

A RECALIBRATION OF THE ABSOLUTE MAGNITUDES OF SUPERGIANTS

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Because of the usefulness of supergiants in studies of galactic structure, an improved calibration of the visual absolute magnitudes for luminosity class I stars is derived. Significant differences from the results of Blaauw and Schmidt-Kaler are found at some MK classes.

Key words: supergiants — absolute magnitudes

I. Introduction

Absolute magnitudes of O and B main-sequence stars have been derived by many authors and may be considered well known, although small unexplained discrepancies still subsist among the various determinations (Lloyd Evans 1972). But the absolute magnitudes of the derivatives of O and B main-sequence stars — the bright supergiants — are still poorly known because of their relative rarity. Improved statistics will be compiled in the present paper to obtain a reliable luminosity calibration for supergiants, since absolute magnitudes of supergiants are useful for galactic kinematical studies, for gauging the distances of remote associations, and for delineating spiral arms.

Luminosities of supergiants are obtained most accurately by the method of group membership. Accordingly, we shall use distance moduli of stellar groups that have been determined from the magnitudes of B-type main-sequence stars, using an extinction ratio of $A_V/E_{B-V} = 3.0$. But, for the supergiants, A_V/E_{B-V} will be taken to be a function of spectral type as follows (Schmidt-Kaler 1965; Lee 1970)

Sp.	A_V/E_{B-V}
O, B, A	3.0
F, G, K	3.3
M	3.6

Intrinsic ($B-V$) colors have been adopted from Johnson (1966) for O-K supergiants and from Lee (1970) for M supergiants. The major uncertainty in the derived values of M_v for the supergiants arises from (1) the assumed difference of extinction ratio between the supergiants and the B-type main-sequence stars and (2) the adopted

M_v of the latter stars. Supergiant data used in the construction of mean absolute magnitudes are discussed below, but it is convenient here to present the final results in Table I. Unlike some earlier calibrations, we have found no significant trend of visual absolute magnitude within a spectral subclass for the supergiants.

II. O, B, and A Supergiants

Earlier work on the absolute magnitudes of blue supergiants depended on relatively few members of clusters and associations (Keenan and Morgan 1951; Kopylov 1955, 1959; Johnson and Hiltner 1956; Johnson and Iriarte 1958; Keenan 1963; Blaauw 1963; Feast 1963; Weaver and Ebert 1964; Schmidt-Kaler 1963, 1965; Lesh 1968*b*; FitzGerald 1969). Our newly enlarged compilation of reliable members of clusters and subgroups of associations (Stothers 1969*a*, 1972*a*) can be used to improve the luminosity calibration of these supergiants. The adopted list of OB members is given in Stothers (1972*b*), but we shall add here the members of the inner group of Perseus OB1. Unfortunately, for the A-type supergiants we have available in the quoted list only the star in IC 2581, which we shall supplement by probable members of associations (Humphreys 1970*a*), adopting Ruprecht's (1966) association boundaries and Humphreys's (1970*a*) tabulated radial velocities as criteria of membership; these stars are listed in Table II, where we note that HD 164514 may be a member of NGC 6514 in Sagittarius OB1, and HD 111613 (A1 Ia) has been omitted from the table but may possibly belong to NGC 4755 (Feast 1963). Distance moduli of the groups have been taken from Stothers (1969*a*, 1972*a*), section IV, or as follows: Cygnus OB7 (Schmidt 1958) and Camelopardalis

TABLE I
MEAN VISUAL ABSOLUTE MAGNITUDES FOR MASSIVE SUPERGIANTS

Sp.	M_v			Standard Deviation of a Single Star (number of stars in parentheses)		
	<i>Ib</i>	<i>Iab</i>	<i>Ia</i>	<i>Ib</i>	<i>Iab</i>	<i>Ia</i>
OB	-6.0	-6.3	-7.0	± 0.5 (29)	± 0.6 (5)	± 0.5 (14)
A	—	-6.3	-7.0	—	± 0.2 (4)	± 0.7 (15)
F	—	-6.9	-7.8	—	— (2)	± 1.0 (5)
G	—	-6.9	-7.9	—	— (1)	± 1.0 (4)
M	-5.0	-5.6	-6.5	± 0.3 (6)	± 0.4 (20)	± 0.6 (10)

