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Luminosities, Masses and Periodicities of Massive Red Supergiants

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Luminosities and masses of M-type supergiants belonging to extreme Population I have been derived from (*a*) cluster membership and other strictly empirical luminosity and mass criteria and (*b*) the pulsational Q value method applied to the known variables. New periods for eight variables have been determined from AAVSO data. It is found, in general, that the primary period and the long secondary period both increase with brighter luminosity class and later spectral type. This reflects, partly, the increase of mass with increasing luminosity if the primary period is interpreted as being due to the fundamental mode of radial pulsation and if the appropriate theoretical mass-luminosity relation is adopted. The long secondary period is tentatively interpreted as the convective turnover time of giant convection cells in the stellar envelope. A possible alternative interpretation is that the primary and secondary periods represent, respectively, the first overtone and the fundamental mode of radial pulsation. Luminosities based on methods (*a*) and (*b*) are in good agreement, and yield a self-consistent calibration of the mean absolute visual magnitudes of the different luminosity classes. The pulsational masses of variables in associations are in satisfactory agreement with the masses inferred from the main-sequence turnoffs, thereby implying that little mass has been lost. The evidence is very strong that the majority of red supergiants are in the early stages of core helium burning.

Key words: red supergiants — periodicities — luminosities — masses

I. Introduction

The question of the luminosities and masses of M-type supergiants associated with young population groups has never been answered with sufficient accuracy for definitive conclusions to be drawn concerning the details of their galactic distribution, the amount of mass loss which they may have suffered, and (to a lesser degree of uncertainty) their present evolutionary status. It is the aim of the present paper to attempt to derive, more accurately than heretofore, self-consistent values for the luminosities of individual red supergiants, based on (*a*) cluster membership and other luminosity criteria and (*b*) the pulsational Q value method applied to the variables. The latter method is also able to yield individual masses. To some extent, these matters were considered in a previous paper (Stothers, 1969b, hereinafter cited as "Paper I"), but the point of the previous paper was chiefly to establish the pulsational nature of the observed light variations.

Section II of the present paper is concerned primarily with the first method of determining luminosities. Sections III and IV take up the matter of the Q value and the observed primary and secondary periods of light variability. In Section V, the second method is used to derive luminosities and masses from the observed primary periods, while, in Section VI, the secondary periods are given a tentative interpretation. The already large body of evidence that the majority of red supergiants are in the early stages of core helium burning is further substantiated by an argument in Section VII. The final Section VIII summarizes and concludes the paper.

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II. Empirical Absolute Magnitudes

Cluster membership is the most accurate method of finding the absolute magnitudes of red supergiants. The preliminary list of M-type supergiants which are known to be members of clusters and associations (Stothers, 1969a) can be extended now. Lee (1970) has given a number of additional assignments and Humphreys (1970b) has made an even more extensive survey. Nevertheless, the most secure membership and absolute magnitude attach to the supergiants of the best-studied groups among those in the preliminary list of Stothers. These are listed

Table 1. *Absolute visual magnitudes of* M-*type supergiants based on cluster membership*

Cluster	Star	Sp.		M_v	Cluster	Star	Sp.		M_v
I Per	HD 14580	M0	Iab	−5.3	χ Per	RS Per	M4.5	Iab−Ib	−5.4
(inner)	BD + 56°595	M0.5	Iab	−5.3	NGC 457	BD + 57°258	M1.5	Ib	−4.9
	FZ Per	M1	Iab	−5.8	Cr 121	σ CMa	M0	Iab	−5.6
	HD 14404	M1	Iab−Ib	−5.8	NGC 3293	CPD−57°3502	M0	Iab	−5.8
	AD Per	M2.5	Iab	−5.7	NGC 3766	CD−60°3621	M1	Ib:	−4.9
	HD 14826	M3	Iab	−5.3		CD−60°3636	M1	Ib−II:	−4.4
	SU Per	M3.5	Iab	−5.6	NGC 4755	ϰ Cru−D	M2	Iab−Ib	−5.4
	BU Per	M3.5	Ib	−5.1	II Sco	α Sco	M1	Iab−Ib	−5.2

I Per (inner group). Humphreys's (1970a) luminosity classifications agree with Wildey's (1964), except in the case of HD 14404, for which Iab and Ib are given by these authors, respectively. Wildey's spectral types have been adopted.

χ Per. Humphreys (1970a) gives Iab−Ib as the infrared luminosity class of RS Per, which is possibly more compatible with the presence of a B2 Ib supergiant in the cluster than Wildey's (1964) luminosity class of Iab. Wildey's spectral type, however, is adopted.

NGC 457. Kraft (Pesch, 1959) gives M0 Ib−II for the spectral type and luminosity class of the red supergiant, but Bidelman and Humphreys (1970a) give M2 Ib, essentially in line with Lee's (1970) classification, which is adopted. A luminosity class of Ib is compatible with the presence of a B6 Ib supergiant in the cluster.

Cr 121. Lee's (1970) classification is adopted.

NGC 3293. Feast's (1958) classification is adopted.

