AN EQUATION OF STATE AT SUBNUCLEAR DENSITIES

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Abstract. An equation of state for cold matter above white dwarf densities is evaluated. The gas is considered to be a mixture of degenerate neutrons, protons and electrons combined with nuclei of one type (that is only one A and Z value). We derive the equilibrium equations for the mixture and calculate the number densities as well as the A and Z of the nucleus. Finally we calculate an equation of state, which smoothly goes over to that of a neutron, proton electron gas mixture at a density of $\sim 5 \times 10^{13}$ g/cm^3.

1. Introduction

It has only been recently that attempts have been made at developing an equation of state for matter denser than white dwarf matter ($\varrho \sim 3 \times 10^{11}$ g/cm^3) and less dense than nuclear matter. At high densities a mixture of degenerate elementary particles is assumed to exist, and at lower densities heavy nuclei are in equilibrium with a degenerate electron gas. Salpeter (1961) calculated the nuclear species which would be in equilibrium with the electron gas (which prevents the nuclei from breaking up by beta decay because of the high energy required of the electron to fill an unoccupied state in the fermi sea). Cameron and Cohen (1969) extended this region by considering also the equilibrium with a degenerate neutron gas when the electron fermi level, $E_f(e^-)$, is greater than 23 MeV. The high fermi level of the neutron gas prevents the neutron-rich nuclei from decaying via neutron emission. The degenerate neutron gas has shifted the ground state of the medium surrounding the nucleus permitting positive energy neutrons to exist in the nucleus. These nuclei would be unbound in a vacuum.

In previous equations of state, (Tsuruta and Cameron, 1966) the electron level was held constant in the region $3.7 \times 10^{11} < \varrho < 4 \times 10^{14}$ and the nuclei were allowed to break up slowly until at nuclear densities only neutrons, protons, and electrons existed. In the scheme of Cameron and Cohen (1969) the electron fermi level increases to 94 MeV and the neutron level (including rest mass) to about 948 MeV. The nuclei are expected to grow larger and more neutron-rich until they merge with the outside gas at nuclear densities.

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In this paper we treat the problem in detail, deriving Cameron and Cohen's equations from both an equilibrium relation and from a thermodynamic potential. We also derive an additional relationship such that the full set of equations may be solved as a function of a single parameter.

The second important addition has been the inclusion of a two-body interaction between the nucleons in the degenerate gas. This interaction makes it possible for protons to exist in the gas outside the nucleus.

As the density increases the protons in the degenerate gas become numerous and the nuclei decrease in number density. At $\varrho \simeq 5 \times 10^{13}$ g/cm^3 the protons in the gas balance the electrons (charge neutrality) and no nuclei are present. We have considered our system to be a cold degenerate, $T=0°$K, gas.

It should be emphasized that without the nucleon interaction no protons would appear and nuclei would be present until the entire system was at nuclear densities.

2. Nuclear Equilibrium at Zero Temperature

Previous calculations of the nuclei which exist in equilibrium at zero temperature in a dense gas have employed the minimization of the following function (Salpeter, 1961; Tsuruta, 1964):

$$b = B(A, Z) - ZE_f'(e^-), \tag{1}$$

$B(A, Z)$ is the binding energy of the nucleus with mass number A and charge Z, and $E_f'(e^-)$ is the fermi energy less the neutron-proton mass difference of the gas of electrons surrounding the nucleus. To find the minimum energy nucleus at a given fermi level of the electron, one first substitutes into Equation (1) the binding energy, $B(A, Z)$, from the mass formula and then minimizes the total function b with respect to A and to Z. This method yields nuclei which become more and more neutron-rich, until at densities of approximately 3×10^{11} gm/cm^3, corresponding to a fermi level of about 23 MeV, the function b turns negative, so that as a result of electron capture the nucleus breaks up into A free neutrons.

The function b represents the binding energy of a nucleus with respect to A free non-degenerate, non-interacting neutrons. It may be derived from the electron capture reaction equation, considering only the effects of the degenerate electrons,

$$(A, Z) + Ze^- \rightleftarrows AN + Z\nu_e. \tag{2}$$

However, as pointed out by Cameron and Cohen (1969), the effect of the degenerate sea of neutrons will be to suppress the ejection of neutrons from a neutron-rich nucleus because of the reduced phase space allowed to the emitted neutron.

One may derive a new function to be minimized from Equation (2) which includes now the effects of the fermi levels of the electron and neutron gases,

$$g = B(A, Z) - ZE_f'(e^-) + A\mu_f(n), \tag{3}$$

where $\mu_f(n)$ is the fermi energy of the neutron gas (usually written $E_f(n)$ when no binding effects are included).