TABLE II
PROBABLE A SUPERGIANT MEMBERS OF VERY
YOUNG CLUSTERS AND ASSOCIATIONS

Assoc.	$(m-M)_0$	Star	Sp.
Sgr OB1	11.0	HD 165784	A2 <i>Ia</i>
		HD 164514	A5 <i>Ia</i>
Cyg OB7	9.3	α Cyg	A2 <i>Ia</i>
Cep OB2	9.5	ν Cep	A2 <i>Ia</i>
Cep OB1	12.4	HD 213470	A3 <i>Ia</i>
Cas OB5	12.5	HD 223960	A0 <i>Ia-0</i>
		HD 223385	A3 <i>Ia-0</i>
Cas OB7	12.1	HD 3940	A1 <i>Ia</i>
Per OB1 (inner)	11.5	HD 14433	A1 <i>Ia</i>
		HD 14535	A2 <i>Ia</i>
Per OB1 (outer)	11.5	HD 13744	A0 <i>Iab</i>
		HD 12953	A1 <i>Ia</i>
		HD 14489	A2 <i>Ia</i>
		HD 16778	A2 <i>Ia</i>
		HD 13476	A3 <i>Iab</i>
		HD 15316	A3 <i>Iab</i>
Cam OB1	9.9	HD 17378	A5 <i>Ia</i>
		HD 20041	A0 <i>Ia</i>
IC 2581	12.0	HD 21389	A0 <i>Ia</i>
		HD 90772	A7 <i>Ia</i>

OB1 (Purgathofer 1961). The outer group of Per OB1 is of uncertain spatial extent (as are the other associations—see §IV) but will be used here in order to calibrate the A *Iab* supergiants; reassuringly, the mean absolute magnitude of the A *Ia* supergiants in the outer group, based on a distance modulus of 11.5, agrees very well with the mean value for the other A *Ia* supergiants; and the absolute magnitude of the A2 *Iab* supergiant BD +60°2546 (Buscombe 1970), which probably belongs in Cassiopeia OB2, agrees perfectly with the mean value for the Perseus A *Iab* supergiants.

The mean absolute magnitudes of OB and A supergiants in Table I are somewhat brighter and fainter, respectively, than the absolute magnitudes most widely quoted at present, those of Schmidt-Kaler (Blaauw 1963; Schmidt-Kaler 1965), although they agree well with the values given by other authors. Blue supergiants in the two Magellanic Clouds (Feast, Thackeray, and Wesselink 1960) are, of course, systematically brighter than our group members (by about 1 magnitude) because of the obvious selection effect involved. However, other methods can be used to check our results. Bouigue (1959) used the amount of interstellar reddening to derive $\langle M_v \rangle = -7.0$ for O9.5–F2 *Ia* supergiants. Murphy (1969) used main-sequence companions to derive individual M_v for supergiants in visual binary systems (HD 9311, HD 164353, ζ Ori, and ζ Per); although his method can, in principle, be extended to spectroscopic binary systems, it does not achieve accurate results when both components are of early spectral type (Batten 1967). For the eclipsing binary system V448 Cyg, the derived radius of the B1 *Ib-II* component, $16.5 R_\odot$ (Sahade 1962), combined with its effective temperature, $22,600^\circ \text{K}$, and bolometric correction, -2.0 (Morton and Adams 1968), yields $M_v = -5.3$. Comparison of absolute magnitudes based on membership in a binary system with those based on group membership is found to be satisfactory.

III. F, G, and K Supergiants

Yellow supergiants are extremely rare among massive stars. Nevertheless, earlier work on their luminosities (Bidelman 1958; Schmidt-Kaler 1961*b*, 1963, 1965; Blaauw 1963) can now be checked by using (1) Ruprecht's (1966) recent

revision of the boundaries of associations, (2) published lists of yellow supergiants (Schmidt-Kaler 1961*a,b*; Humphreys 1970*a*), (3) published radial velocities as a criterion of membership (Humphreys 1970*a*), and (4) improved distance moduli of associations as listed in section IV, but here including Cyg OB7 and Cepheus OB5 (Schmidt 1958) and Centaurus OB2 (Thackeray and Wesselink 1965).