NGC 3766. Keenan (private communication) and Evans (private communication) give M1 Ib for CD−60°3621, while Schild (1970) gives M0 Iab. Keenan states that CD−60°3636 has a low luminosity spectroscopically, and, in fact, Evans obtains M1 Ib−II; Schild, however, gives M0 Ib. The B5 Ia supergiant near the cluster is a background star, according to Schild, and so Keenan's and Evans's luminosity classifications of the two red supergiants are more compatible with the presence of the single A0 Ib blue supergiant in the cluster.

NGC 4755. Hernandez (1960) and Schild (1970) give M2 Iab for the red supergiant, but Feast (1963b) gives M2 Ib.

II Sco. Lee's (1970) classification of α Sco is M1 Iab, although the usual luminosity class given is Ib (Jaschek *et al.*, 1964).

in Table 1. Such a limited selection will actually facilitate the discrimination of expected differences in M_v between the various luminosity classes, which was not apparent in the work of Stothers because he used all the supergiants in his list.

The absolute visual magnitudes of Table 1 were obtained from the following data: cluster moduli listed by Stothers (1969a), apparent visual magnitudes and $B-V$ colors from the source references listed therein, $(B-V)_0 = +1.71$ (Lee, 1970), and a ratio of total to selective extinction $A_V/E_{B-V} = 3.0$. Possible sources of error, and their estimated sizes, have been discussed previously by Stothers (1969a). Lee's (1970) intrinsic $B-V$ color for the average spectral subdivision of M supergiants (M2) has been adopted for all spectral subdivisions, on the grounds that the color differences among the subdivisions are of the same magnitude as the uncertainties of the colors themselves. Lee has also suggested that $A_V/E_{B-V} = 3.6$ for distant reddened M supergiants, but finds that the ratio in the Perseus region is still unsettled. In view of the location of over half the M supergiants of Table 1 in the I Per association and the possible variation of A_V/E_{B-V} over different regions of space both for red (Lee, 1970) and blue (Johnson, 1968) stars, with the attendant uncertainty in the cluster distance moduli, we have adopted for simplicity the uniform value $A_V/E_{B-V} = 3.0$. The major remaining source of error is light variability, which is occasionally present in these stars, being cyclic on a long time scale although usually erratic on a short time scale. By redetermining the absolute visual magnitudes of the nine red supergiants in the inner group of I Per with the help of apparent visual magnitudes and $B-V$ colors due to Johnson and Mendoza (1966), we find, from comparison with results based on Wildey's (1964) observations, that the mean error in M_v due to variability of light and color and to photometric errors is likely to be about ± 0.3. Probably the total mean error in absolute magnitude for an individual red supergiant is about ± 0.4 mag.

On the basis of Table 1, we obtain the following mean absolute magnitudes as a function of luminosity class[1]):

Class	$\langle M_v \rangle$	
Iab	-5.6	(8 stars)
Iab − Ib	-5.4	(4 stars)
Ib	-5.0	(3 stars) .

These values are (not surprisingly) close to those derived in earlier work (e.g. Keenan and Morgan, 1951; Blaauw, 1963; but see Lee, 1970) since most modern determinations, including the present one, have relied heavily on the I Per association, starting with Keenan's (1942) work. However, the present determination has the advantages of excluding the stars in the uncertain outer group of I Per, adding a significant number of members of other stellar groups besides the inner group of I Per, and yielding sufficient resolution to isolate an intermediate luminosity group Iab−Ib (although the luminosity discriminants begin to be blurred at this level of resolution).

Other empirical methods of determining the absolute magnitude of red supergiants are available, but they are less reliable. Wilson (1942) used radial velocities and proper motions to obtain a statistical parallax for thirteen variable red "supergiants" (now known to be a mixture of luminosity classes I and II), and determined $\langle M_v \rangle = -3.4$ at maximum light. Keenan and Morgan (1951) proceeded in a similar fashion for a combination of variable and nonvariable yellow and red supergiants, but used only radial velocities (and therefore strictly the method of differential galactic rotation) in obtaining $\langle M_v \rangle = -4.2$ (Ib) and -7.2 (Ia). Trigonometric parallaxes (Jenkins, 1963) are available for eight red supergiants brighter than the sixth magnitude, but they are much too uncertain to be useful except in the case of α Ori. The "moving group" method has been used successfully for α Sco (Bertiau, 1958). For a red supergiant having a blue companion, the spectroscopic parallax of the blue star can be used to estimate the absolute magnitude of the red star (Bidelman, 1958). The strength of the emission reversal in the K line of ionized calcium has been found to be correlated with absolute magnitude (Wilson and Bappu, 1957; Wilson, 1959); and, importantly, the calibration can be made in terms of stars which are not supergiants. Finally, since the linear diameter can be directly determined for the red-supergiant member of the eclipsing binary VV Cep, the absolute magnitude follows either from the astrometric (in lieu of an interferometric) angular diameter[2]) or from the spectroscopic effective temperature (Peery, 1966).