Minimization of the function g with respect to first Z and then A yields

$$E_f'(e^-) = \frac{\partial B(A, Z)}{\partial Z}, \tag{4}$$

$$\mu_f(n) = -\frac{\partial B(A, Z)}{\partial A}. \tag{5}$$

In discrete form, $\partial B(A, Z)/\partial Z$ and $\partial B(A, Z)/\partial A$ are the beta decay energy and the binding energy per last neutron, respectively, of the (A, Z) nucleus. It is interesting to note that we arrive at the same results as did Cameron and Cohen (1969), but that Equations (4) and (5) were derived by a different technique.

The binding energy of the nucleus, $B(A, Z)$, used in these calculations is found from the mass formula of Green (1954).

$$B(A, Z) = \alpha A + \beta\left(\frac{A - 2Z}{A}\right)^2 A + \gamma A^{2/3} + \frac{\delta Z^2}{A^{1/3}}, \tag{6}$$

where

$$\begin{aligned} \alpha &= -15.756 \text{ MeV}. \\ \beta &= 23.694 \text{ MeV}. \\ \gamma &= 17.794 \text{ MeV}. \\ \delta &= 0.710 \text{ MeV}. \end{aligned}$$

It should be noted that nuclear shell effects have not been included in these considerations. Inclusion of an asymmetry surface term in the mass formula will yield a more neutron rich equilibrium nuclei at a given density but will not significantly change the equation of state.

3. Statistical Equilibrium

Minimizing the equilibrium equation determines which nucleus (A and Z numbers) will be in equilibrium with a given set of fermi levels for a degenerate neutron and electron gas. To solve the equation of state of the gas we must determine what the number densities of the constituents are.

To find the conditions necessary for equilibrium of a gas of degenerate neutrons, protons and electrons and a single species of nuclei we minimize the thermodynamic potential $\Phi = \Phi(P, T, N_i)$ where T is the temperature, P the pressure and N_i the number density of particle i. For fixed T (here taken as zero) and P, Φ is a minimum when $d\Phi = 0$, or

$$\frac{\partial \Phi}{\partial N_1} + \frac{\partial \Phi}{\partial N_2}\frac{\partial N_2}{\partial N_1} + \cdots = 0. \tag{7}$$

The chemical potential is defined as

$$\mu_i = \left(\frac{\partial \Phi}{\partial N_i}\right)_{T,P}. \tag{8}$$

As constraints on the gas we also require charge neutrality

$$\sum_i q_i n_i = 0, \quad i = \text{all particles} \tag{9}$$

and baryon conservation

$$\sum_k \int n_k = \text{constant}, \quad k = \text{all baryons}, \tag{10}$$

where n_i is the number density and q_i the charge of particle type i. We now have to minimize $\Phi' = \Phi + \text{constraints}$ where Φ will be the total energy density of the system as a function of number density. Φ' can be written,

$$\Phi' = \sum_i \int \left(E_i(n) + B_i(n)\right) \mathrm{d}n_i + E(\text{nuc.})\, N + \alpha\left(ZN + n(p) - n(e^-)\right) + \beta\left(n(p) + n(n) + AN\right), \tag{11}$$

where $\sum_i$ is a sum over all the degenerate elementary particles in the gas neutrons, protons and electrons.

The first term in Φ' is an integral over the energy of the degenerate particles in the gas, where the energy is a function of number density n. $E_i(n)$ is the free energy of a particle (rest mass + kinetic energy) as a function of n, and $B_i(n)$ is the binding energy due to the nucleon–nucleon interactions in the free gas. The derivation of the binding energy from a nucleon potential is described in the next section. We use here the results for the average value of the binding energy per particle $\langle B_i \rangle$. Now

$$\int B_i(n)\, \mathrm{d}n_i = n(i) \langle B_i \rangle, \tag{12}$$

$n(i)$ is the number of nuclei of species i and $E(\text{nuc.})$ is the total energy of the nucleus in the gas. The third and fourth terms in (11) are charge neutrality and baryon conservation respectively. Setting

$$\frac{\partial \Phi'}{\partial n_i} = 0, \tag{13}$$

we have

$$\alpha = E_f(e^-), \tag{14}$$

$$\beta = -\left(E_f(n) + \langle B(n) \rangle + \sum_i n_i \frac{\partial \langle B(i) \rangle}{\partial n_n}\right) = -\mu_f(n), \tag{15}$$

and

$$\mu_f(n) = \mu_f(p) + \mu_f(e^-),$$

or

$$E_f(n) + \langle B(n) \rangle + \sum_i n_i \frac{\partial \langle B(i) \rangle}{\partial n_n} = E_f(p) + \langle B(p) \rangle + \sum_i n_i \frac{\partial \langle B(i) \rangle}{\partial n_p} n_p + E_f(e^-). \tag{16}$$

