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Shape of the Crab Pulsar and its Period Fluctuations

RICHARDS *et al.*[1] have recently recorded sinusoidal variations in the arrival times of radio pulses from NP 0532 with a period of about three months and an amplitude $\Delta\tau = 6\times10^{-4}$ s. This effect can be interpreted in terms of genuine sinusoidal variations of the circular frequency of the Crab pulsar

$$\Omega(t) = \Omega(1 + A\cos\omega t) \tag{1}$$

where $\Omega \simeq 200$ s^{-1}, $\omega \simeq 10^{-6}$ s^{-1}, $A \simeq 10^{-10}$. Equation (1) can be explained by the widely accepted oblique rotator model[2-4] simply by investigating an interesting consequence of the possible ellipticity of the neutron star. To account for the history and energy balance of the Crab Nebula[3], an upper limit must be placed[4,5] to this ellipticity, such that the gravitational quadrupole radiation dominates the magnetic dipole radiation only in the early life of the pulsar. It turns out that the explanation given here for the anomalies in the pulse arrival times provides in the end an even stronger limit to the deviations from spherical symmetry allowed in a neutron star.

In the oblique rotator, because of the emission of low frequency electromagnetic waves, a non-conservative torque is applied to the neutron star. The torque is responsible for slowing down[4] the rotation at a rate in good agreement with the observations, and probably for aligning the magnetic axis with the rotation axis (F. C. Michel at the Princeton pulsar meeting, November 1969). Both effects are secular (with time scales of thousands of years for the Crab pulsar), so that over periods of the order of months, the angular momentum vector **M** of the pulsar may be considered constant in direction as well as in magnitude. The pulsar accordingly behaves as a free top.

The model assumed here for NP 0532 is a slightly aspherical ellipsoid instead of a perfect sphere. Examples of such configurations in rotating bodies are well known in Newtonian gravitation (Jacobi ellipsoids) and no *a priori* argument (except for a lifetime which is possibly much too short because of gravitational radiation) can rule out their existence in general relativity. The motivation adopted here, however, for the lack of axial symmetry is magnetic, not rotational. Let x_1, x_2 and x_3 be the three principal axes of inertia of the neutron stars and let the corresponding moments of inertia satisfy the inequalities

$$I_1 < I_2 < I_3 \tag{2}$$

With the magnetic field axis along the direction of greater elongation, x_1, let **M** be at a small angle with x_3. In other words, let the rotation be nearly around x_3, but not exactly (off axis rotation has already been investigated in connexion with other phenomena in pulsars[6]). An oblique rotator is thereby realized, the free motion of which is well known[7]. A new frequency, ω', is associated with the system in addition to the main frequency of rotation, $\Omega \simeq \Omega_3$

$$\omega' = \Omega\varepsilon_1\varepsilon_2 \tag{3}$$

where

$$\varepsilon_1^2 = \frac{I_3}{I_1} - 1,\ \varepsilon_2^2 = \frac{I_3}{I_2} - 1 \tag{4}$$

are two small quantities. ω' appears in a variety of effects: in particular, it turns out[7] that the components of **M** along x_1 and x_2 are given by

$$M_1 = aM\varepsilon_2\cos\omega' t$$
$$M_2 = aM\varepsilon_1\sin\omega' t \tag{5}$$

where a is the small-amplitude constant related to the mean square angle θ between **M** and x_3 by

$$a^2 = \frac{2\theta_2}{\varepsilon_1^2 + \varepsilon_2^2} \tag{6}$$

Obviously Ω_3 differs from $|\mathbf{\Omega}| = \Omega$ for terms of the order a^2, which can be evaluated by means of the appropriate Euler's equation. By using equation (5), one finds that

$$\Omega_3 = \Omega\left[1 - \frac{1}{4}a^2(\varepsilon_1^2 + \varepsilon_2^2) + \frac{1}{4}a^2(\varepsilon_1^2 - \varepsilon_2^2)\cos 2\omega' t\right] \tag{7}$$

The constant of integration of order a^2 is really immaterial: the choice in equation (7) satisfies $M_1^2 + M_2^2 + M_3^2 = M^2$. The kinetic energy of rotation is thus

$$K = \frac{1}{2}I_3\Omega^2(1 + a^2\varepsilon_1^2\varepsilon_2^2)$$

According to equation (7) the ellipsoidal pulsar does not rotate on itself at a fixed rate $\Omega_3 < \Omega$, but slower or faster with frequency $2\omega'$, which may, of course, account for equation (1). The assumption $\varepsilon_1 \neq \varepsilon_2$, or relations (2), is then crucial, whereas a flattened spheroid with axial symmetry would not be adequate. It is assumed here that, although ε_1^2 and ε_2^2 are mainly rotational deformations, $(\varepsilon_1^2 - \varepsilon_2^2)$ is determined by magnetic effects.

