in a somewhat larger error.

c) Supergiants

In order to date clusters by the average luminosity of their supergiants (Stothers 1969), it is necessary to identify the particular stage of evolution attained by each supergiant—a task not easy in the case of supergiants located "above" the tip of the main sequence on the H-R diagram (Hayashi and Cameron 1962; Stothers and Lloyd Evans 1970). The theoretical main-sequence turnups derived in § II suggest that many of the early-type supergiants in the youngest stellar groups are still in the core-hydrogen-burning phase of evolution; their apparent displacement from the other stars on the turnup is simply due to the decreasing number of stars expected in the rapid last stages of core hydrogen burning. But other early-type supergiants seem to have luminosities appropriate to core helium burning, although their spectral types place them near the main-sequence turnup. The situation is complicated by the fact that the predicted difference in luminosity between the TAMS and the main phase of core helium burning is rather small:

$\mathfrak{M}/\mathfrak{M}_{\odot}$	$\Delta M_{\rm bol}$
9	0.6
15	0.4
30	0.3

Furthermore, both the predicted and observed effective temperatures of supergiants are very uncertain. Johnson's (1966) observational work suggests that the effective temperatures of supergiants are cooler than those of main-sequence stars of the same spectral type. If so, some published model sequences predict that the earliest spectral type attained during core helium burning should be almost equal to the spectral type of the main-sequence turnup. Other published models, however, predict that the earliest type attained should be somewhat later. At the present time we are confronted with a class of supergiants whose evolutionary stage remains uncertain, separated on the H-R diagram from those supergiants that are definitely in a post–main-sequence stage of evolution.

All the stellar groups of table 7 are known to contain supergiants, which can now be divided into three major classes of stars: (1) main-sequence stars, (2) possible post–main-sequence stars, and (3) definite post–main-sequence stars. (Very few of the listed supergiants are expected to be pre–main-sequence stars.) The three adopted classes are differentiated in table 8, where $\langle M_{\rm bol}\rangle$ refers to the definitely post–main-sequence stars: "blue" and "red" refer to spectral types of O, B, A and of M, respectively. The collected data have been taken from Stothers (1969, 1972*a*, *b*). The trend of the properties of these supergiants with spectral type of the main-sequence turnup confirms the empirical relationships suggested by Schild (1970); but our larger statistics indicate, further, that main-sequence supergiants are confined to spectral types B1 and earlier, and that the average luminosity class of the M-type supergiants changes with spectral type of the turnup approximately as follows (supplementary data from Stothers 1972*a*): Ia being found at B0, Iab at B0.5, and Ib at B2. Fifty-five of the 60 supergiants of table 8 are shown in a composite H-R diagram in figure 4, in which the relative distributions of supergiants are expected to be more useful for detailed evolutionary studies than are those in previous composite H-R diagrams that contain only specific kinds of supergiants and/or supergiants of uncertain group membership (Kopylov 1955, 1959; Johnson *et al.* 1969; Barbaro *et al.* 1966; Sedyakina 1969; Stothers 1969; Humphreys 1970; Eggen 1971).

Depending on the number and extent of the evolutionary "loops" on the H-R diagram (fig. 1), the luminosity ascribed to a supergiant of specified mass and spectral type may vary. If we reckon the beginning of "post–main-sequence" evolution from the point of complete exhaustion of hydrogen at the stellar center (point *g* in fig. 1), it becomes convenient to consider the post–main-sequence supergiants as being distributed (mostly) among three broad subclasses: (1) "blue" supergiants burning core helium, (2) "red" supergiants burning core helium, and (3) "red" supergiants burning core carbon. Since the local segment of an evolutionary track for a massive star in the H-R diagram is *approximately* horizontal, the age of a supergiant can, in each case, be almost uniquely correlated with its luminosity. Theoretical models are available for evolved stars with masses of 9–100 $\mathfrak{M}_{\odot}$ (Hayashi, Hōshi, and Sugimoto 1962; Iben 1966*a*, *b;* Stothers 1966; Hofmeister 1967; Stothers and Chin 1968, 1969; Chiosi and Summa 1970; Paczyński 1970; Simpson 1971). By interpolation among these models, the ages can be