Finally, from $\partial\Phi'/\partial N = 0$,

$$E(\text{nuc}) = A\mu_f(n) - ZE_f(e^-), \tag{17}$$

where $E_f(n) = (p_f^2(n)\, c^2 + m^2(n)\, c^4)^{1/2}$ is the fermi level of the neutron gas without binding, similarly for p the proton and e^- the electron, and m is the mass of each particle. $p_f(n)$ is the fermi momentum of the neutron gas and its relationship to the number density in a completely degenerate gas ($T = 0°\,\text{K}$) is

$$n(i) = \frac{(2S+1)\, a_0}{m_{e^-}^3}\, p_f^3(i), \tag{18}$$

where $a_0 = 8\pi (m_{e^-} c^2/hc)^3 = 1.76 \times 10^{30}\ \text{cm}^{-3}$ and $(2S+1)$ is the spin multiplicity.

We have rederived the standard results for an n, p, e^- gas in equilibrium, but with two important differences. Equation (17) is a new result, allowing us to determine which nucleus will be in equilibrium with the gas. Remembering that our energies are relativistic (i.e. include the rest mass) we rewrite $E(\text{nuc.})$ in terms of the nuclear binding energy per particle $b(A, Z)$ $(= B(A, Z)/A)$

$$\begin{aligned}(A - Z)\, m(n) + Zm(p) - Ab(A, Z) \\ = A\left(E_f(n) + \langle B(n)\rangle + \sum_i n_i \frac{\partial \langle B(i)\rangle}{\partial n_n}\right) - ZE_f(e^-).\end{aligned} \tag{19}$$

Rewriting this in terms of the kinetic energy Fermi level, K_f, we have

$$\begin{aligned}E_f(e^-) = \frac{A}{Z}\left(K_f(n) + \langle B(n)\rangle + \sum_i n_i \frac{\partial \langle B(i)\rangle}{\partial n_n} + b(A, Z)\right) \\ + (m(n) - m(p)).\end{aligned} \tag{20}$$

Looking at Equation (17) we note that we have

$$E(\text{nuc}) - A\mu_f(n) + ZE_f(e^-) = 0, \tag{21}$$

and that this is the equilibrium equation which was minimized in Section 2 to find A and Z. Not only must g be a minimum with respect to A and Z, but it must also equal zero. If $g < 0$ then the system is unstable and the nuclei will break up. It now appears that the neutron-rich nuclei in equilibrium with the gas are quasi stable, just existing in equilibrium. These quasi nuclei should be continually breaking and forming in the gas.

The results of section two can be derived from minimizing the thermodynamic potential. The equations will result from considering the addition to Equation (11) of two more species of nuclei, instead of $E(\text{nuc.})\, N$ we would have

$$\begin{aligned}E(N(A, Z))\, N(A, Z) + E(N(A+1, Z))\, N(A+1, Z) \\ + E(N(A, Z-1))\, N(A, Z-1),\end{aligned} \tag{22}$$

plus the appropriate additions to the α and β term. If we minimize the thermodynamic potential with respect to the two new nuclei, we have two more equilibrium relations which, with Equation (17), can be rewritten to give Equations (4) and (5) in finite difference form. These are

$$E(A, Z) - E(A, Z-1) = -E'_f(e^-) \tag{23}$$

$$E(A+1, Z) - E(A, Z) = \mu_f(n). \tag{24}$$

The second important difference in our result is the inclusion of potential effects in Equation (16). Without it no protons would be present until nuclear densities. With it protons in the degenerate gas can be present in number larges enough to insure charge equality and a reduction of nuclear number density with increased density. In this way the nuclei go away, leaving a neutron, proton, electron gas, and the equation of state is continuous (i.e. pressure and energy) during the transition from nuclei to no nuclei.

4. Nucleon Binding

To determine correctly the last state filled for the neutron gas the effects of nucleon-nucleon interactions must be included in the chemical potential (top of the fermi sea). When binding is included many more neutrons will be present for a given Fermi level than if binding were neglected. Without binding, protons would not be present in the neutron-electron mixture. Typically $E_f(e^-) > 24$ MeV and the following condition would hold

$$E_f(n) - E_f(e^-) < m(p). \tag{25}$$

With a potential the relatively few protons interact with many neutrons, producing a large binding, allowing Equation (25) to be satisfied.