Results based on the foregoing methods are collected in Table 2. Comparison can be made with the results based on the cluster method for the four stars in common with Table 1. The agreement is seen to be satisfactory for α Sco, AD Per and HD 14404, but poor in the case of σ CMa. However, if Warner's (1969) two discrepant values are disregarded, then the agreement is generally quite good. The absolute magnitude of VV Cep may not be trustworthy since the probable membership of this star in the I Cep association would suggest $M_v = -4$ to -6 (Simonson, 1968). Similarly, α Ori ought to have $M_v = -8$ if a member of I Ori, but this possibility is unlikely (Stothers, 1969a).

III. Pulsational Q Value

In Paper I, pulsational Q values were obtained in two ways, where, as usual, we define

$$Q = P\,(\langle\varrho\rangle/\langle\varrho_\odot\rangle)^{1/2}\,. \qquad (1)$$

The first way involved theoretical calculation of the fundamental mode of radial pulsation in models of red-supergiant envelopes. These envelopes are in convective equilibrium, and, for simplicity, their structure and pulsational motions were taken to be strictly adiabatic, and ionization was assumed to be complete. It was found that, over a very wide range

[1]) A less critical selection of data can enlarge the statistics of Table 1. By using Humphreys's (1970b) comprehensive list of M supergiants designated by her as probable members of clusters and associations, but omitting all composite objects as well as RW Cep, μ Cep and α Ori, and adopting distance moduli for the various groups from Humphreys's unpublished compilation, we have derived the following mean absolute visual magnitudes: −6.3 (Ia, 5 stars); −5.2 (Iab, 29 stars); −4.8 (Iab−Ib, 7 stars); and −4.9 (Ib, 7 stars). Omission of the numerous stars in the I Per association does not alter these means. (Humphreys used 11.8 as the true distance modulus for I Per whereas in Table 1 we have used 12.0.)

[2]) Fredrick (1960) has attempted to measure the trigonometric parallax of VV Cep. His result, 0.005 arc-sec, is found to be irreconcilable with the spectroscopic parallax, which is an order of magnitude smaller. If, instead, we be permitted here to interpret his measurement as a direct astrometric determination of some measure of the red star's angular diameter (in the spirit of Evans's [1957] suggestions), then the distance to VV Cep follows immediately from the red star's linear diameter as determined from the orbit. Tentatively, we adopt an angular diameter of 0.005 arc-sec and Peery's (1966) linear diameter of 3240 $R_\odot$.

Table 2. *Absolute visual magnitudes of* M-*type supergiants based on purely empirical criteria other than cluster membership*

Star	M_v	Method	Ref.
HD 13136	−6.0	Ca II reversal	[a])
AD Per	−5.7	Ca II reversal	[a])
HD 14404	−6.0	Ca II reversal	[a])
CE Tau	−5.1	Ca II reversal	[b])
α Ori	−5.7	Ca II reversal	[b])
	−6.1	Trigonometric parallax	[c])
ψ^1 Aur	−5.7	Ca II reversal	[b])
σ CMa	−4.5	Ca II reversal	[d])
α Sco	−4.9	Ca II reversal	[b])
	−5.6	Ca II reversal	[d])
	−5.4	Sco-Cen moving group	[e])
	−5.3	Blue companion	[f])
HR 7475	−4.9	Blue companion	[f])
HR 8164	−5.2	Blue companion	[f])
VV Cep	−6.9	Interstellar polarization	[g])
	−7.5	Radius and effective temperature	[h])
	−7.6	Radius and angular diameter	[i])

[a]) Wilson (1959).
[b]) Wilson and Bappu (1957).
[c]) Jenkins (1963).
[d]) Warner (1969).
[e]) Bertiau (1958).
[f]) Bidelman (1958).
[g]) Cowley (1969).
[h]) Peery (1966) — adjusted value based on Lee's (1970) T_e and B.C. for an M2 supergiant.
[i]) This paper.

of mass and chemical composition, $Q = 0.05 - 0.07$ day.

The second way of obtaining the Q value depended on variable red supergiants that are members of clusters and associations. These stars, ten in number, were drawn from the tabulation of Stothers (1969a). Group membership permits the determination of luminosities and also of masses if one assumes that the red supergiants have approximately the same mass as blue supergiants in the same group. While this is certainly justifiable on the grounds of the relatively fast evolution time involved between the two stages (Stothers, 1969a), the additional assumption has to be made that no significant mass loss has occurred during or after the main-sequence phase. Then the Q value is formed:

$$Q = P\left(\frac{L}{L_\odot}\right)^{-3/4}\left(\frac{\mathscr{M}}{\mathscr{M}_\odot}\right)^{1/2}\left(\frac{T_e}{T_{e\odot}}\right)^3 . \qquad (2)$$

This method was used provisionally by Abt (1957) but with better and more numerous data in Paper I. In the latter paper, half the red variables had observational Q values which were discrepant with the theoretical values, but this result was shown to be probably due to errors or confusion concerning the observed primary periods or luminosities (distances). Therefore the pulsational hypothesis was tentatively supported.