TABLE 8

SUPERGIANT MEMBERS OF YOUNG CLUSTERS

Cluster	Turnup Sp	Supergiant Sp Main Sequence	Supergiant Sp Post-Main Sequence?	Supergiant Sp Post-Main Sequence	Blue ⟨M_{bol}⟩	Red ⟨M_{bol}⟩
Sco OB1	O7	...	...	O9 Ia, B1.5 Ia-0	−10.4	...
NGC 6231	O9–O9.5	O9 Ib, O9.5 Ib, O9.5 Ib, O9.5 Ib	O7f, O8f, O8f, B0.5 Ia	B0.5 Ia, B1 Ia	− 9.3	...
Ori OB1b	O9–O9.5	...	O9.5 Ib, B0 Ia, B0.5 Ia	...	...	...
NGC 6910	O9.5::	...	...	B1.5 Ia	− 9.7	...
NGC 6913	O9.5–B0	B0: I:, B0.5 Ib	B1 Iab	...	...	...
IC 1805	B0:	...	O5f, B0.5 Ia	...	...	...
Pi 20	B0:	O9.5 Ib, B0 Ib	...	B1 Ia:	−10.9	...
NGC 6823:	B0:	B0.5 Ib	...	...	...	...
NGC 6871	B0(e)	O9.5 I, B0 Ib, B1 Ib–II	B1 Ib, B1 Ib	...	...	...
Cr 107	B0–B0.5	...	B1 Ib	...	...	...
IC 2581	B0.5::(e)	...	...	B2.5 Ib, A7 Ia	− 8.4	...
h Per	B0.5:	...	...	B2 Ia, B3 Ia	− 8.2	...
Ori OB1a	B0.5:(e)	...	...	B8 Ia, M2 Iab–Ib	− 8.6	−7.5:
NGC 4755	B0.5–B1(e)	...	...	B1.5 Ib, B2 Ib, B3 Ia, B9 Iab, M2 Iab–Ib	− 7.8	−7.2
NGC 3293	B0.5–B1	...	B0 Ib, B0.5 Ib	M0 Iab	...	−7.3
Sco OB2	B0.5–B1	...	...	M1 Iab–Ib	...	−6.7
NGC 957	B1::	...	...	B1.5 Ib	− 8.0	...
NGC 457	B1::(e)	...	...	B6 Ib, F0 Ia, M1.5 Ib	− 7.5	−6.6
χ Per	B1:(e)	...	...	B2 Ib, M4.5 Iab–Ib	− 6.5	−7.5:
Per OB2	B1:	...	B1 Ib	...	...	...
NGC 3766	B1.5–B2(e)	...	...	M1 Ib:, M1 Ib–II:	...	−5.8
NGC 2129	B2:(e)	...	...	B3 Ib	− 7.5	...
NGC 581	B2:	...	...	B5 Ib, M0.5 Iab–Ib	− 6.9	−6.1
NGC 7235	B2:	...	...	B9 Iab	− 6.7	...
NGC 6530	B2(e)	...	...	B9 Iab	− 6.4	...

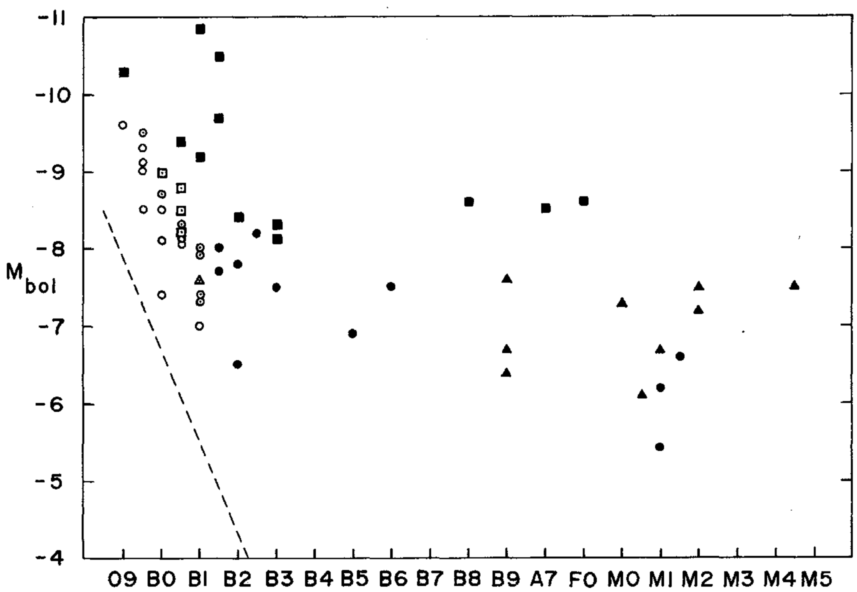

FIG. 4.—Bolometric H-R diagram for observed supergiants in young clusters and subgroups of associations. The shape of a symbol refers to the stellar luminosity class: *squares*, Ia; *triangles*, Iab; and *circles*, Ib. The degree of filling of a symbol refers to the stellar evolutionary phase: *open*, main sequence; *dotted*, possibly post-main sequence; *filled*, definitely post-main sequence. The dashed line represents the mean locus of luminosity class V stars. The scale of spectral types is contracted between B9 and M0.