To calculate the binding we have used the results of Weiss and Cameron (1969). Here we briefly outline their procedure for determining the binding energy of a neutron-proton mixture.

Weiss and Cameron (1969) based their work on the V_α and V_γ nucleon potentials of Levinger and Simmons (1961). We use only the results for the V_α potential (for the density region considered in this paper, the V_α, V_β and V_γ potentials yield the same results within a few percent); this velocity dependent potential has the following form in each of the four possible interaction states

$$V_\alpha = -V_0 J_1(r) - \frac{\lambda}{M} \mathbf{P} \cdot J_2(r) \mathbf{P} \tag{26}$$

$$J_1(r) = J_2(r) = \begin{cases} 1, & r < b \\ \tfrac{1}{2}, & r = b \\ 0, & r > b \end{cases}$$

$$\mathbf{P} = i\hbar\nabla, \qquad V_0 = 16.9 \text{ MeV.}, \qquad \lambda = -0.21 \quad \text{and} \quad b = 2.4 \text{ F}.$$

The four possible spin–parity states are singlet-even (se), triplet-even (te), triplet-odd (to) and singlet-odd (so); all four are present when unlike particles interact, but when two like nucleons interact only the (se) and (to) are involved.

It was still necessary to adjust the various interaction strengths in each state. Weiss and Cameron (1969) set all of the strengths in the velocity part equal to the original Levinger and Simmons value, as well as the static (se). The other static interaction strengths were varied to agree with nuclear matter results, specifically to reproduce the saturation density, and the volume and symmetry energy coefficients in mass formulae for nuclei.

To lowest order the interaction part of the ground state energy is

$$\langle V\rangle = \tfrac{1}{2}\sum_{ij}\{\langle ij|\,V\,|ij\rangle - \langle ij|\,V\,|ji\rangle\}. \tag{27}$$

The sum is over all single particle quantum numbers and V is the two-nucleon potential energy operator. The antisymmetric nature of the two body wave function for fermions gives rise to the two terms; the first is called the normal term and the second the exchange term. The i and j refer to the single particle wave function

$$\psi_i = \varphi_{k_i}(r_i)\,\alpha_{m_\sigma}(\sigma)\,\lambda_{m_\tau}(\tau), \tag{28}$$

where $\varphi_{k_i} = \Omega^{-1/2}\exp(i\mathbf{k}_i\cdot\mathbf{r}_i)$ is a plane wave orbital wavefunction, and α and λ are the spin and isospin factors respectively. For example, the exchange term appears as follows

$$\langle ij|\,V\,|ji\rangle = \sum_{\sigma,\,\tau}\int \psi_i^*(r_i)\,\psi_j^*(r_j)\,V\psi_i(r_j)\,\psi_j(r_i)\,\mathrm{d}^3r_i\,\mathrm{d}^3r_j, \tag{29}$$

and the sum is over the spin and isospin variables. Only two types of matrix elements

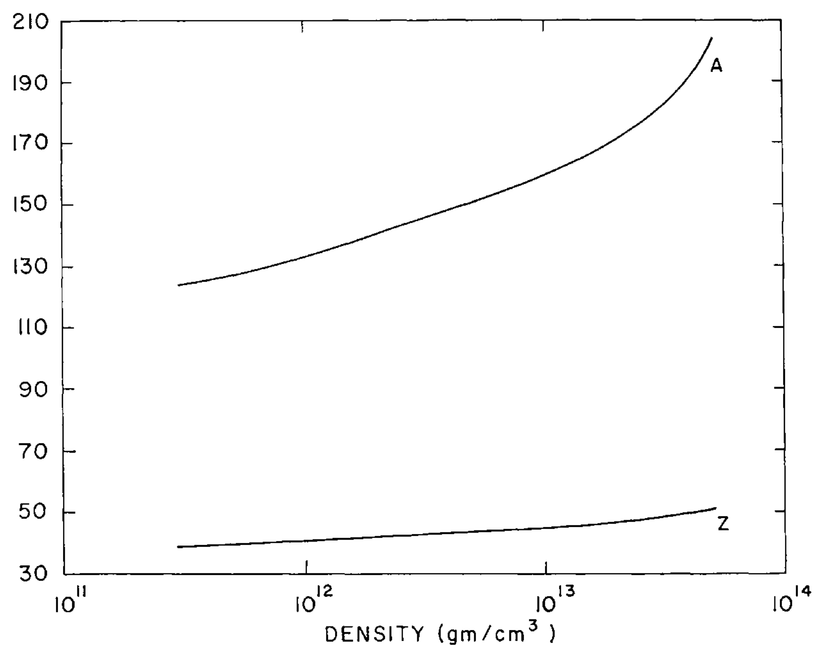

Fig. 1. The mass number A and charge Z of the equilibrium nucleus as a function of energy density in units of g/cm³.

