1970Ap&SS...6..228C

NEUTRON STAR MODELS BASED ON AN IMPROVED EQUATION OF STATE

JEFFREY M. COHEN and WILLIAM D. LANGER,
Goddard Institute for Space Studies, NASA, New York, N.Y., U.S.A.

LEONARD C. ROSEN,
Belfer Graduate School of Science, Yeshiva University, New York, N.Y., U.S.A.

and

A. G. W. CAMERON
Belfer Graduate School of Science, Yeshiva University,
and Goddard Institute for Space Studies, NASA, New York, N.Y., U.S.A.

(Received 4 August, 1969)

Abstract. Using an equation of state for cold degenerate matter which takes nuclear forces and nuclear clustering into account, neutron star models are constructed. Stable models were obtained in the mass range above 0.065 $M_\odot$ and density range $10^{14.08}$ to $10^{15.4}$ gm/cm^3. All of these models were found to be bound. The outer crystalline layer of the star was found to have a thickness of 200 m or more depending on the mass of the model.

1. Introduction

In a recent paper (Cameron and Cohen, 1969), it was shown that stellar models intermediate between white dwarfs and neutron stars are dynamically unstable. In particular, it was found that stellar models beyond the white dwarf peak ($\sim 10^{10.5}$ gm/cm^3 for a white dwarf composed of pure carbon) and below 10^{14} gm/cm^3 are dynamically unstable. These calculations were carried out using an equation of state which takes nuclear clustering into account.

The nuclear clustering problem can be stated as follows: if a box is filled with free neutrons, protons, and electrons of a given density and zero net charge, what will be the final equilibrium composition? Will the box remain filled with free particles or will clusters (nuclei) form? At typical white dwarf density ($\sim 10^6$ gm/cm^3), the final equilibrium composition is: clusters (nuclei) and electrons if the temperature is sufficiently low. In fact, if $T \lesssim 10^6$ K, the nuclei form a crystalline lattice (Van Horn, 1968). The lattice forces alter the equation of state from that of a degenerate electron gas (Salpeter, 1961) which in turn alters the equilibrium white dwarf models (Hamada and Salpeter, 1961) from those of Chandrasekhar (1935) and lowers the pulsation frequency (Cohen *et al.*, 1969). In the latter calculations the contribution of the lattice structure to both the energy density and pressure was included. On the other hand, at typical neutron star densities ($\sim 10^{15}$ gm/cm^3), the equilibrium composition is: neutrons, protons, electrons, muons, and hyperons. At 10^{14} gm/cm^3, only neutrons, protons, and electrons are present at low temperatures.

In the region intermediate between white dwarf and neutron star densities ($\sim 10^{13}$

Astrophysics and Space Science **6** (1970) 228–239; *Copyright © 1970 by D. Reidel Publishing Company, Dordrecht - Holland. All Rights Reserved.*

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1970Ap&SS...6..228C

gm/cm^3), the equilibrium composition is (Cameron and Cohen, 1969): electrons, neutrons, and clusters (nuclei). The clusters become more and more neutron rich as the density rises. The existence of the clusters drastically alters the equation of state from that of a free neutron, proton, and electron gas. Whereas, neutron stars composed of such a gas are dynamically stable, the stellar structures beyond the white dwarf peak and below 10^{14} gm/cm^3 were found to be dynamically unstable (Cameron and Cohen, 1969) when clustering was taken into account. This instability is because clusters, electrons, and neutrons supply less pressure at low temperatures than a free Fermi gas of neutrons, protons, and electrons.

In the calculations of Cameron and Cohen (1969) the nuclear potential energy between the nucleons in the cluster was taken into account via a semi-empirical mass formula but nuclear interactions in the neutron gas were neglected. In this paper, we include these interactions using the velocity dependent V_α potential of Levinger and Simmons (1961) in the manner described by Weiss and Cameron (1969). In a previous paper, the equation of state was given in the region between 3×10^{11} gm/cm^3 and 10^{14} gm/cm^3 (Langer *et al.*, 1969). Here we extend the equation of state up to 10^{16} gm/cm^3, including muon production but neglecting hyperon production; the equation of state is then used to construct general relativistic neutron star models. The numerical method used to construct general relativistic equilibrium models and to determine their stability is discussed elsewhere (Cohen *et al.*, 1969).