adjusted to fit an initial hydrogen abundance of $X_e = 0.70$ (to an accuracy of ± 0.04 in $\log \tau$). Thus:

(1) For helium-burning "blue" supergiants ($-11.3 \leq M_{\rm bol} \leq -5.5$),

$$\log \tau \text{ (years)} = 8.37 + 0.18\, M_{\rm bol}\,.$$

(2) For *young* helium-burning "red" supergiants ($-9.0 \leq M_{\rm bol} \leq -5.5$),

$$\log \tau \text{ (years)} = 8.29 + 0.18\, M_{\rm bol}\,.$$

(3) For carbon-burning "red" supergiants (with or without neutrino emission, $-9.0 \leq M_{\rm bol} \leq -5.5$),

$$\log \tau \text{ (years)} = 8.42 + 0.18\, M_{\rm bol}\,.$$

For *old* helium-burning "red" supergiants, the expression for the "blue" supergiants can be used. The quoted ranges of $M_{\rm bol}$ correspond to a mass range of 9–100 $\mathfrak{M}_\odot$ (for blue supergiants) and of 9–30 $\mathfrak{M}_\odot$ (for red supergiants). For any other choice of X_e, the ages should be adjusted by using the approximate expression $\Delta \log \tau / \Delta X_e = 1.0$. Although the adopted models do not include the effects of rotation (Iben 1967; Kippenhahn, Meyer-Hofmeister, and Thomas 1970), mass loss (Hayashi *et al.* 1962; Forbes 1968; Stothers and Chin 1970), or the "convective" mode of evolution (§ II), these effects are found to change the luminosity very little, and the main source of error in dating actual supergiants is observational, amounting to ± 0.2 in $\log \tau$.

Ages of stellar groups, based on the luminosities of their supergiant members, are listed in columns (4) and (5) of table 7. We have assumed that all of the definitely post-main-sequence supergiants are burning core helium, partly on the basis of the slightly greater brilliance of the blue supergiants compared with the red supergiants in the same stellar group and partly on the basis of evidence presented elsewhere (Stothers 1972*b* and references therein). If the red supergiants were burning core carbon, their ages would be systematically older by $\Delta \log \tau = 0.13$, i.e., in the direction away from agreement with the ages of the blue supergiants (except in χ Per). As far as the stellar groups are concerned, ages based on supergiant luminosities agree quite well with the ages based on the spectral type of the main-sequence turnup, except for a slight systematic tendency for the former ages to fall below the latter. The assumption of core carbon burning or of mass loss in the red supergiants would only increase the discrepancy. However, a smaller initial hydrogen abundance (but see § III), or a larger initial metals abundance, or a significant uniform rotation on the main sequence would produce better agreement between the ages in the discrepant cases. A quantitative evaluation of these effects is difficult to make because the standard calibration of the B-type main-sequence stars, on which the supergiant luminosities depend, is itself uncertain and dependent on these effects in an unknown way.

d) Kinematics of Expanding Groups

The rates of expansion of the subgroups of several nearby associations have been measured accurately enough to derive reliable kinematic ages (Blaauw 1964; Lesh 1968*a*, 1969). Furthermore, a number of individual high-velocity stars of early spectral type have been identified by their space motions as having originated from associations (Blaauw 1961, 1964); these stars also yield kinematic ages. In table 9 (an updated version of Blaauw's [1964] table 2), kinematic and revised nuclear ages are compared for six stellar groups. We confirm that, in all cases but one, the nuclear age is much longer than the kinematic age. Since this fact is often ignored in theories of the origin of expanding associations (and in theories of stellar evolution, too!), it deserves some brief explanation.