In order to try to refine the observational Q value, we shall here use: only the variables in Table 1 with well-determined primary periods, Lee's (1970) effective temperatures and bolometric corrections (which are slightly smaller than those of Johnson, 1966), and the masses of Paper I. The variables involved are SU Per, AD Per and FZ Per — all in the I Per association. Adopting a mass of $14\,\mathscr{M}_\odot$, one finds an average $Q = 0.047$ day, with little dispersion. However, the average mass could be as high as $20\,\mathscr{M}_\odot$, or one might adopt Johnson's (1966) effective temperatures and bolometric corrections, or the distance modulus could be 0.3 mag smaller (Blaauw, 1963), or one might take into account that the average semiamplitude of light variability in these four stars exceeds 0.3 mag. Each one of these four factors alone is found to increase the Q value to $Q = 0.056$. Tentatively, the closeness of the empirical Q value to the theoretically predicted value suggests that the red supergiants cannot have lost too much mass, and (but not independently) that the adiabatic calculations neglecting ionization are adequate to determine the theoretical periods[3]).

In lieu of any more definite evidence, we shall henceforth adopt the uniform Q value used in Paper I, namely,

$$Q = 0.06 \text{ day} . \qquad (3)$$

Although we have here assumed that the primary observed period is due to the fundamental mode of radial pulsation, a possibility exists that it is rather due to the first overtone (Section VIII).

IV. Periods

Periods (or, more correctly, mean cycle times) of variable M-type supergiants have been taken from the following sources: Payne-Gaposchkin and Gaposchkin (1952); Payne-Gaposchkin (1954); Kukarkin *et al.* (1958); and Fredrick (1960). For several of the brighter variables we have been able to derive periods using magnitude estimates published *passim* in the *Harvard Annals* and the AAVSO *Quarterly* by the American Association of Variable Star Observers (AAVSO). Although these data are of markedly nonuniform quality, they are often available in large quantity, and surprisingly good light curves can sometimes be obtained when averages of a sufficiently large number of observations over several nights are taken. The procedure adopted in order to determine the "period" is to perform a standard periodogram analysis (Wehlau and Leung, 1964) followed by a searching routine which simultaneously determines the period and shape of the light curve (Leung, unpublished). In the case of the red semiregular variables, erratic changes in both period and shape of the light curve are common, and the periods are therefore only approximately determined by any technique. Our results are listed in Table 3. Agreement with previously published periods is generally good; the deviations never exceed 10 per cent of the average of the periods for any star. The periods for α Ori, XX Per and FZ Per are believed to be new.

Table 3. *Periods (mean cycle times) of variable red supergiants derived from* AAVSO *data*[a])

Variable	P_1(day)	P_2(day)	Remarks
α Ori	200−400	2335	Irregular cycles
S Per	968, 807		Two close periods with nearly regular light curves
T Per	260:		Small amplitude
W Per	465		
SU Per	533		
XX Per	415	4100	
AD Per	330:		Large variation in mean magnitude
FZ Per	184		

[a]) AAVSO data are available but inadequate to derive periods, for μ Cep, TV Gem, WY Gem and RS Per. Although a reliable period for RS Per cannot be obtained, the AAVSO data do allow us to say that the Gaposchkins' period of 152 days is apparently too short.

All the variables for which reasonably well-determined cycle times are known to us are listed in Table 4. Here, as in Table 3, the long secondary period which is often observed is given in addition to the primary period. In certain cases, an average of the periods from different sources has been taken. Probably the secondary period is present in all the variables, but it is difficult to detect because of its great length and relatively small amplitude. In the case of ST Cep, VV Cep, μ Cep, α Ori and α Sco, it is the *primary* period that is difficult to determine, because of the constantly changing cycle time and erratic fluctuations; the prototype for this kind of behavior is μ Cep (Ashbrook *et al.*, 1954[4]). But, to some extent, all of the massive red variables share this behavior.

Spectral types and luminosity classes have been adopted mostly from Lee (1970), with the major

[3]) In this connection, we refer to the adiabatic calculation of Stothers and Schwarzschild (1961) for red supergiants in old population groups; agreement between the observed and theoretical Q values of long-period variables at the tip of the giant branch in globular clusters was found to be satisfactory, considering the uncertainties (see Reddish, 1955; Feast, 1963a; Keeley, 1970). However, alternative explanations for the cyclic variability of red stars have been proposed and have involved the following mechanisms: veiling (Merrill, 1940), progressive waves in the atmosphere (Kamijo, 1962), and relaxation oscillations (Paczynski and Ziolkowski, 1968), among other possibilities.

[4]) It may be more than coincidental that ST Cep, VV Cep, μ Cep and α Ori all have a spectral type of M 2 (and α Sco is close, with M 1). The other variables of spectral type M 1 − M 2 for which primary periods are available have, significantly, small light amplitudes (except possibly RW Cyg).