2. Equation of State

For degenerate matter, the equation of state takes the simple form $p=p(\varrho)$ where p is the pressure and ϱ is the energy density. This pressure can be obtained from the thermodynamic relation (Landau and Lifshitz, 1958; Chiu, 1968).

$$T\,\mathrm{d}S = \mathrm{d}E + p\,\mathrm{d}V - \sum_i \mu_i\,\mathrm{d}N_i\,, \tag{1}$$

where T is the temperature, S the entropy, E the energy of the system, V the volume, μ_i the chemical potential of the ith particle, and N_i the number of particles i. The chemical potential is given by

$$\mu_i = \left.\frac{\partial E}{\partial N_i}\right|_{S,\,V} = \left.\frac{\partial \varphi}{\partial N_i}\right|_{p,\,T}, \tag{2}$$

where φ is the thermodynamic potential (or Gibbs free energy) defined by

$$\varphi = E - TS + pV\,. \tag{3}$$

At first sight it seems as if the pressure p is given by

$$p = -\left.\frac{\partial E}{\partial V}\right|_{S,\,N_i}; \tag{4}$$

however, it can be shown that it is not necessary to hold the particle numbers N_i

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1970Ap&SS...6..228C

constant. This follows from the requirement that, for an equilibrium configuration, the thermodynamic potential φ takes its minimum value for fixed p and T, given by

$$\sum_i \frac{\mathrm{d}N_i}{\mathrm{d}N_j} \frac{\partial \varphi}{\partial N_i}\bigg|_{p,\,T} = 0\,. \tag{5}$$

The derivative of the thermodynamic potential with respect to each type of particle j vanishes. Substitution of the definition of the chemical potential μ_i (Equation (2)) into Equation (5) yields the relation

$$\sum_i \mu_i \,\mathrm{d}N_i = 0 \tag{6}$$

after multiplying by the arbitrary quantity $\mathrm{d}N_j$. Because of Equation (6), Equation (1) takes the familiar form

$$T\,\mathrm{d}S = \mathrm{d}E + p\,\mathrm{d}V\,, \tag{7}$$

and the expression for the pressure becomes likewise

$$p = -\frac{\partial E}{\partial V}\bigg|_S\,. \tag{8}$$

Although the Expression (8) for the pressure is the same as that when particle numbers are constant, the contribution of created or captured particles is automatically taken into account via their contribution to the energy E.

The Expression (8) can be brought into a simple form containing only the baryon number density $n = N/V$ and the energy density $\varrho = E/V$ by substituting these expressions into Equation (8) and observing that baryons are conserved:

$$p = n\,\frac{\partial \varrho}{\partial n}\bigg|_S - \varrho\,. \tag{9}$$

Since we have considered elsewhere (Langer *et al.*, 1969) the region where neutrons, nuclei and electrons are the equilibrium composition, we will restrict ourselves here to the region above 6×10^{13} gm/cm^3 where the equilibrium composition is neutrons, protons and electrons with muons appearing when their threshold is exceeded. Hyperon production will be considered elsewhere.

The equilibrium composition can be obtained by minimizing the energy density ϱ of the system of particles (keeping S and V constant) subject to the constraints that baryons and charge be conserved:

$$\delta I|_{S,\,V} = 0\,, \tag{10}$$

where

$$I = \varrho + \alpha(n_p - n_e - n_\mu) + \beta(n_p + n_n)\,. \tag{11}$$

Here α and β are the Lagrange multipliers associated with charge equality and baryon conservation, respectively.

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1970Ap&SS...6..228C

In the region below the muon threshold, n_μ vanishes and there is no muon contribution to ϱ; consequently, differentiation of Equation (11) yields

$$O = \mu_e - \alpha, \tag{12a}$$

$$O = \mu_p + \alpha + \beta, \tag{12b}$$

$$O = \mu_n + \beta, \tag{12c}$$

where the μ_i are the chemical potentials defined in Equation (2). Elimination of the Lagrange multipliers yields

$$\mu_n = \mu_p + \mu_e; \tag{13}$$

while charge conservation yields the requirement that there be the same number of protons and electrons

$$n_p = n_e. \tag{14}$$

If one of the three quantities n_p, n_e, n_n is chosen, the other two can be determined from Equations (13) and (14).