The smallness of the kinematic age in the case of the "runaway" stars is possibly understandable, in Blaauw's picture, if his mechanism for producing them from sudden catastrophic disruption of a binary system by explosion of the primary is correct. However, in the case of the *general expansion* of a stellar group, the expansion might be expected, on the simplest theoretical grounds, to have started early—at (or before) the epoch of formation of the brightest stars (Blaauw 1961, 1964, and references therein). But it may be, according to van Albada's (1968) picture, that an association is gravitationally bound at first, experiencing a slow initial phase of disintegration due to the escape of an occasional star (see Spitzer 1940; King 1958*a*), and only later undergoes a more rapid phase of overall expansion due to the formation and disintegration of multiple systems of stars; this process could also form, every once in a while, a runaway star of high velocity. Alternatively, a close tidal encounter with a massive interstellar cloud may also set the association into rather sudden expansion (Spitzer 1958; King

TABLE 9

KINEMATIC AGES OF SUBGROUPS OF ASSOCIATIONS
(in Units of Years)

Subgroup	Turnup Sp	Turnup log τ	Expansion log τ	Runaway log τ	Runaway Stars	Ref.
Ori OB1d......	<O6	<6.5	5.5	...	...	1
NGC 1502.....	B0:	6.8:	...	6.3	α Cam	1, 2
Ori OB1a......	B0.5: (e)	7.0:	6.7	6.5	AE Aur, μ Col, 53 Ari, α Ori?	1, 3
Sco OB2......	B0.5–B1	7.1	7.3	6.0	ζ Oph	1, 4
Per OB2......	B1:	7.2:	6.1	6.2	ξ Per	1, 5
Lac OB1b.....	B1.5(e)	7.3	6.4	...	...	1, 5

REFERENCES.—(1) Blaauw 1964; (2) Purgathofer 1961; (3) Lesh 1968*a*; (4) Garrison 1967; (5) Lesh 1969.

1958*b*). Therefore it is perhaps not so puzzling that the kinematic age of an association is often so much shorter than its nuclear age.

V. CONCLUSION

A detailed grid of stellar models for the upper main sequence (5–120 $\mathfrak{M}_\odot$) has been computed with the following new features: (1) a variety of initial chemical compositions, (2) complete evolutionary tracks to the end of the main phase of core hydrogen burning, and (3) a variety of assumed modes of evolution. The evolutionary tracks have been converted into loci of constant time, for which the most characteristic parameter is found to be the effective temperature (spectral type) of the turnup on the H-R diagram.

To test observationally the theoretical stellar models (including published models for the post–main-sequence phases), reliable data for eclipsing binary systems and for members of young stellar groups have been collected. The empirical (mass, spectral type)-diagram suggests that (1) currently adopted initial chemical compositions are reasonable, (2) evolution proceeds off the upper main sequence with a growing chemical inhomogeneity in the stellar interior, and (3) axial rotation and/or some variation of initial hydrogen or metals abundance is important for massive stars. The empirical mass-luminosity diagram provides the following additional information: (1) the rotation of normal upper-main-sequence stars is approximately uniform, (2) mass accretion by a star in a close binary system and subsequent tidal friction due to the gravitational field of the companion may induce highly nonuniform rotation in the star, and (3) mass loss seems to be rather small for normal massive stars (including supergiants, for which orbital masses have been supplemented by masses determined by four other methods).

The distribution of stars on the H-R diagram of young stellar groups suggests that the most likely mode of evolution is either the "radiative" mode or one of the "semiconvective" modes; the "convective" mode is probably ruled out by available observational data for the supergiants (see Stothers 1970). Ages of the stellar groups have been determined by (1) the spectral type of the main-sequence turnup (which is independent of distance) and (2) the average luminosity of the supergiants. Both methods yield ages for a group that probably have a maximum error of 60 percent and are in fairly good agreement with each other. A slight systematic difference seems to exist for some stellar groups, and may be due either to a larger initial metals abundance than normal or to a fast uniform rotation of the upper-main-sequence stars (but a correlation with the number of Be stars present is not apparent). The published kinematic ages of the stellar groups (with one exception) are much shorter than their nuclear ages—as previously noted by Blaauw (1964)—but a definitive explanation of this is likely to come only from further dynamical studies of these groups.

Further research is desirable in virtually all areas of the present investigation. Particularly useful would be improved spectral types, luminosity classes, and rotational velocities for the components of those eclipsing binary systems with the most reliably determined masses. Many more clusters and subgroups of associations should be studied with a view toward their possible supergiant membership (especially the rare yellow supergiants) and the precise location of their main-sequence turnups on the H-R diagram. Accurate model atmospheres could improve our knowledge of the metals abundance and masses of extreme Population I stars (especially supergiants). Finally, studies of stellar evolution could usefully concentrate on the effects of small changes of initial chemical composition on the properties of supergiants, the role of semiconvection in the interior, the evolution off the main sequence of stars with initially uniform rotation, and the effects of accretion of mass and angular momentum (with tidal friction included) on stars in close binary systems.

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