A list of probable members of clusters and associations is provided in Table III, subject to the uncertainties mentioned in section IV. The two K *Iab* supergiants may belong to a somewhat older population in Cyg OB7 and Cr 121, and HD 10494 may belong to NGC 654 in Cas OB8 (Pesch 1960). In forming $\langle M_v \rangle$ for the supergiants in Table III, objects that are composite or have a strongly variable spectral type will be omitted. Unfortunately, virtually all the massive yellow supergiants have an uncertain spectral type and luminosity class, partly due to intrinsic variability (Abt 1957) and partly to a lack of suitable standards. The resulting absolute magnitudes in Table I are therefore uncertain. (They would be 0^m2 fainter if $A_V/E_{B-V} = 3.0$.)

Agreement with the results of Schmidt-Kaler is, nevertheless, satisfactory for luminosity class *Ia*; but, for luminosity class *Iab*, our results are about half a magnitude fainter than his (probably

due to the small number of stars involved). Yellow supergiants in the Magellanic Clouds (Feast et al. 1960) are, of course, selectively brighter than our group members in most cases. For further comparison, the statistical method of differential galactic rotation has been used by Keenan and Morgan (1951) for eight A5-M2 *Ia* supergiants, yielding $\langle M_v \rangle = -7.2$. The strength of the reversal of the K-line in Ca II is correlated with M_v for individual supergiants (Wilson and Bappu 1957; Warner 1969), but only a few bright supergiants have been measured (ψ^1 Aurigae, α^1 Canis Majoris, and HD 101947). The derived radii of the supergiants in the eclipsing binary systems 32 Cyg, ϵ Aur, and BL Telescopii (Feast 1967; Wright 1970) are not entirely empirical and so will not be used here to infer luminosities. The pulsation-constant method (Stothers and Leung 1971) can, in principle, be used for the variables RW Cep, W Cep, and AX Sgr (if they are pulsating), but their primary periods and spectral types are too uncertain to be useful here.

IV. M Supergiants

Red supergiants that are definite members of galactic clusters and the inner regions of associations are very few in number. Of the available calibrations of their absolute magnitudes (Keenan 1942, 1963; Bidelman 1947; Keenan and Morgan 1951; Johnson and Hiltner 1956; Blaauw 1963; Schmidt-Kaler 1963, 1965; FitzGerald 1967; Stothers and Leung 1971) the most recent calibration depended on eight members with luminosity class *Iab*, four with class *Iab-Ib*, and three with class *Ib*; there were none with class *Ia*. One member that can now be added is Zug 4 (BD +59°274) in NGC 581 (Stothers 1969*a*), with a modern spectral classification of M0.5 *Iab-Ib* (Humphreys 1970*a*; Lee 1970). A number of supergiants are available with absolute magnitudes determined by independent, but less reliable, empirical methods (Stothers and Leung 1971). To this list can now be added CPD -56°3586, with $M_v = -5.3$ calibrated from its blue companion (Keenan 1970).*

In order to improve the present unsatisfactory situation, we have reexamined the lists of M supergiants given by Johnson and Mendoza

*Keenan (1970) has also revised the luminosity of HR 8164, which is now $M_v = -4.9$.