Above the muon threshold, muons will be present and their number density as well as their threshold can be determined from Equations (10) and (11) giving the relations

$$O = \mu_e - \alpha, \tag{15a}$$

$$O = \mu_p + \alpha + \beta, \tag{15b}$$

$$O = \mu_n + \beta, \tag{15c}$$

$$O = \mu_\mu - \alpha. \tag{15d}$$

Elimination of the Lagrange multipliers yields

$$\mu_n = \mu_p + \mu_e, \tag{16a}$$

$$\mu_\mu = \mu_e, \tag{16b}$$

and charge equality yields

$$n_p = n_e + n_\mu. \tag{16c}$$

Although the muon chemical potential goes from zero to a finite value of 105.7 MeV when the electron Fermi level reaches this value, the muon number density n increases smoothly with electron Fermi level E_{fe}. This is because the muon number density

$$n_\mu = 8\pi p_\mu^3 / 3h^3 \tag{17}$$

and

$$p_\mu = (\mu_\mu^2 - m_\mu^2)^{1/2}. \tag{18}$$

Consequently, at the muon threshold, the muon number density is zero.

In Table I, the number densities of the different particles are given in the density range above 10^{14} gm/cm^3. The effect of hyperons has been neglected here and will be treated elsewhere. The equation of state will be affected by the presence of hyperons

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1970Ap&SS...6..228C

above densities in the vicinity of 3×10^{14} gm/cm^3. The numbers in the table above the hyperon threshold are used in this paper, but since hyperons are not included, points above the hyperon threshold are of provisional interest since they are used in the construction of the models given in this paper.

TABLE I

Equation of State Variables

ϱ g/cm^3	$P\times10^{30}$ dyne/cm^2	Γ	$N(n)$ 10^{30} cm^{-3}	$N(p)$ 10^{30} cm^{-3}	$N(e^-)$ 10^{30} cm^{-3}	$N(\mu^-)$ 10^{30} cm^{-3}
1.01×10^{14}	4.27×10^{2}	2.92	5.93×10^{7}	9.81×10^{5}	9.81×10^{5}	0.0
1.51×10^{14}	1.41×10^{3}	2.98	8.75×10^{7}	2.40×10^{6}	2.40×10^{6}	0.0
2.00×10^{14}	3.17×10^{3}	2.96	1.14×10^{8}	4.38×10^{6}	4.38×10^{6}	0.0
2.19×10^{14}	4.18×10^{3}	2.95	1.25×10^{8}	5.38×10^{6}	5.37×10^{6}	1.16×10^{4}
2.51×10^{14}	6.19×10^{3}	2.94	1.41×10^{8}	7.59×10^{6}	7.05×10^{6}	5.39×10^{5}
3.02×10^{14}	1.05×10^{4}	2.93	1.67×10^{8}	1.21×10^{7}	1.00×10^{7}	2.10×10^{6}
4.00×10^{14}	2.29×10^{4}	2.91	2.12×10^{8}	2.36×10^{7}	1.69×10^{7}	6.74×10^{6}
5.03×10^{14}	4.24×10^{4}	2.89	2.55×10^{8}	3.88×10^{7}	2.54×10^{7}	1.34×10^{7}
6.02×10^{14}	6.75×10^{4}	2.88	2.92×10^{8}	5.55×10^{7}	3.46×10^{7}	2.09×10^{7}
7.00×10^{14}	9.87×10^{4}	2.87	3.26×10^{8}	7.36×10^{7}	4.43×10^{7}	2.93×10^{7}
8.06×10^{14}	1.40×10^{5}	2.87	3.59×10^{8}	9.43×10^{7}	5.53×10^{7}	3.90×10^{7}
9.04×10^{14}	1.83×10^{5}	2.87	3.87×10^{8}	1.13×10^{8}	6.54×10^{7}	4.81×10^{7}
1.00×10^{15}	2.32×10^{5}	2.87	4.14×10^{8}	1.33×10^{8}	7.57×10^{7}	5.74×10^{7}
1.50×10^{15}	5.56×10^{5}	2.89	5.27×10^{8}	2.28×10^{8}	1.25×10^{8}	1.03×10^{8}
2.00×10^{15}	9.89×10^{5}	2.89	6.18×10^{8}	3.12×10^{8}	1.68×10^{8}	1.44×10^{8}
3.01×10^{15}	2.00×10^{6}	2.87	7.56×10^{8}	4.48×10^{8}	2.38×10^{8}	2.10×10^{8}
4.01×10^{15}	3.14×10^{6}	2.85	8.62×10^{8}	5.57×10^{8}	2.93×10^{8}	2.64×10^{8}
5.00×10^{15}	4.35×10^{6}	2.83	9.50×10^{8}	6.47×10^{8}	3.39×10^{8}	3.08×10^{8}
5.35×10^{15}	4.80×10^{6}	2.83	9.78×10^{8}	6.77×10^{8}	3.54×10^{8}	3.23×10^{8}