Table 4. *Periods of* M-*type supergiants with masses and luminosities derived by the semiempirical Q value method*

Variable	Sp.		P_1(day)	P_2(day)	T_e(°K)	$\mathcal{M}/\mathcal{M}_\odot$	M_{bol}	M_v	Association
PZ Cas	M3.5	Ia	900		3050	29	−8.8	−6.5	I Cas
ST Cep	M2:	Ib		2050	3450				
VV Cep	M2p	Ia		5000	3450				I Cep
μ Cep	M2	Ia		4500	3450				I Cep ?
RW Cyg	M2.0	Ia:	586		3450	27	−8.7	−7.2	II Cyg
AZ Cyg	M3:	Ia	459		3200	20	−7.8	−5.8	
TV Gem	M1	Iab	182		3550	14	−6.7	−5.4	I Gem
U Lac	M4	Iab	689	2900	2950	22	−8.1	−5.6	II Cep
α Ori	M2	Iab:		2200	3450				I Ori ?
S Per	M4	Ia	968		2950	28	−8.8	−6.2	I Per
T Per	M2	Iab	290	2800	3450	18	−7.4	−5.8	I Per
W Per	M3	Iab:	467	3060	3200	20	−7.8	−5.8	I Per
SU Per	M3.5	Iab	500		3050	19	−7.7	−5.4	I Per
XX Per	M4	Ib	415	4100	2950	16	−7.2	−4.6	I Per
YZ Per	M2.5	Iab	378		3350	19	−7.7	−5.9	I Per
AD Per	M2.5	Iab	325		3350	18	−7.4	−5.6	I Per
BU Per	M3.5	Ib	365	2950	3050	16	−7.1	−4.8	I Per
FZ Per	M1	Iab	184		3550	14	−6.7	−5.4	I Per
KW Sgr	M3.0	Ia	670		3200	26	−8.5	−6.5	
α Sco	M1	Iab:		1733	3550				II Sco
CE Tau	M2.0	Ib	165		3450	13	−6.3	−4.8	
WY Vel	M3	Iab:	460		3200	20	−7.8	−5.8	

PZ Cas. Lee (private communication) averages M4 from his photometry with M3 from the modern spectral type (Humphreys, 1970a).
ST Cep. Keenan (1942) gives M0 Ib.
μ Cep. Possibly $P_1 = 900$ days.
RW Cyg. Keenan (1942) gives M3 Ia.
AZ Cyg. Keenan (1942) gives M2: Ia.
α Ori. A luminosity classification of Ib is sometimes given (Jaschek *et al.*, 1964).
W Per. Bidelman (Humphreys, 1970a) gives M3 Ia, while Humphreys (1970a) gives an infrared luminosity class of Iab.
α Sco. The usual luminosity classification is Ib (Jaschek *et al.*, 1964).
WY Vel. Cowley (1969) gives M3 ep Ib, but Humphreys (private communication) gives an infrared luminosity class of Iab.

Table 5. *Primary periods (in days) of variable* M-*type supergiants*

Class	M1	M2	M2.5	M3	M3.5	M4
Ia	...	586	...	670 459	900	968
Iab	184 182	290	378 325	467 460	500	689
Ib	...	165	...	...	365	415

Table 6. *Secondary periods (in days) of variable* M-*type supergiants*

Class	M1	M2	M2.5	M3	M3.5	M4
Ia	...	5000 4500	...	...	...	...
Iab	1733	2800 2200	...	3060	...	2900
Ib	...	2050	...	...	2950	4100

exception of the supergiants in the I Per association, for which Wildey's (1964) classifications have been used. Of course, it should be recognized that the spectral type often varies slightly during a cycle, and it is a little uncertain to what extent the listed classifications are representative of the mean spectral types.

Five of the twenty-two red variables in Table 4 are known to have blue companions and are therefore of the VV Cephei class (Cowley, 1969). The five variables are: VV Cep, U Lac, α Sco, WY Vel and XX Per. It is believed, however, that the spectral classifications of the red components are nonetheless reliable (Lee, 1970).

A variant of the usual Hess diagram is presented in Table 5, in terms of spectral type and luminosity class. Keenan (1942) was the first to present such a diagram for the variable red supergiants. Here however, we have added the values of period. It is immediately clear that the primary period increases both with later spectral type and with brighter luminosity class, and therefore also with increasing radius if the variation of mass is small. This is in accord with the pulsation hypothesis.

In Table 6, a similar diagram is presented for the secondary periods. The trends of period with spectral type and luminosity class are the same as for the primary periods. It is possible that the long secondary period is associated with a convective mode in the stellar envelope (Section VI) rather than with a radial-pulsation mode. In either case, the trends of period can be qualitatively understood.

If one realizes that the assigned spectral types and luminosity classes may be in error by at least half a subclass, the monotonicity of the trends of period is remarkable. The only obvious exception is the secondary period of U Lac (2900 days). In view of the greater uncertainty of the secondary periods in general, the otherwise clear monotonicity of the secondary periods should probably be considered as real.