TABLE III

PROBABLE F-K SUPERGIANT MEMBERS OF VERY YOUNG CLUSTERS AND ASSOCIATIONS

Assoc.	$(m-M)_0$	Star	Sp.
Sgr OB1	11.0	AX Sgr	G8-M2 <i>Ia</i>
Vul OB1	12.1	HD 187299	G5 <i>Iab</i>
Cyg OB1	11.0	HD 195593	F5 <i>Iab</i>
Cyg OB7	9.3	32 Cyg	K5 <i>Iab</i> +B
Cep OB1	12.4	RW Cep	G8-M0 <i>Ia-0</i>
		W Cep	K0-M0 <i>Ia</i> +B
Cep OB5	11.6	HD 217476	G0-G4 <i>Ia</i> :
Cas OB7	12.1	HD 6474	G0 <i>Ia</i>
NGC 457	12.2	ϕ Cas	F0: <i>Ia</i>
Cas OB8	12.1	HD 10494	F5 <i>Ia</i>
		HD 9973	F5: <i>Iab</i>
Cas OB6	11.7	HD 17971	F5: <i>Ia</i>
		HD 18391	G0 <i>Ia</i>
Aur OB1	10.6	ϵ Aur	F0:p <i>Ia</i>
Cr 121	9.0	δ CMa	F8 <i>Ia</i>
		α^1 CMa	K5: <i>Iab</i>
Cen OB2	11.5	HD 101947	G0 <i>Ia</i>

(1966), Lee (1970), and Humphreys (1970*b*), and have compared anew the positions of all stars not known to be cluster members with the boundaries of associations tabulated by Ruprecht (1966). Radial velocity as a criterion of membership was used whenever possible (Stothers 1969*a*; Humphreys 1970*b*); new radial velocities are now available for the M supergiants in NGC 457, NGC 581, NGC 3766, and Per OB1 (Humphreys 1970*a*; Gahm and Arkling 1971; Humphreys, Strecker, and Ney 1972). Proper motions have also been used in a few cases (Stothers 1969*a*); the proper motions of supergiants in Per OB1 have been recently remeasured (Lavdovskii 1965) and rediscussed (Bronnikova 1968; Ferrari et al. 1970). Apparent magnitude and color excess were almost never used as criteria for membership, except in a very few obvious cases. Kopylov (1958) has emphasized the frequent confusion arising from different aggregates lying along the line of sight, and Ruprecht (1966) has discussed the analogous problem of occasional ambiguity and overlap of associations against the sky. Therefore our assignment of a particular supergiant to a particular association is not always definitive.

Distance moduli for the clusters containing M supergiants have already been listed in a previous paper (Stothers 1969*a*); a recent redetermination of the distances of some of these clusters (Schild 1970) gives moduli that do not differ very much from the earlier values. However, the new distance moduli for χ Per and the inner group of Per OB1 (Crawford, Glaspey, and Perry 1970) are substantially smaller than the large values of Schild (1967) used previously, or even the classical value (Ruprecht 1966). Since Vogt's (1971), and to some extent Lloyd Evans's (1972), independent investigations confirm the work of Crawford et al. (1970) (when allowance is made for the different extinction ratio for blue stars assumed by Vogt), we shall adopt the new distance moduli for χ Per and the inner group of Per OB1. Unfortunately the outer group in Perseus remains a problem due to the persistent difficulty of segregating background stars (Schild 1967; Bronnikova 1968; Stothers 1969*b*; Humphreys 1970*a*) and we shall not use this group. For the other associations, we have adopted distance moduli from the following sources: Sgr OB1 (Hiltner, Morgan, and Neff 1965); Ser-

pens OB1, Cas OB2 (Hoag and Applequist 1965); Vulpecula OB1, Cep OB1 (Hoag and Applequist 1965; Walker and Hodge 1968); Cyg OB1, Cyg OB9 (Walker and Hodge 1968); Cep OB2 (Simonson 1968); Cas OB5 (Ampel 1964; Reddish 1967); Cas OB4 (Ampel 1964); Cas OB7 (Ampel 1964; Ruprecht 1966 — Notes); Cas OB8 (Pesch 1960; Hoag and Applequist 1965); Cas OB6 (Walker and Hodge 1968; Ishida 1970); Aur OB1 (Stothers 1971); Gemini OB1 (Crawford et al. 1955; Stothers 1969*a*); Cr 121 (Feinstein 1967); Scorpius OB2 (Garrison 1967); and Sgr OB5 (Hiltner 1954; revised in this paper). Consideration was also given to distance moduli determined previously by Morgan, Whitford, and Code (1953), Schmidt (1958), Johnson et al. (1961), Becker (1963), and Buscombe (1963). Ruprecht's (1966) listed distance moduli are usually those of Morgan et al. or Schmidt; the difference between his and our distance moduli is only $\pm 0^m.2$ on the average (if two very discrepant cases are ignored).