From Table I it can be seen that the muon threshold is about 2.2×10^{14} gm/cm^3. This is lower than the threshold obtained by Tsuruta (1964) who assumes a free Fermi gas when computing the composition. The depressed muon threshold obtained here may seem surprising since muons do not seem to participate in strong interactions. However, the depression of the muon threshold is an indirect effect of nuclear interactions. The number density of protons is higher when nuclear forces are taken into account. Consequently, the electron number density and Fermi level, given in Table II also increases because of charge equality. Thus the electron Fermi level reaches the muon threshold 105.7 MeV at a lower mass density when nuclear forces are taken into account.

The energy density of the system of particles is given by

$$\varrho = \sum_i \int_0^{n_{fi}} \left(m_i^2 + \varrho_i^2(n_i)\right)^{1/2} \mathrm{d}n_i + \tfrac{1}{2}\sum_{i,j} \int_0^{n_{fi}} \int_0^{n_{fj}} B(n_i, n_j)\, \mathrm{d}n_i\, \mathrm{d}n_j\,. \tag{19}$$

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1970Ap&SS...6..228C

TABLE II

Particle energies

ϱ gm/cm^3	$K_f(n)$ MeV	$\mu_f(n)$ MeV	$K_f(p)$ MeV	$\mu_f(p)$ MeV	$\mu_f(e^-)$ MeV
1.01×10^{14}	29.4	946.52	1.92	886.5	60.6
1.51×10^{14}	28.3	952.6	3.50	871.0	81.7
2.00×10^{14}	45.5	960.5	5.35	860.6	99.8
2.19×10^{14}	48.3	964.3	6.02	857.4	106.9
2.51×10^{14}	52.4	970.7	7.68	853.7	117.1
3.02×10^{14}	58.3	982.1	10.4	850.4	131.7
4.00×10^{14}	68.0	1008	16.2	851.1	156.6
5.03×10^{14}	76.6	1038	22.4	859.0	179.5
6.02×10^{14}	83.6	1071	28.5	872.0	198.9
7.00×10^{14}	89.6	1105	34.2	888.9	216.0
8.06×10^{14}	95.4	1144	40.3	911.1	232.6
9.04×10^{14}	100.1	1180	45.4	933.9	245.9
1.00×10^{15}	104.4	1218	50.4	959.4	258.2
1.50×10^{15}	121.5	1415	71.3	1110	305.0
2.01×10^{15}	134.3	1621	87.4	1284	337.1
3.01×10^{15}	151 7	2005	109.8	1627	378.1
4.01×10^{15}	165.1	2362	125.9	1956	405.4
5.00×10^{15}	175.2	2691	138.4	2265	425.6
5.35×10^{15}	178.5	2806	142.3	2374	431.9

The first integral is the sum of the kinetic energies of the particles while the latter given the sum of the nuclear potential energies assuming a two particle interaction. For this energy density, the expression for the electron chemical potential μ_e becomes

$$\mu_e = (m_e^2 + p_{fe}^2)^{1/2} = E_f\,, \tag{20}$$

where p_{fe} is the Fermi momentum and E_f is the Fermi energy of the electron. Similarly for the neutrons, we obtain

$$\mu_n = (m_n^2 + p_{fn}^2)^{1/2} + \sum_j \int_0^{n_{fj}} B(n_{fn}, n_j)\, \mathrm{d}n_j\,. \tag{21}$$

Here the neutron chemical potential is given by the sum of the neutron kinetic energy at the top of the neutron Fermi sea and the sum of the nuclear potential energies between a neutron at the top of the neutron Fermi sea and all the other baryons in the system.