V. Semiempirical Masses and Absolute Magnitudes Based on the Primary Periods

Masses of red supergiants derived directly from binary orbits are few in number and are rather poorly determined. Cowley (1969) has summarized most of the pertinent orbital data. For the red component of VV Cep, Peery (1966) has derived a mass of 84 $\mathcal{M}_\odot$, expressing considerable uncertainty as to its reliability. Wright and Larson (1969) have recently redetermined the orbit using superior observational data and now find a much smaller mass ratio, which will reduce considerably the mass of the red star. Cowley (1965, 1969) has obtained a lower limit of 34 $\mathcal{M}_\odot$ for the red component of Boss 1985.

Masses of the variable red supergiants can be derived from their observed primary periods and effective temperatures if the mass-luminosity relation is known. We have already determined a number of accurate luminosities for red supergiants in well-observed clusters. However, even the rather small error in M_v and effective temperature for these supergiants will cause a large uncertainty in the derived masses if Eq. (2) is used, because $\mathcal{M}$ is so sensitive to the various parameters in this expression. Nevertheless, Takeuti (1958) has used it to establish at least the order of magnitude of the mass of a typical red supergiant.

In view of this paucity of data on the masses of red supergiants (except that, qualitatively, the data confirm the suspected high masses of these stars), we shall adopt a mass-luminosity relation based on theory. Elsewhere (Stothers, 1969a), it has been pointed out that the red supergiants are primarily in the early stages of core helium burning; only a tiny fraction of these stars is expected to be in the phase of core carbon burning and later phases. The average luminosity L of a star beginning to deplete core helium is (on the assumption of evolution without mass loss)

$$\frac{L}{L_\odot} = \left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)^\gamma, \quad \text{with} \quad \gamma = 4.0 \left(\frac{\mathcal{M}/\mathcal{M}_\odot}{13}\right)^{-0.09}, \tag{4}$$

valid in the range $9 - 30\ \mathcal{M}_\odot$. We shall assume that this (equilibrium) luminosity is equivalent to the mean luminosity of the pulsating star. Fortunately, the uncertainty of γ due to the dispersion of luminosity during the early stages of core helium burning and to our present uncertainty regarding the zero-age chemical composition of massive stars produces an insignificant uncertainty in the mass derived from Eq. (4). According to the definition of the Q value,

$$Q = P\left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)^{1/2}\left(\frac{R}{R_\odot}\right)^{-3/2}. \tag{5}$$

Since

$$L = 4\pi R^2 \sigma T_e^4, \tag{6}$$

we find

$$\frac{\mathcal{M}}{\mathcal{M}_\odot} = \left(\frac{P}{Q}\right)^{4/(3\gamma-2)}\left(\frac{T_e}{T_{e\odot}}\right)^{12/(3\gamma-2)}. \tag{7}$$

With the adoption of Lee's (1970) relation between spectral type and effective temperature and $Q = 0.06$ day, we may determine the masses of the variable red supergiants in Table 4 by iterating Eq. (7) and the second of Eqs. (4). The absolute visual magnitudes follow immediately from Eqs. (4) and the application of Lee's bolometric corrections.

With $\gamma \approx 4$, one has approximately that $\mathcal{M} \sim (P/Q)^{0.4} T_e^{1.2}$. Since the mean errors in effective temperature, period, and Q value are not expected to exceed 10, 10 and 15 per cent, respectively, the masses should be accurate within 15 per cent (as long as extensive mass loss has not occurred). The masses, then, have a surprisingly high accuracy. However, the corresponding error in absolute bolometric magnitude of an individual star is ± 0.6 mag.

Within the resolution available, there is no obvious trend of absolute visual magnitude with spectral type at a given luminosity class. The reason for this is chiefly that the mass (and therefore the bolometric luminosity) increase slightly with advancing spectral type while the bolometric correction decreases (algebraically). For each luminosity class, an average mass and absolute visual magnitude has been formed:

Class	$\langle \mathcal{M}/\mathcal{M}_\odot \rangle$	$\langle M_v \rangle$	
Ia	26	− 6.5	(5 stars)
Iab	18	− 5.6	(9 stars)
Ib	15	− 4.7	(3 stars) .

The agreement of the semiempirical absolute magnitude for luminosity classes Iab and Ib with that based on cluster members (− 5.6 and − 5.0, respectively) is very good. Even more useful, perhaps, is the determination of an absolute magnitude for luminosity class Ia, for which no well-established cluster members are available. For the four stars common to Tables 1 and 4, we find:

Star	Cluster M_v	Pulsational M_v
SU Per	− 5.6	− 5.4
AD Per	− 5.7	− 5.6
BU Per	− 5.1	− 4.8
FZ Per	− 5.8	− 5.4

The four stars cover the broad spectral range M1 − M3.5 and Iab − Ib, and agreement is again satisfactory, considering the large mean error of the individual pulsational M_v.[5]) We should note, moreover, that the derived masses of those red supergiants of Table 4 which have been assigned membership in associations show good agreement with the estimated masses of associated blue supergiants and, more meaningfully, with the masses of stars at the tip of the associated main-sequence turnoffs. This implies that extensive mass loss cannot have taken place.