Table IV contains our final list of M supergiants that are considered to be probable members of clusters and associations.* For a few stars with blue companions the reddening correction is uncertain, and their absolute magnitudes have not been used for the calibration in Table I. Our new calibration does not differ appreciably from other recent calibrations except for luminosity class Ia, for which the older calibrations depended chiefly on the method of differential galactic rotation (Keenan and Morgan 1951). A comparison of absolute magnitudes based on group membership with those based on other empirical methods has already been published (Stothers and Leung 1971); agreement was found to be very good. The semiempirical pulsation-constant method also *predicts* absolute magnitudes (Stothers and Leung 1971) that are in very good agreement with the values derived here.

Although there is considerable room for improvement of the absolute magnitudes of the bright F-K supergiants, it is believed that the

*For reasons given in the cited references, μ Cep (Simonson 1968), α Ori (Lesh 1968*a*; Stothers 1969*a*), and HD 101712 (Schild 1970) have been rejected from membership in Cep OB2, Ori OB1, and IC 2944, respectively, although they still remain as possible members.

TABLE IV

PROBABLE M SUPERGIANT MEMBERS OF VERY YOUNG CLUSTERS AND ASSOCIATIONS				
Assoc.	$(m-M)_0$	Star	Sp.	
Sgr OB1	11.0	HD 163428	M1	Ib
Ser OB1	11.7	Case 49	M2.0	Iab
		BD - 14° 5105	M2.0	Iab
		FR Sct	M2.5	Iab + B
Vul OB1	12.1	BD + 24° 3902	M1.5	Ia
Cyg OB1	11.0	BD + 35° 4077	M1.5	Iab
		BI Cyg	M3.0	Iab
		BC Cyg	M3.5	Ia
		KY Cyg	M3.5	Ia
Cyg OB9	11.3	RW Cyg	M2.0	Ia:
Cep OB2	9.5	BD + 58° 2249	M1p	Ib + B
		VV Cep	M2p	Ia + B
		HD 202380	M3	Ib
Cep OB1	12.4	AZ Cep	M2.0	Ia
		V 358 Cas	M2.5	Ia:
		Case 75	M3.5	Ia
		SW Cep	M3.5	Ia-Iab
		U Lac	M4	Iab + B
Cas OB2	12.0	Case 80	M2	Iab
		Case 81	M2.0	Iab
Cas OB5	12.5	TZ Cas	M2.5	Iab
		PZ Cas	M3.5	Ia
Cas OB4	12.1	KN Cas	M1p	Ib + B
		MZ Cas	M2	Iab
Cas OB7	12.1	Case 23	M2.0	Iab
NGC 457	12.2	BD + 57° 258	M1.5	Ib
NGC 581	12.1	BD + 59° 274	M0.5	Iab-Ib
Cas OB8	12.1	AZ Cas	M0	Ib + B
		WX Cas	M2	Iab-Ib
		BD + 60° 335	M3	Iab-Ib
		HD 236871	M3	Iab-Ib
x Per	11.5	RS Per	M4.5	Iab-Ib
Per OB1 (inner)	11.5	HD 14580	M0	Iab
		BD + 56° 595	M0.5	Iab
		FZ Per	M1	Iab
		HD 14404	M1	Iab-Ib
		AD Per	M2.5	Iab
		HD 14826	M3	Iab
		SU Per	M3.5	Iab
		BU Per	M3.5	Ib
Per OB1 (outer)	11.5 - 12.7	BD + 57° 530a	M0	Iab-Ib
		BD + 57° 524	M1	Iab
		HD 237006	M1	Ib + B
		BD + 57° 647	M2.0	Ia-Iab
		HD 236915	M2	Iab
		T Per	M2	Iab
		BD + 58° 445	M2	Iab
		BD + 60° 478	M2	Iab
		BD + 58° 373	M2	Iab-Ib
		HD 13136	M2	Iab-Ib
		YZ Per	M2.5	Iab
		W Per	M3	Iab:
		S Per	M4e	Ia
		XX Per	M4	Ib + B
Cas OB6	11.7	GP Cas	M2.0	Iab