For the Crab pulsar $\omega/\Omega \simeq 10^{-8}$, so that in order to justify $\omega \simeq 2\omega'$ one must have from equation (3) $\varepsilon_1\varepsilon_2 \simeq 10^{-8}$, or

$$\varepsilon_1^2 \text{ and } \varepsilon_2^2 \simeq 10^{-8} \tag{8}$$

The rotational deformations of a general relativistic neutron star have been evaluated under the assumption of axial symmetry[8]. For an ellipsoid such computations turn out to be forbiddingly difficult. In a Newtonian approximation every fluid model will have rotational deformations of the moment of inertia, $\Delta I/I \simeq \varepsilon_1^2 \simeq \varepsilon_2^2$ of the order[9] of $\Omega^2/(2\pi G\rho_c)$. This implies for our case

$$\varepsilon_1^2 \text{ and } \varepsilon_2^2 \simeq 10^{-4} \tag{9}$$

rather than equation (8).

Strictly speaking, the rotational deformations in a general relativistic star are to a certain extent smaller than in their Newtonian counterpart[10] (because the dragging of the inertial frame implies, among other things, a reduction of the centrifugal force[8]). This effect can hardly be enough to reconcile equations (8) and (9), however.

If the present explanation for the period fluctuations proves correct, therefore, the discrepancy between equations (8) and (9) will by itself point to the existence of strong forces in a neutron star forbidding relatively wide deformations. A very thick solid crust[11] might perhaps be adequate. The geometrical deformations, $\Delta R/R$, implied by equation (8) are exceedingly small, the equatorial radius being only 10^{-2} cm longer than the polar radius. In a sense it is gratifying that the recent quake[12,13] on the Crab pulsar implies[11] only a minor crust readjustment of the order of 10^{-3} cm.

In terms of the assumed magnetic perturbation $\delta_M \ll 1$

$$\varepsilon_1^2 = \varepsilon_2^2(1 + \delta_M) \tag{10}$$

the magnitude of the variable part of the quadrupole moment is clearly $D \simeq I_3\varepsilon_2^2\delta_M$. The corresponding gravitational radiation energy loss[14] becomes

$$-\frac{dK}{dt} \simeq \frac{G}{c^5}(I_3\varepsilon_2^2\delta_M)^2\Omega^6 \simeq 10^{28}\delta_M^2 \text{ erg s}^{-1}$$

within an irrelevant numerical factor. This is so small in comparison with the energy requirement of the Crab Nebula and assumed magnetic dipole loss[2–4] as to make the gravitational damping totally insignificant at the present time. No contradiction arises therefore with the general model (see also ref. 15).

By making use of equations (6), (7) and (10) the amplitude A in equation (1) becomes

$$A = \frac{1}{2}\,\theta^2\,\frac{\varepsilon_1{}^2 - \varepsilon_2{}^2}{\varepsilon_1{}^2 + \varepsilon_2{}^2} = \frac{1}{4}\,\theta^2 \delta_M \qquad (11)$$

For a field of 10^{12} G the magnetic effects are likely to be rather superficial. They may be calculated within a certain model[4] or, very roughly, simply by taking δ_M equal to the ratio of the magnetic to the ordinary pressure. For an indicative density of 10^9 g cm^{-3} in white dwarf material, $\delta_M \simeq 10^{-4}$. This in turn reproduces from equation (11) an amplitude $A \simeq 10^{-10}$ for a sensible deviation $\theta \gtrsim 10^{-3}$.

Much more remains to be seen before definite conclusions can be reached, but it seems that the phenomenon in itself emphasizes the precision of the pulsar clock and supports the notion[16] of a beacon on the neutron star surface rather than farther out, accurate timing being harder in the second case. Our explanation of equation (1) is appealing because it does not require new assumptions about pulsar structure and it has a considerable flexibility.

A difficulty, however, may be that the present state of the crust theory does not seem to yield the required stiffness (equation (8)). It is also possible that the frozen crust retains much of the flattening of the original fluid configuration.

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Possible Clues to the Early History of the Solar System

ONE problem in connexion with the nucleosynthesis of the light elements, D, Li, Be and B, has been to decide whether preplanetary matter at an early stage of the solar system was intensely irradiated by solar particles. First put forward by Hayakawa[1] and Fowler, Greenstein and Hoyle[2], and later modified by others[3–5], this notion has motivated searches for anomalies in the isotopic abundances of several elements in meteorites[6–16]. Within experimental accuracies, however, these isotopic abundances were identical with those of terrestrial matter except for lithium[11,13,16], and because of recent theoretical considerations the tendency is to abandon the irradiation mechanisms of Fowler, Greenstein and Hoyle[17,18]. But it seems to us worthwhile to continue the study, on one hand by improving the experimental accuracy, and on the other by looking for abundance anomalies caused by other types of nuclear reactions.