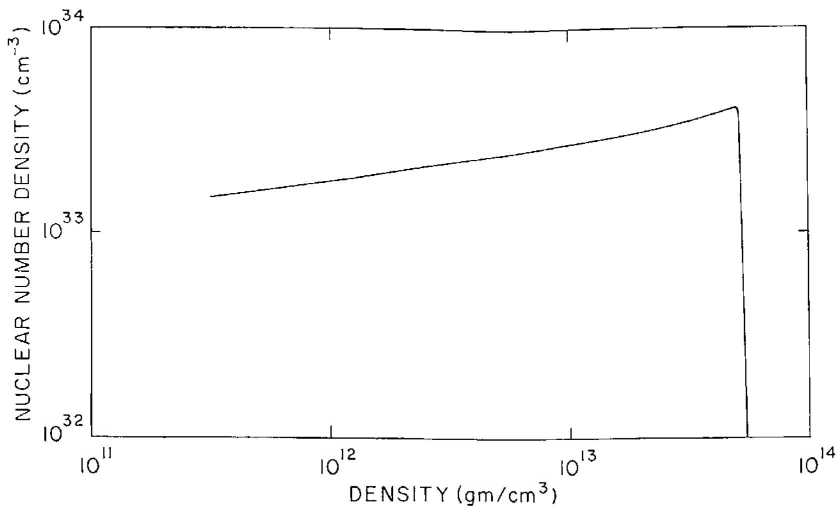

Fig. 2. Number density of nuclei as a function of energy density in units of g/cm^3.

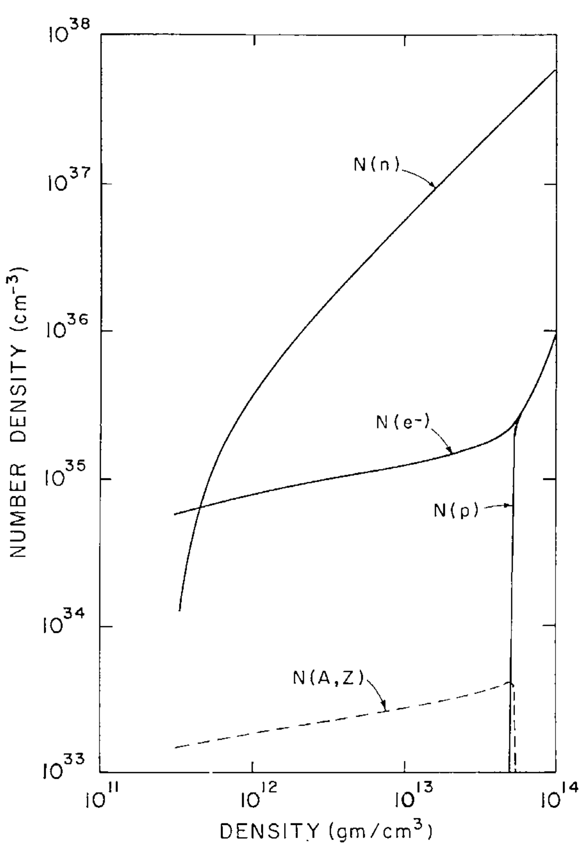

Fig. 3. Number density of neutrons, $N(n)$, electrons, $N(e^-)$, protons, $N(p)$ and nuclei, $N(A, Z)$ as a function of energy density in units of g/cm^3.

are present: the normal term

$$\langle + \mid V \mid + \rangle = \Omega^{-1} \int V(k, r) \, \mathrm{d}^3 r \tag{30}$$

and the exchange term

$$\langle + \mid V \mid - \rangle = \Omega^{-1} \int V(k, r) \, \mathrm{e}^{-2i\mathbf{k}\cdot\mathbf{r}} \, \mathrm{d}^3 r \, . \tag{31}$$

To find the average interaction energy per particle in the gas, $\overline{\langle | V | \rangle}$, we evaluate the following matrices

$$\overline{\langle + \mid V \mid \pm \rangle}_{\mu\mu} = \int_0^{k_\mu} \langle + \mid V \mid \pm \rangle \, \mathbf{P}_{\mu\mu}(k, k_\mu) \, \mathrm{d}^3 k \, , \tag{32}$$

$$\overline{\langle + \mid V \mid \pm \rangle}_{\mu\nu} = \int_0^{1/2\,(k_\mu + k_\nu)} \langle + \mid V \mid \pm \rangle \, \mathbf{P}_{\mu\nu}(k, k_\mu, k_\nu) \, \mathrm{d}^3 k \, , \tag{33}$$