TABLE IV (Continued)

Assoc.	$(m-M)_0$	Star	Sp.	
Aur OB1	10.6	HD 35601	M1	Ib
		HD 37536	M2	Iab
Gem OB1	10.5	BU Gem	M1	Ia
		TV Gem	M1	Iab
		WY Gem	M2p	Iab + B
Cr 121	9.0	σ CMa	M0	Iab
NGC 3293	12.1	CPD - 57° 3502	M0	Iab
NGC 3766	11.4	CD - 60° 3621	M1	Ib:
		CD - 60° 3636	M1	Ib-II:
NGC 4755	11.9	κ Cru-D	M2	Iab-Ib
Sco OB2	6.2	α Sco	M1	Iab-Ib + B
Sgr OB5	11.8	KW Sgr	M3.0	Ia

present luminosity calibration for massive supergiants can be considered reliable. It should be emphasized, however, that the present improvement is due only to the large number of group members used and to the rejection of probable nonmembers or of members of groups with uncertain distance moduli, rather than to increased accuracy in the absolute magnitudes of B-type main-sequence stars that are required for the calibration.

REFERENCES

- Abt, H. A. 1957, *Ap. J.* **126**, 138.
 Ampel, R. 1964, *Acta Astr.* **14**, 52.
 Batten, A. H. 1967, *Pub. Dominion Astrophys. Obs.* **13**, 119.
 Becker, W. 1963, *Zs. f. Ap.* **57**, 117.
 Bidelman, W. P. 1947, *Ap. J.* **105**, 492.
 — 1958, in *Comparison of the Large-Scale Structure of the Galactic System with that of Other Stellar Systems*, I.A.U. Symposium No. 5, N. G. Roman, ed. (Cambridge: Cambridge University Press), p. 54.
 Blaauw, A. 1963, in *Basic Astronomical Data*, K. Aa. Strand, ed. (Chicago: University of Chicago Press), p. 383.
 Bouigue, R. 1959, *Toulouse Obs. Ann.* **27**, 47.
 Bronnikova, N. M. 1968, *Soviet Astr.* **12**, 445.
 Buscombe, W. 1963, *Mount Stromlo Mimeogram* No. 6.
 — 1970, *ibid.* No. 8 (revised edition).
 Crawford, D. L., Glaspey, J. W., and Perry, C. L. 1970, *A.J.* **75**, 822.
 Crawford, D., Limber, D. N., Mendoza, E., Schulte, D., Steinman, H., and Swihart, T. 1955, *Ap. J.* **121**, 24.
 Feast, M. W. 1963, *M.N.R.A.S.* **126**, 11.
 — 1967, *ibid.* **135**, 287.
 Feast, M. W., Thackeray, A. D., and Wesselink, A. J. 1960, *M.N.R.A.S.* **121**, 337.
 Feinstein, A. 1967, *Ap. J.* **149**, 107.
 Ferrari, A., Galeotti, P., Silvestro, G., and Trussoni, E. 1970, *Ap. and Space Sci.* **9**, 181.
 FitzGerald, M. P. 1967, Dissertation, Case Western Reserve University.