The second term in Equation (19) can also be expressed as the average nuclear potential energy per baryon of type i times the number of baryons of type i giving

$$\varrho = \sum_i \int_0^{n_{fi}} (m_i^2 + p_i^2(n_i))^{1/2}\, \mathrm{d}n_i + \sum_i n_i \langle B_i \rangle\,. \tag{22}$$

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1970Ap&SS...6..228C

Consequently, the neutron chemical potential also takes the form

$$\mu_n = (m_n^2 + p_{fn}^2)^{1/2} + \langle B_n \rangle + \sum_i n_i \partial_n \langle B_i \rangle . \tag{23}$$

For a detailed discussion of this see Langer *et al.* (1969).

Once the number densities of the constituents have been determined, the equation of state can be obtained from Equation (9) and the expression for the energy density. The equation of state is tabulated in Table I.

3. Model Calculation

In this section we discuss the construction of equilibrium models and the determination of their stability. For equilibrium models, Einstein's equations take the well-

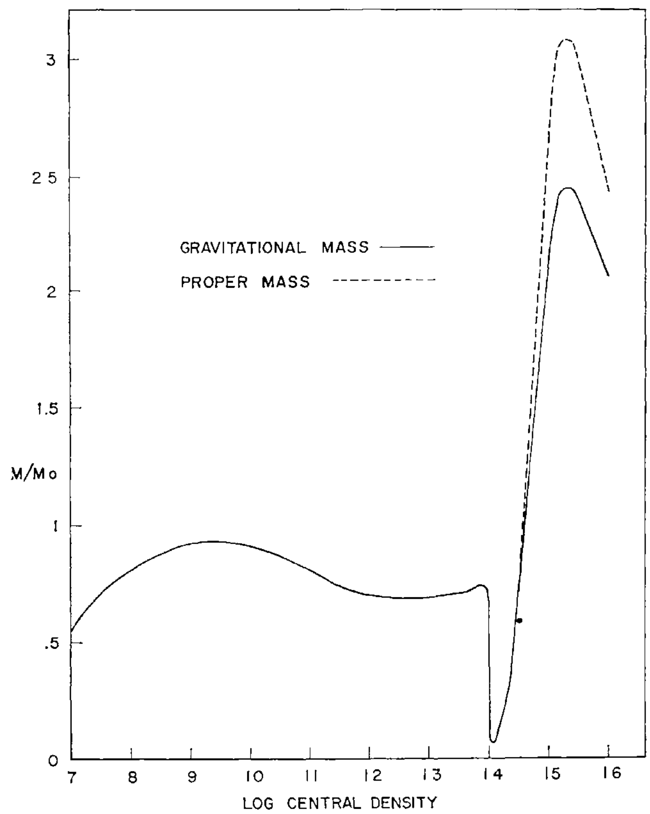

Fig. 1. Gravitational mass and proper mass vs. central density.

known form (Landau and Lifshitz, 1962; Cohen *et al.*, 1969; Oppenheimer and Volkoff, 1939)

$$\partial_r m = 4\pi r^2 \varrho , \tag{24}$$

$$c^2 \partial_r \varphi = G(m + 4\pi r^3 p/c^2)\, r^{-1} (r - 2Gm/c^2)^{-1} , \tag{25}$$

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1970Ap&SS...6..228C

$$\partial_r p = -\left(\varrho + p/c^2\right) c^2 \partial_r \varphi . \tag{26}$$

Here the radial coordinate r is defined (Cohen and Cohen, 1969) in such a way that the surface area of a sphere of radius r is $4\pi r^2$, φ corresponds in the weak field limit to the gravitational potential, and m is the gravitational mass. Machine integration of these equations, using the equation of state described in the previous section, gives the gravitational mass and radius of neutron star models as a function of central density. This is shown in Table III and Figure 1.

The proper mass of a star is the sum of the masses of the particles in the star when the star is broken up into particles and all the particles are given infinite separation.

TABLE III

Log central density gm/cm³	mass/10^{33} gm	Proper mass/10^{33} gm	Radius km	T_0 ms	Binding energy/10^{33} gm
16	4.10	4.85	9.39	U	
15.8	4.35	5.30	9.84	U	
15.6	4.62	5.77	10.49	U	
15.4	4.83	6.11	11.36	U	
15.3	4.87	6.14	11.87	5.05	1.27
15.2	4.82	6.02	12.41	0.83	1.20
15	4.28	5.14	13.39	0.49	0.86
14.8	3.11	3.52	13.91	0.41	0.41
14.6	1.73	1.85	13.79	0.40	0.12
14.4	0.754	0.778	13.80	0.44	0.024
14.2	0.279	0.283	17.72	1.31	0.004
14.1	0.164	0.166	33.41	5.25	0.002
14.06	0.134	0.135	76.4	22.6	0.001
14.04	0.124	0.125	305.0	222.2	0.001
14.02	1.45	1.45	2233	U	
14	1.45	1.45	2233	U	
13.8	1.46	1.46	661.4	U	
13.6	1.41	1.41	566.4	U	
13.4	1.39	1.39	564.1	U	
13.2	1.39	1.39	563.5	U	