Equation (32) is for like particles μ-μ interacting, and Equation (33) is for unlike particles ν-μ interacting. $\mathbf{P}$ is the normalized pair probability function (Brueckner, 1961; Tabakin, 1964) and it relates the distribution of states available to two interacting fermions in a degenerate gas. For example, the distribution for like particles is

$$\mathbf{P}_{\mu\mu}(k, k_\mu) = \frac{24}{k_\mu^3} k^2 \left[1 - \frac{3}{2} \frac{k}{k_\mu} + \frac{1}{2} \frac{k^3}{k_\mu^3} \right] . \tag{34}$$

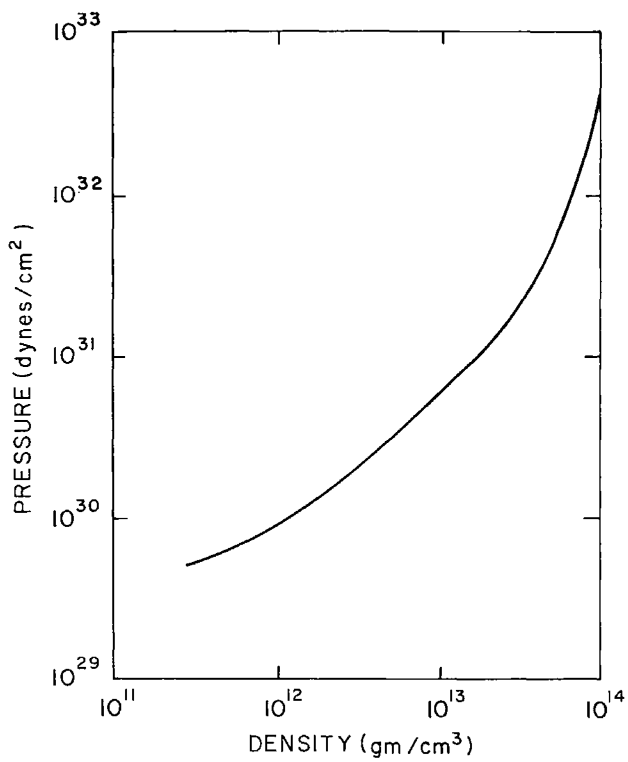

Fig. 4. Pressure versus energy density.

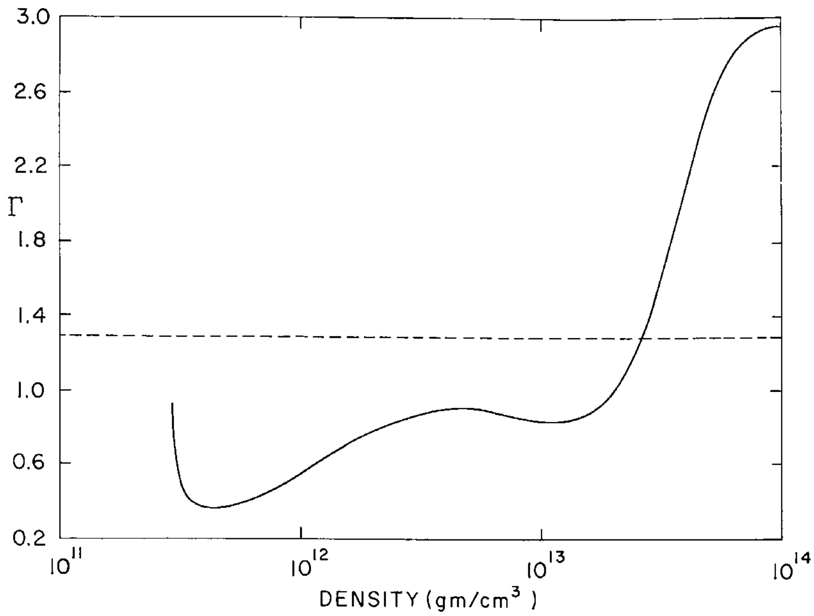

Fig. 5. The dependence of $\Gamma = (P + U)/P\ \partial P/\partial U$ as a function of energy density in units of g/cm³. The dashed line is drawn at $\Gamma = 4/3$.

The binding energy is now the number of particles interacted with, times the sum over the average interaction between that type of particle in all the states available.

Weiss and Cameron (1969) have analytic results for the binding energies of neutrons and protons in a degenerate gas; it is these results which we have used here. Their analytic forms are too long to be repeated here and we refer the interested reader to their paper for the results as well as details of the calculation.