- 1969, *Pub. A.S.P.* 81, 71.
 Gahm, G. F., and Arkling, J. G. 1971, *Astr. and Ap.* 10, 414.
 Garrison, R. F. 1967, *Ap. J.* 147, 1003.
 Hiltner, W. A. 1954, *Ap. J.* 120, 41.
 Hiltner, W. A., Morgan, W. W., and Neff, J. S. 1965, *Ap. J.* 141, 183.
 Hoag, A. A., and Applequist, N. L. 1965, *Ap. J. Suppl.* 12, 215 (No. 107).
 Humphreys, R. M. 1970a, *Ap. J.* 160, 1149.
 — 1970b, *A.J.* 75, 602.
 Humphreys, R. M., Strecker, D. W., and Ney, E. P. 1972, *Ap. J.* 172, 75.
 Ishida, K. 1970, *Pub. Astr. Soc. Japan* 22, 277.
 Johnson, H. L. 1966, *Annual Rev. of Astr. and Astrophysics* 4, 193.
 Johnson, H. L., and Hiltner, W. A. 1956, *Ap. J.* 123, 267.
 Johnson, H. L., and Iriarte, B. 1958, *Lowell Obs. Bull.* 4, 47.
 Johnson, H. L., and Mendoza, E. E. 1966, *Ann. d'Ap.* 29, 525.
 Johnson, H. L., Hoag, A. A., Iriarte, B., Mitchell, R. I., and Hallam, K. L. 1961, *Lowell Obs. Bull.* 5, 133.
 Keenan, P. C. 1942, *Ap. J.* 95, 461.
 — 1963, in *Basic Astronomical Data*, K. Aa. Strand, ed. (Chicago: University of Chicago Press), p. 78.
 — 1970, *Ap. J.* 162, 199.
 Keenan, P. C., and Morgan, W. W. 1951, in *Astrophysics*, J. A. Hynek, ed. (New York: McGraw-Hill Book Co.), p. 12.
 Kopylov, I. M. 1955, *Izv. Crimean Ap. Obs.* 15, 153.
 — 1958, *Soviet Astr.* 2, 359.
 — 1959, *Ann. d'Ap. Suppl.* 8, 41.
 Lavdovskii, V. V. 1965, *Izv. Glav. Astr. Obs. Pulkova* 176, 138.
 Lee, T. A. 1970, *Ap. J.* 162, 217.
 Lesh, J. R. 1968a, *Ap. J.* 152, 905.
 — 1968b, *Ap. J. Suppl.* 17, 371 (No. 151).
 Lloyd Evans, T. 1972, *Quart. J.R.A.S.* (in press).
 Morgan, W. W., Whitford, A. E., and Code, A. D. 1953, *Ap. J.* 118, 318.
 Morton, D. C., and Adams, T. F. 1968, *Ap. J.* 151, 611.
 Murphy, R. E. 1969, *A.J.* 74, 1082.
 Pesch, P. 1960, *Ap. J.* 132, 696.
 Purgathofer, A. 1961, *Zs. f. Ap.* 52, 186.
 Reddish, V. C. 1967, *M.N.R.A.S.* 135, 251.
 Ruprecht, J. 1966, *Transactions I.A.U.* 12B, 348.
 Sahade, J. 1962, *Symposium on Stellar Evolution* (La Plata: Observatorio Astronomico, Universidad Nacional de La Plata), p. 185.
 Schild, R. E. 1967, *Ap. J.* 148, 449.
 — 1970, *ibid.* 161, 855.
 Schmidt, K. H. 1958, *A.N.* 284, 76.
 Schmidt-Kaler, T. 1961a, *Zs. f. Ap.* 53, 1.
 — 1961b, *ibid.* 53, 28.
 — 1963, *Mitt. Astr. Gesell.*, p. 25.
 — 1965, in *Landolt-Bornstein: Astronomie und Astrophysik*, H. H. Voigt, ed. (Berlin: Springer-Verlag), p. 284.
 Simonson, S. C. 1968, *Ap. J.* 154, 923.
 Stothers, R. 1969a, *Ap. J.* 155, 935.
 — 1969b, *ibid.* 156, 541.
 — 1971, *Nature*, 229, 180.
 — 1972a, *Ap. J.* (in press).
 — 1972b, *Ap. J.* (in press).
 Stothers, R., and Leung, K. C. 1971, *Astr. and Ap.* 10, 290.
 Thackeray, A. D., and Wesselink, A. J. 1965, *M.N.R.A.S.* 131, 121.
 Vogt, N. 1971, *Astr. and Ap.* 11, 359.
 Walker, G. A. H., and Hodge, S. M. 1968, *Pub. A.S.P.* 80, 290.
 Warner, B. 1969, *M.N.R.A.S.* 144, 333.
 Weaver, H., and Ebert, A. 1964, *Pub. A.S.P.* 76, 6.
 Wilson, O. C., and Bappu, M. K. V. 1957, *Ap. J.* 125, 661.
 Wright, K. O. 1970 in *Vistas in Astronomy*, Vol. 12, A. Beer, ed. (Oxford: Pergamon Press), p. 147.