When this is done many particles decay but the number of baryons remains constant. The remaining particles are protons and an equal number of electrons. Consequently, the proper mass can be found by adding the mass of the proton and the electron and multiplying by the baryon number. The proper mass is given in column 3 of Table III.

The adiabatic index $\Gamma_1 = \left(c^{-2} + \varrho/p\right) \partial_\varrho p|_s$, (27)

is given in Table I. The pulsation period for small oscillations of a stellar model can be obtained by perturbing the star and solving an eigenvalue equation for the perturbations. The pulsation equation has been derived directly from Einstein's equations (Taub, 1962; Chandrasekhar, 1964; Cohen, 1969) and via variational principles

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1970Ap&SS...6..228C

(Chandrasekhar, 1964; Cocke, 1965; Harrison *et al.*, 1965). A numerical method of obtaining the pulsation periods, valid even for models with density discontinuities, is given by Cohen *et al.* (1969). Since the age of the star is much larger than the electron capture, beta decay, nuclear reaction, and elementary particle interactions, these lifetimes can be assumed to be zero relative to the age of the star. Consequently, by computing the stellar pulsation period assuming instantaneous electron capture, etc., it can be determined whether or not the model exhibits secular instability. The fundamental pulsation period is shown in Figure 2. If the fundamental pulsation

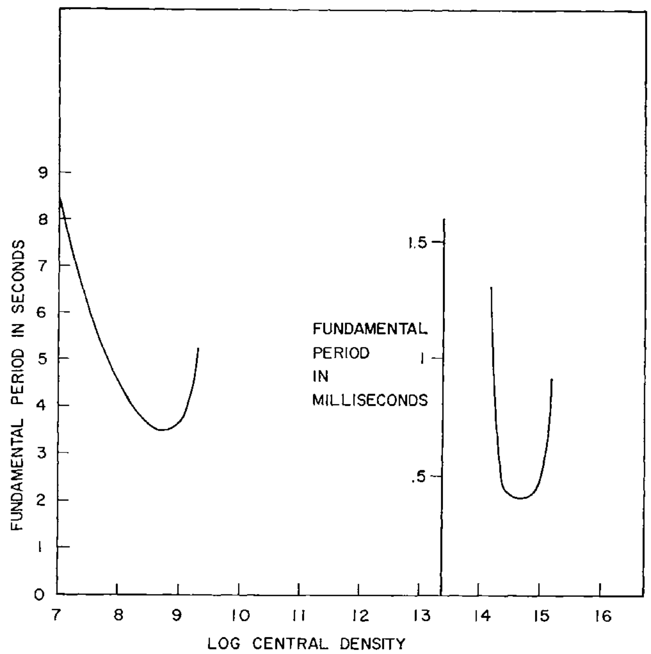

Fig. 2. Fundamental period vs. log central density.

period is imaginary, the star exhibits secular instability unless non-linear effects stabilize the model for large pulsations. This can be determined using a full non-linear hydrodynamic computer code (see, e.g. Cocke and Cohen, 1968, 1969, and the references cited there). For a discussion of the effects of electron capture and beta decay rates on stellar pulsation periods and on the stability of stellar models see, e.g., Cohen *et al.*, (1969) and Chiu and Cohen (1969).

The maximum mass of these neutron star models was found to be about 2.4 $M_{\odot}$ while the minimum mass is about 0.065 $M_{\odot}$. Unlike some neutron star models given by others (Meltzer and Thorne, 1966), all of the stable neutron star models given here are bound including those with mass less than 0.15 $M_{\odot}$. The models intermediate between white dwarf and nuclear density which were believed to be stable but unbound, are found to be unstable when nuclear clustering is taken into account. These results are in agreement with those obtained using an equation of state which neglected the

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1970Ap&SS...6..228C

nuclear interaction between the neutrons in the neutron gas between the clusters (Cameron and Cohen, 1969). The gravitational mass and proper mass of the neutron star models are plotted in Figure 1 as a function of central density.