5. Results and Conclusions

The equation of state and composition of the subnuclear gas has been found by iterating Equations (4, 5; 20) through the use of a computer. The results of Weiss and Cameron (1969) have been applied to find the appropriate fermi levels of the particles in the degenerate gas outside the nucleus. One finds the number density of the nuclei $N(A, Z)$ from charge conservation

$$N(A, Z) = (n(e^-) - n(p))/Z. \tag{35}$$

The equation of state then follows from the total energy

$$U = \sum u_i = \sum n_i E_i, \tag{36}$$

$$P = \frac{-\partial U}{\partial V}. \tag{37}$$

The mass number and charge of the nucleus to be found at a given density is plotted

TABLE I
Equation of State Variables

ϱ (g/cm^3)	$P \times 10^{30}$ dyn/cm^2	$N(n)$ 10^{30} cm^{-3}	$N(p)$ 10^{30} cm^{-3}	$N(e^-)$ 10^{30} cm^{-3}	N(nuc) 10^{30} cm^{-3}	A	Z	Γ
3.03×10^{11}	.547	3.51×10^{1}		5.79×10^{4}	1.47×10^{3}	124	39	.938
3.36×10^{11}	.575	1.21×10^{4}		6.00×10^{4}	1.52×10^{3}	125	40	.395
3.88×10^{11}	.605	3.51×10^{4}		6.22×10^{4}	1.56×10^{3}	126	40	.351
4.41×10^{11}	.633	6.08×10^{4}		6.39×10^{4}	1.60×10^{3}	127	40	.354
5.81×10^{11}	.703	1.31×10^{5}		6.75×10^{4}	1.68×10^{3}	129	40	.402
7.56×10^{11}	.790	2.23×10^{5}		7.12×10^{4}	1.76×10^{3}	131	41	.475
9.94×10^{11}	.912	3.50×10^{5}		7.53×10^{4}	1.84×10^{3}	134	41	.564
1.19×10^{12}	1.02	4.56×10^{5}		7.82×10^{4}	1.90×10^{3}	135	41	.627
1.69×10^{12}	1.30	7.31×10^{5}		8.43×10^{4}	2.03×10^{3}	138	42	.744
2.26×10^{12}	1.63	1.05×10^{6}		9.00×10^{4}	2.14×10^{3}	141	42	.825
3.31×10^{12}	2.27	1.64×10^{6}		9.83×10^{4}	2.30×10^{3}	146	43	.895
4.23×10^{12}	2.83	2.17×10^{6}		1.04×10^{5}	2.41×10^{3}	149	43	.914
5.32×10^{12}	3.50	2.80×10^{6}		1.10×10^{5}	2.52×10^{3}	152	44	.915
6.21×10^{12}	4.03	3.31×10^{6}		1.13×10^{5}	2.59×10^{3}	154	44	.906
8.30×10^{12}	5.21	4.53×10^{6}		1.21×10^{5}	2.72×10^{3}	157	44	.878
1.03×10^{13}	6.28	5.69×10^{6}		1.26×10^{5}	2.82×10^{3}	160	45	.854
1.55×10^{13}	8.91	8.80×10^{6}		1.35×10^{5}	2.98×10^{3}	165	45	.864
2.06×10^{13}	1.16×10^{1}	1.18×10^{7}		1.43×10^{5}	3.11×10^{3}	169	46	1.00
4.00×10^{13}	3.06×10^{1}	2.32×10^{7}	7.77×10^{-4}	1.77×10^{5}	3.67×10^{3}	186	48	2.05
4.96×10^{13}	4.98×10^{1}	2.88×10^{7}	3.52×10^{-2}	2.06×10^{5}	4.10×10^{3}	201	50	2.43
5.03×10^{13}	5.17×10^{1}	2.92×10^{7}	1.69×10^{-1}	2.08×10^{5}	4.14×10^{3}	203	50	2.44
5.08×10^{13}	5.30×10^{1}	2.95×10^{7}	1.97×10^{3}	2.10×10^{5}	4.09×10^{3}	204	51	2.47
5.12×10^{13}	5.33×10^{1}	2.97×10^{7}	5.26×10^{4}	2.11×10^{5}	3.11×10^{3}	204	51	2.50
6.03×10^{13}	8.36×10^{1}	3.57×10^{7}	3.02×10^{5}	3.02×10^{5}	0.0			2.76
8.05×10^{13}	1.89×10^{2}	4.75×10^{7}	5.86×10^{5}	5.86×10^{5}	0.0			2.89
1.01×10^{14}	3.66×10^{2}	5.93×10^{7}	9.81×10^{5}	9.81×10^{5}	0.0			2.92