[5]) The slight systematic difference of about 0.25 mag between the cluster and pulsational values of M_v is probably not meaningful, and could be made to disappear by making any one of the following changes: (a) $Q = 0.052$ day, (b) effective temperatures which are hotter than Lee's by 5 per cent, (c) a distance modulus of I Per which is 0.25 mag smaller than ours, (d) adoption of a correspondingly fainter phase of the fluctuating light cycle, (e) intrinsic colors which are redder than ours by 0.08 mag, or (f) $A_V/E_{B-V} = 2.5$.

It is our belief that a valid identification has been made, in the foregoing, of the results based on known variables with results based on (supposed) nonvariables. Two reasons for our belief are offered, in addition to the general agreement found above. First, there appears to be no obvious systematic difference in M_v (as based on cluster membership) between the variables and nonvariables in the inner group of I Per (Table 1). Second, the observed light amplitudes of the known variables are generally quite small and average about 1 mag photographic; since some of the known variables have much smaller amplitudes than this, and since some of the supposed nonvariables show slightly different magnitudes when observed at different times (usually by different observers), it is rather likely that all the red supergiants are varying to some extent. The suggestion of a universal variability among these stars is not new (see, e.g. Abt, 1957), but the suspicion has never been really checked and a careful study would be useful. If correct, the suggestion may explain why the variable and "nonvariable" red supergiants are intermixed on the H − R diagram. Although we have not been able to determine any definitely preferred locus of maximum light amplitude on the H − R diagram, there does exist a trend of increasing photographic light amplitude both with brighter luminosity class and with advancing spectral type (i.e. with increasing period).

VI. Interpretation of the Secondary Periods

The long secondary periods which are observed in variable red supergiants are interpreted here tentatively as the convective turnover time of giant convection cells in the stellar envelope. The theoretical and observational basis for this suggestion and an analytical expression for the relevant mixing time are given in the Appendix. At present, it is uncertain whether one should use one or two times the travel time of an element from the base to the top of the envelope, and whether the proper "mixing length" is the whole or only part of the geometrical extent of the envelope, apart from other uncertainties. Therefore the derived turnover times are expected to be accurate only within a factor of 2, at best. Using the mean pulsational masses found earlier for each luminosity class and Lee's effective temperatures, we obtain the following turnover times (in days):

Class	M 1	M 2	M 3	M 4
Ia	3900	4100	4500	5100
Iab	3500	3600	4000	4500
Ib	3300	3400	3800	4200

Surprisingly good qualitative agreement is obtained with the secondary periods of Table 6.

A possible alternative interpretation of the secondary periods is that they are due to the fundamental mode of radial pulsation (Section VIII). Clearly, the problem of the secondary periods deserves more attention because of their importance in establishing the detailed characteristics of extended convective envelopes.

VII. Evolutionary Status of Red Supergiants

The agreement of the masses and absolute magnitudes which have been derived by the Q-value method and cluster membership with those which have been derived from theoretical models of stars burning core helium is quite striking. To be absolutely certain of our conclusion, we have adopted the theoretical mass-luminosity relation for stars burning core carbon, as based on the models of Hayashi *et al.* (1962), Kippenhahn *et al.* (1966), and Stothers and Chin (1969), for the mass range $4-30\,\mathscr{M}_\odot$. By the use of this relation, it is found that, over the whole range represented in Table 4, the masses based on core carbon burning are about $8\,\mathscr{M}_\odot$ smaller than those based on core helium burning; and, furthermore, that the absolute magnitudes range from 0.2 to 0.7 mag fainter (going from the highest to the lowest masses). On the supposition that the stars have evolved with mass loss, the calculated pulsational masses would be even smaller, but the luminosity would remain approximately constant. However, even the maximum (zero-age) mass based on core carbon burning is unquestionably too small in most cases (notably among the supergiants in I Gem and in the inner group of I Per) to be compatible with the mass of the stars at the associated main-sequence turnoffs. Since much additional evidence that the majority of red supergiants are burning core helium has been assembled by Stothers and Evans (1970), we conclude that little doubt remains any longer as to the general correctness of this conclusion.

VIII. Conclusion

Luminosities and masses of M-type supergiants have been derived for (a) cluster members and (b) known variables on the assumption that their primary periods are due to radial pulsation with $Q = 0.06$ day (interpreted as representing the fundamental mode of pulsation). The luminosities based on the two methods are in good agreement, and the means for the various luminosity classes are:

Class	$\langle M_v \rangle$
Ia	-6.5 ± 0.3 (m.e.)
Iab	-5.6 ± 0.1 (m.e.)
Ib	-4.8 ± 0.2 (m.e.)

The pulsational masses of variables in associations are in satisfactory agreement with the masses inferred from the main-sequence turnoffs, thereby implying that little mass has been lost. The long secondary periods are tentatively interpreted as the convective turnover time of giant cells in the stellar envelope. Since it is likely that all the red supergiants later than M 0 are at least slightly variable, the quasi-periodic phenomena of the known variables constitute a useful probe into the structure of all these stars. The evidence is very strong that they are in the early stages of core helium burning.