TABLE IV

Muon Threshold	2.2×10^{14} gm/cm^3
Nuclear Break-up	6.0×10^{13} gm/cm^3
Proton Threshold	4.0×10^{13} gm/cm^3
Neutron Threshold	3.0×10^{11} gm/cm^3

TABLE V

Density distribution. Central density $= 10^{14.5}$ gm/cm^3

Density gm/cm^3	Mass gm	Radius km
3.16×10^{14}	0	0
3.15×10^{14}	1.37×10^{30}	1.01
3.10×10^{14}	1.10×10^{31}	2.04
3.03×10^{14}	3.33×10^{31}	2.96
2.90×10^{14}	8.67×10^{31}	4.10
2.74×10^{14}	1.70×10^{32}	5.18
2.60×10^{14}	2.51×10^{32}	5.97
2.39×10^{14}	3.80×10^{32}	6.95
2.2×10^{14}	5.01×10^{32}	7.72
2.15×10^{14}	5.57×10^{32}	8.06
1.82×10^{14}	7.26×10^{32}	9.00
1.45×10^{14}	9.14×10^{32}	10.0
9.64×10^{13}	1.08×10^{33}	11.0
6.0×10^{13}	1.148×10^{33}	11.52
4.0×10^{13}	1.164×10^{33}	11.70
1.94×10^{12}	1.172×10^{33}	12.0
2.85×10^{11}	1.172×10^{33}	12.18
1.49×10^{10}	1.172×10^{33}	13.0
2.97×10^{8}	1.172×10^{33}	13.5
2.04×10^{1}	1.172×10^{33}	13.68

In Table IV, the density at the muon threshold, etc. is tabulated. In Tables V to VIII, the mass and density distributions of various models are given. The difference between the radius at the outer boundary of the neutron star model and that at the neutron drip line gives the minimum size of the outer crystalline layer of the model.

Acknowledgements

For helpful discussion we are indebted to Prof. B. Koharitz, S. Ungar, and Dr. R. Weiss. We should also like to thank Dr. R. Weiss for the use of his binding energy subroutines. The work of two of us (J.M.C. and W.L.) supported, in part, by NAS-NRC Research Associateships sponsored by the National Aeronautics and Space Administration. This work was also supported, in part, by the National Science Foundation, National Aeronautics and Space Administration, and the U.S. Atomic Energy Commission.

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1970Ap&SS...6..228C

TABLE VI

Density distribution. Central density $= 10^{14.06}$ gm/cm^3

Density gm/cm^3	Mass gm	Radius km
1.148×10^{14}	0	0
1.137×10^{14}	5.12×10^{29}	1.02
1.103×10^{14}	3.57×10^{30}	1.97
1.042×10^{14}	1.22×10^{31}	2.99
9.60×10^{13}	2.69×10^{31}	3.96
8.35×10^{13}	5.14×10^{31}	5.03
6.88×10^{13}	7.89×10^{31}	5.98
6.0×10^{13}	9.28×10^{31}	6.42
4.78×10^{13}	1.10×10^{32}	6.99
4.0×10^{13}	1.16×10^{32}	7.21
7.11×10^{12}	1.29×10^{32}	8.01
8.30×10^{11}	1.31×10^{32}	9.03
2.85×10^{11}	1.315×10^{32}	9.64
2.40×10^{11}	1.317×10^{32}	10.0
4.41×10^{10}	1.327×10^{32}	15.1
1.34×10^{10}	1.331×10^{32}	20.1
2.31×10^{9}	1.336×10^{32}	30.0
5.36×10^{8}	1.337×10^{32}	40.0
1.32×10^{8}	1.338×10^{32}	50.1
2.91×10^{7}	1.338×10^{32}	60.1
3.29×10^{6}	1.338×10^{32}	70.1
1.95×10^{1}	1.338×10^{32}	76.4