in Figure 1. It may be noted that the equilibrium nuclei become more and more neutron-rich as A/Z increases with increasing density due to the suppression of neutron emission by the degenerate sea of neutrons outside the nuclei. The maximum A and Z of the nucleus given by (204, 51) is reached at 5.1×10^{13} g/cm^3. At this point the number density of protons rapidly builds up as the number density of nuclei drops. This effect may be seen in Figure 2 in which the number density of nuclei is plotted as a function of density. After a density of approximately 5.1×10^{13} g/cm^3, only a mixture of neutrons, electrons and protons exists. The number densities of the neutrons, protons and electron are given for all densities considered in the work in Figure 3. The pressure of the mixture of electrons, protons, neutrons and nuclei is given as a function of density up to 1.0×10^{14} g/cm^3 in Figure 4. At the point of disappearance of the nuclei at 5.0×10^{13} g/cm^3, the equation of state goes over continuously to that of a gas of electrons, protons and neutrons. The dependence of $((P+U)/P)\,\partial P/\partial U = \Gamma$ on density is illustrated in Figure 5. The value of $\Gamma = \frac{4}{3}$, which may represent the lower limit for stability of a stable star, is drawn as a dashed line. The numerical values of the variables in Figures 1–5 and the relevant energies of the particles and nuclei are given in Tables I and II. The energies of the particles are given

TABLE II

Particle energies

ϱ (g/cm^3)	$K_f(e^-)$ MeV	$K_f(n)$ MeV	$\mu_f(n)$ MeV	$K_f(p)$ MeV	$\mu_f(p)$ MeV
3.03×10^{11}	23.1	5.3×10^{-4}	939.5		
3.36×10^{11}	23.4	1.04×10^{-1}	939.6		
3.88×10^{11}	23.7	.22	939.7		
4.41×10^{11}	23.9	.31	939.8		
5.81×10^{11}	24.3	.50	939.9		
7.56×10^{11}	24.8	.72	940.0		
9.94×10^{11}	25.3	.98	940.2		
1.19×10^{12}	25.6	1.17	940.3		
1.69×10^{12}	26.3	1.61	940.5		
2.26×10^{12}	26.9	2.04	940.7		
3.31×10^{12}	27.7	2.75	940.9		
4.23×10^{12}	28.2	3.32	941.1		
5.32×10^{12}	28.7	3.93	941.2		
6.21×10^{12}	29.0	4.40	941.3		
8.30×10^{12}	29.6	5.41	941.5		
1.03×10^{13}	30.1	6.30	941.6		
1.55×10^{13}	30.8	8.41	941.8		
2.06×10^{13}	31.4	10.2	942.0		
4.00×10^{13}	33.8	16.0	942.6	1.4×10^{-6}	908.3
4.96×10^{13}	35.5	18.4	943.0	2.0×10^{-5}	907.0
5.03×10^{13}	35.7	18.6	943.1	5.5×10^{-5}	906.8
5.05×10^{13}	35.75	18.7	943.1	8.0×10^{-3}	906.8
5.10×10^{13}	35.8	18.8	943.1	.11	906.7
6.03×10^{13}	40.4	21.3	943.5	0.89	902.5
8.05×10^{13}	51.1	25.6	944.9	1.39	893.8
1.01×10^{14}	60.1	29.4	946.5	1.92	886.5

in Table II as free fermi energies, $K_F(n_i) = \sqrt{(P_{F_i}^2 + m_i^2)} - m_i$ and as the chemical potentials $\mu_f(n_i) = E_f(n_i) + B(n_i)$ where P_{F_i} and $B(n_i)$ represent the Fermi momentum and binding energy at the top of the Fermi sea respectively for the ith component.

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References

Brueckner, K. A., Locket, A. M., and Rotenberg, M.: 1961, *Phys. Rev.* **121**, 225.
Cameron, A. G. W. and Cohen, J. M.: 1969, *Astrophys. Letters* **3**, 3.
Green, A. E. S.: 1954, *Phys. Rev.* **95**, 1006.
Levinger, J. S. and Simmons, L. M.: 1961, *Phys. Rev.* **124**, 916.
Salpeter, E. E.: 1961, *Astrophys. J.* **134**, 669.
Tabakin, F.: 1964, *Ann. Phys.* **11**, 65.
Tsuruta, S.: 1964, Thesis, Columbia University, New York, New York.
Tsuruta, S. and Cameron, A. G. W.: 1966, *Can. J. Phys.* **44**, 1985.
Weiss, R. A. and Cameron, A. G. W.: 1969, to be published.