It is important to point out one possible alternative interpretation of the observed periods. If the ionization zones of hydrogen and helium (which were not included in the calculations of Paper I) are favorably situated in the envelope of these stars, it is possible to increase the Q value, and hence the period, of the radial-pulsation modes by virtue of the reduced size of the effective adiabatic exponent (see the results of Keeley [1970] for red giants of small mass). In that case, the primary observed period may represent the first radial overtone, while the secondary period may represent the fundamental mode itself. However, none of the other conclusions of the present paper would be changed by this interpretation as long as the primary period can be represented approximately by $Q = 0.06$ day.

Refinement of the observed periods and discovery of new periods, accurate distance determinations, and detailed theoretical models of pulsating convective envelopes are all useful areas for future work on the M-type supergiants.

Appendix
Mixing Time of Stellar Envelopes in Convective Equilibrium

Two possible modes of convective transport have been suggested to describe the rate of turbulent mixing in a star according to the mixing-length theory. The first suggested mode is a direct trajectory of the major flux-carrying elements between top and bottom of the convective zone, with the local velocity evaluated in terms of one of the local thermodynamical scale heights. The second suggested mode is a random walk of the major flux-carrying elements, with a "mean free path" given by the local mixing length, which is usually taken to be of the same order as a scale height.

The maximum size of elements responsible for transporting the flux is of interest here. Any relevant observational data refer, of course, strictly to the stellar surface layers, but inferences concerning the underlying layers can often be drawn from these observations, which are now summarized. In the case of the red supergiants, large condensations are known to exist in their chromospheres on the basis of eclipse spectra taken of members of eclipsing binary systems (Wilson, 1960; Cowley, 1969). These condensations, however, seem to occur on a scale much smaller than the stellar radius, and condensations of a larger size (and therefore longer lifetime) would be difficult, if not impossible, to detect. In the case of the sun, however, there is direct observational (and some theoretical) evidence for the existence of *giant cells*, whose scale is comparable with the depth of the convection zone and whose lifetime is a few months (Bumba, 1967; Simon and Weiss, 1968); the observed lifetime of these cells is comparable with the theoretical convective turnover time. If the giant cells comprise the elements which transport most of the energy, then their turnover time may have other observable consequences in the pulsating envelopes of red supergiants, for example by modulating the fundamental period of pulsation. This is to be considered at least possible because the e-folding time of the decay of pulsations is relatively long in supergiants with convective envelopes (Rabinowitz, 1957). Furthermore, it is possible that the movement of a giant cell produces a kind of nonradial oscillation which distorts the star's shape and affects its brightness, thereby accounting for the asymmetry of the lunar-occultation curve observed for α Sco (Evans, 1957) as well as the lack of strict periodicity of the primary periods in general. Consequently, we shall investigate here the time scale of convective turnover based on the first suggested mode of transport (as is proper for a cell whose size is comparable with the extent of the convection zone).

A slight modification of the standard mixing-length theory of turbulent convection (Böhm-Vitense, 1958) to take account of radiation pressure yields for the average local velocity of a convecting element

$$v^3 = \alpha\,(\Gamma_3 - 1)\,H_{\rm conv}/2\varrho\,,$$

where $H_{\rm conv}$ is the convective flux, ϱ the mass density, α the ratio of mixing length to density scale height, and Γ_3 the third generalized adiabatic exponent for a mixture of perfect gas and radiation. We shall assume $H_{\rm conv} = \sigma\, T_e^4$ throughout the convective envelope and $\alpha = 0.5$ (although, formally, the "mixing length" for the first mode of transport is equal to the total extent of the convection zone). Material transport from the base of the convective envelope to the top occurs in a time given by the integral of v^{-1} over the envelope radius, multiplied by the number of mixing lengths between top and bottom (ς). In the non-dimensional variables of Paper I, we have

$$v^3 = (\alpha\, LR/2\mathscr{M})\,\tilde{v}^3\,, \quad \tilde{v}^3 = (\Gamma_3 - 1)\,x/qU\,,$$

$$\tau_{\rm mix} = \varsigma\, I\,(2\mathscr{M}\,R^2/\alpha L)^{1/3} = \varsigma\, I\,(2\mathscr{M}/4\pi\,\sigma\, T_e^4\,\alpha)^{1/3},$$

where

$$I = \int_{x_e}^{1} \tilde{v}^{-1} dx\,.$$

For the massive red supergiants of concern here, an integration of the relevant adiabatic envelopes of Paper I yields $I \approx 1.4$ in all cases. Adopting $\varsigma = 1$, we find that $\tau_{\rm mix}$ depends only on mass and effective temperature. However, we should remark that, if the second mode of convective transport is adopted, then $\varsigma \approx 50$ over the mass range $10-30\,\mathscr{M}_\odot$, and ς in this case depends slightly on effective temperature, viz. $\Delta\varsigma = (7/\alpha)\,\Delta \log T_e$. In the text of the present paper, we have adopted only the first mode of convective transport (relevant for a giant cell). The time scale of convective turnover is, finally, $2\tau_{\rm mix}$.

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