TABLE VII

Density distribution. Central density $= 10^{14}$ gm/cm^3

Density gm/cm^3	Mass gm	Radius km
1.0×10^{14}	0	0
6.0×10^{13}	5.56×10^{31}	5.57
4.0×10^{13}	7.88×10^{31}	6.55
1.03×10^{13}	9.07×10^{31}	7.35
1.05×10^{12}	9.47×10^{31}	8.70
2.85×10^{11}	9.52×10^{31}	9.70
3.98×10^{10}	9.78×10^{31}	20.2
9.88×10^{9}	1.02×10^{32}	40.7
3.58×10^{9}	1.12×10^{32}	80.8
2.66×10^{9}	1.19×10^{32}	103.1
1.28×10^{9}	1.70×10^{32}	204.0
4.05×10^{8}	4.74×10^{32}	499
1.43×10^{8}	8.57×10^{32}	803
7.08×10^{7}	1.07×10^{33}	1001
1.17×10^{7}	1.37×10^{33}	1505
9.03×10^{5}	1.45×10^{33}	2000
1.61×10^{1}	1.45×10^{33}	2233

TABLE VIII

Density distribution. Central density $= 10^{12.8}$ gm/cm^3

Density gm/cm^3	Mass gm	Radius km
6.31×10^{12}	0	0
6.02×10^{12}	1.43×10^{29}	1.77
3.99×10^{12}	4.32×10^{30}	6.00
2.15×10^{12}	1.46×10^{31}	10.3
1.08×10^{12}	2.71×10^{31}	14.5
4.24×10^{11}	4.31×10^{31}	20.7
2.85×10^{11}	5.32×10^{31}	25.3
2.52×10^{11}	6.68×10^{31}	30.5
1.66×10^{11}	1.49×10^{32}	50.3
5.80×10^{10}	4.97×10^{32}	101
2.05×10^{10}	8.32×10^{32}	151
7.36×10^{9}	1.07×10^{33}	201
1.06×10^{9}	1.30×10^{33}	301
1.40×10^{8}	1.37×10^{33}	401
4.25×10^{7}	1.38×10^{33}	450
8.13×10^{6}	1.38×10^{33}	501
1.51×10^{1}	1.38×10^{33}	552

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1970Ap&SS...6..228C

References

Cameron, A. G. W. and Cohen, J. M.: 1969, *Astrophys. Letters* **3**, 3.
Chandrasekhar, S.: 1935, *Monthly Notices Roy. Astron. Soc.* **95**, 207.
Chandrasekhar, S.: 1964, *Astrophys. J.* **140**, 417.
Chiu, H. Y.: 1968, *Stellar Physics*, Vol. 1, Blaisdell, Waltham, Mass.
Chiu, H. Y. and Cohen, J. M.: 1969, to be published.
Cocke, W. J.: 1965, *Ann. Inst. Henri Poincaré* **4**, 238.
Cocke, W. J. and Cohen, J. M.: 1968, *Nature* **219**, 1009.
Cocke, W. J. and Cohen, J. M.: 1969, *Pulsating Stars*, Plenum Press, New York.
Cohen, J. M.: 1969, to be published.
Cohen, J. M. and Cohen, M. D.: 1969, *Nuovo Cimento* **60B**, 241.
Harrison, B. D., Thorne, K. S., Wakano, M., and Whealers, J. A.: 1965, *Gravitation Theory and Gravitational Collapse*, University of Chicago Press.
Hamada, T. and Salpeter, E. E.: 1961, *Astrophys. J.* **134**, 6381.
Landau, L. and Lifshitz, E.: 1958, *Statistical Physics*, Addison-Wesley, Reading, Mass.
Landau, L. and Lifshitz, E.: 1962, *Classical Theory of Fields*, Addison-Wesley, Reading, Mass.
Langer, W., Rosen, L., Cohen, J. M., and Cameron, A. G. W.: 1969, *Astrophys. Space Sci.* **5**, 259.
Levinger, J. S. and Simmons, L. M.: 1961, *Phys. Rev.* **124**, 916.
Meltzer, D. W. and Thorne, K. S.: 1966, *Astrophys. J.* **145**, 2, 514.
Oppenheimer and Volkoff: 1939, *Phys. Rev.* **55**, 374.
Salpeter, E. E.: 1961, *Astrophys. J.* **134**, 669.
Taub, A. G.: 1962, *Colloques Internationaux de Centre Nationale de la Recherche Scientifique* **91**, 173.
Tsuruta, S.: 1964, Thesis, Columbia University.
Van Horn, H.: 1968, *Astrophys. J.* **151**, 1,227.
Weiss, R. and Cameron, A. G. W.: 1969, *Canad. J. Phys.* (in press).

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System