According to Mitler, the mean proton energy of 570 MeV assumed by Fowler, Greenstein and Hoyle is too high[3], based on recent observations of solar flare particles. Hayakawa has also used, in his latest article[5], the spectra of contemporary solar particles. In this case isotopic anomalies in the product nuclides from high energy nuclear reactions such as ^{50}V or ^{54}Cr would be less probable. The neutron-induced reactions will also be less important because of the lower neutron production with softer primaries. We therefore looked for other nuclear reactions induced by low energy protons and alpha particles and found that ^{138}Ba(p,n)^{138}La and ^{180}Hf(p,n)^{180}Ta are the most suitable, because ^{138}La and ^{180}Ta are extremely low in cosmic abundance compared with the respective target nuclides ^{138}Ba and ^{180}Hf. (Brancazio *et al.*[19] have investigated similar reactions, but their discussion refers to bombardment in stellar surfaces.) Table 1 shows the results of our calculations for these two reactions as well as other possible side reactions that may give the same product nuclei. For the calculation of average cross-section $\bar{\sigma}$, we used Barbier's semi-empirical $\sigma(E, AT)$ function[20] as no measured excitation functions are available. The shape of the integral proton spectrum was assumed to be $\varphi(>P) = \varphi_0 \exp(-P/P_0)$ according to the solar flare observation[21], where φ is the total flux for the event; P is the rigidity (MV/nucleon); and P_0 the characteristic rigidity for the solar event, here taken to be 100 MV. For the neutron spectrum in the range 2 to 20 MeV we used the differential form $\varphi = k(1 + 0{\cdot}01\ E^{-1} + 1{\cdot}1 \times 10^{-5}\ E^{-2})$ given by Arnold *et al.*[22], where E is in BeV. The abundance ratios of product nuclide to target nuclide in the preplanetary matter were taken to be the same as the cosmic abundances for non-volatile elements and Cameron's new cosmic abundances were employed[23]. (The product nuclide here means the same nuclide species present in preplanetary matter as the product of the nuclear reaction on the assumed target.) The last column of Table 1 shows a measure of the sensitivity of each reaction, that is, the integrated flux of irradiating particles that may cause a 1 per cent relative anomaly in isotopic abundance ratio. The calculation assumes uniform irradiation in the same way as done by Burnett *et al.*[9] for ^{40}K. In order to compare with other indicator elements for proton irradiation, we calculated the corresponding total flux φ_0 for each reaction, assuming the same integral rigidity function for the event. Table 2 shows the values of φ_0 with $P_0 = 50$, 100 and 200, covering the characteristic rigidity for present day solar flares. It can be seen that ^{138}La and ^{180}Ta are the most sensitive

Table 1. NUCLEAR REACTIONS POSSIBLY GIVING RISE TO THE ISOTOPIC ABUNDANCE ANOMALY OF ^{138}La AND ^{180}Ta

Reaction	Product/ target	σ (mbarn)	Integrated fluxes for 1 per cent anomaly (cm^{-2})
^{138}Ba(p,n)^{138}La	$9{\cdot}5 \times 10^{-5}$	200 (10–50 MeV p)	$4{\cdot}8 \times 10^{18}$p
^{139}La(n,2n)^{138}La	$8{\cdot}9 \times 10^{-4}$	500 (2–20 MeV n)	$1{\cdot}8 \times 10^{19}$n
^{139}La(p,pn)^{138}La	$8{\cdot}9 \times 10^{-4}$	70 (10–50 MeV p)	$1{\cdot}3 \times 10^{20}$p
^{138}Ce(n,p)^{138}La	0·11	< 100 (2–20 MeV p)	> $1{\cdot}1 \times 10^{22}$n
^{180}Hf(p,n)^{180}Ta	$4{\cdot}8 \times 10^{-5}$	70 (10–50 MeV p)	$6{\cdot}9 \times 10^{18}$p
^{181}Ta(n,2n)^{180}Ta	$1{\cdot}2 \times 10^{-4}$	500 (2–20 MeV n)	$2{\cdot}4 \times 10^{18}$n
^{181}Ta(p,pn)^{180}Ta	$1{\cdot}2 \times 10^{-4}$	50 (10–50 MeV p)	$2{\cdot}4 \times 10^{19}$p
^{180}W(n,p)^{180}Ta	$1{\cdot}25 \times 10^{-2}$	< 50 (2–20 MeV n)	> $2{\cdot}5 \times 10^{21}$n