THE ASTROPHYSICAL JOURNAL, Vol. 154, December 1968

ON THE FLOW OF GAS FROM THE GALACTIC CENTER

D. W. MOORE*
Goddard Institute for Space Studies, New York

AND

E. A. SPIEGEL
Department of Physics, New York University
Received May 6, 1968

ABSTRACT

We consider a steady axisymmetric flow out of the galactic center, neglecting magnetic and turbulent stresses and the self-gravitation of the gas. The flow is in a disk and possesses angular momentum, but otherwise the problem is like solar-wind theory, and we assume that initial conditions are specified at the edge of the nuclear disk. The equations of motion are then quite simple, and their solution indicates that the flow must contain a nearly discontinuous transition from supersonic to subsonic conditions. We argue that this transition must be a compressible hydraulic jump, or bore, which generates intense turbulence and gives rise to H II on the downstream edge of the bore. The bore causes radial deceleration of the gas and hence density enhancement, and we suggest that this phenomenon is associated with the expanding arm.

I. INTRODUCTION

The discovery that gas flows from the galactic center (van Woerden, Rougoor, and Oort 1957) has led to a number of interesting studies of the kinematics of interstellar gas in the inner Galaxy (Rougoor and Oort 1960; Rougoor 1960; Kerr and Westerhout 1965; Kerr 1967). The salient features of the flow are: (1) a central nuclear disk of radius ~800 pc, which rotates rapidly but does not expand appreciably; (2) a relatively dense (in H I) region at about 4 kpc from the galactic center which has an outward velocity of 53 km sec^{-1} superposed on its galactic rotation—the expanding arm; (3) an irregular distribution of less dense gas between 800 pc and 4 kpc which exhibits outward velocities of as much as 200 km sec^{-1}.

These aspects of the flow seem relatively clear on our side of the galactic center; the observations relating to H I on the other side are much harder to interpret. It seems established that there is outflow there, too, but it is likely that the flow is not axisymmetric, and the possibility of inflow from certain directions is not excluded. It is also worth noting that radio observations indicate H II outside the expanding arm, but nowhere inside it (Mezger and Höglund 1967).

Theoretical discussions of the flow have mainly been concerned with the driving mechanisms (Wentzel 1961; Burbidge and Hoyle 1963; Woltjer 1965; Oort 1966). The question that has seemed uppermost is whether the flow is reasonably steady or is the result of an explosion in the galactic center. In the former case, the difficulty seems to be to provide a source of gas, since even if the flow produced a mass flux as low as 0.1 $M_\odot$/yr, the gas in the central disk would be quickly depleted. On the other hand, if the flow resulted from a recent explosion, it seems difficult to explain the presence of a relatively quiescent nuclear disk.

In the present work, we attempt to clarify the assumption of a maintained flow by considering the dynamics of the expansion between 800 pc and 4 kpc. Our approach is closely akin to that of solar-wind theory, and assumes, as known, the properties of the

* Now at Imperial College, London.

gas at the edge of the nuclear disk. Given these properties, we show in § II how a form of shock is to be expected in our simple model, and in § III we indicate that this shock is like a hydraulic jump, or bore, which ionizes the gas and generates turbulence.

II. THE UNDERLYING FLOW PATTERN

a) The Equations of the Model

We consider in this section axisymmetric flow of perfect gas which is confined to a thin sheet by the vertical[1] component of the gravitational field of the stars. We neglect the self-gravitation of the gas, and we regard the gravitational field of the stars as given. We allow the stars to influence the gas motion only through their mean gravitational field, and we neglect the drag exerted on the gas by individual stars or star clusters. We neglect viscosity since the Reynolds numbers are $O(10^3)$, at least, but we assume the motion is laminar. Further, we assume that the process of heat exchange between the gas and its environment maintains its temperature constant at a temperature T of about 100° K, just as for an H I region at rest. At the speeds we contemplate, the gas would travel only $\sim 10^2$ pc before readjusting its temperature, so that, since for the moment we are interested in a flow structure on a scale of 1 kpc, the isothermal assumption seems reasonable.

We use cylindrical polar coordinates (r,θ,z) with velocity components (u,v,w), where $z = 0$ is the galactic plane and $r = 0$ is the galactic center. The gravitational field of the stars is $[g_r(r),0,-\beta z]$, where $g_r(r)$ need not be more precisely specified at present and where β is a constant. The neglect of the z-dependence of g_r is justified, since the sheet of gas is reasonably thin compared with the distribution of the stellar component. The form of the vertical component is chosen to simplify the analysis, and in reality it will depend on r. As will be clear from the calculation, it is not difficult to incorporate a more realistic vertical component into the model, though at this exploratory stage the extra complication is not warranted.

We are now in a position to verify that there is a steady solution of the equations with $w = 0$ and in which u and v depend only on the radial coordinate r. With this form of the velocity field, the equations for steady motion are

$$\rho\left(u\frac{\partial u}{\partial r} - \frac{v^2}{r}\right) = -\frac{\partial p}{\partial r} + \rho g_r , \tag{1}$$

$$\rho\left(u\frac{\partial v}{\partial r} + \frac{uv}{r}\right) = 0 , \tag{2}$$

$$0 = -\beta z\rho - \frac{\partial p}{\partial z} , \tag{3}$$

$$\frac{\partial}{\partial r}(\rho r u) = 0 , \tag{4}$$

and

$$p = \frac{k}{m_{\mathrm{H}}}\rho T , \tag{5}$$

the last equation being the equation of state of neutral hydrogen in the standard notation. (We neglect the presence of other elements and the possibility of molecular hydrogen.)

[1] By "vertical" we mean the direction perpendicular to the galactic plane.

Equations (3) and (5) have the solutions

$$\rho(r,z) = \rho^{\dagger}(r) \exp\left(-\frac{z^2}{2H^2}\right) \tag{6}$$

and

$$p(r,z) = p^{\dagger}(r) \exp\left(-\frac{z^2}{2H^2}\right), \tag{7}$$

where the dagger denotes values in the galactic plane and where the scale height H is given by

$$H^2 = kT/\beta m_{\mathrm{H}} . \tag{8}$$

The equation of continuity (4) now shows that

$$\rho^{\dagger} r u = Q , \tag{9}$$

where Q is a constant. Provided that Q is not zero, so that there is some radial motion, equation (2) forces us to take

$$v = A/r , \tag{10}$$

where A is a constant. This equation is simply a consequence of the fact that no processes for transferring angular momentum other than convection by fluid particles have been included in our model. This dependence of v on r cannot be reconciled with the usually accepted rotation curve (Kerr and Westerhout 1965). There are, of course, uncertainties in the determination of the range 800 pc to 4 kpc, but the values of v at the ends of this range are well determined and cannot be fitted to equation (10) by any choice of A. This discrepancy points to existence of angular-momentum transfer, and as reported elsewhere (Moore and Spiegel, reported in Spiegel 1966; also Mestel, Moore, and Spiegel, to be published) the effect of a magnetic field on this flow has been examined. However, our present interest is in the over-all structure of the flow and, in particular, in the bore to be discussed in § III, and the present simplified model is adequate when only such gross features are being studied.

We can now easily find an equation for the radial velocity u, which is

$$u\frac{\partial u}{\partial r} - \frac{kT}{m_{\mathrm{H}} u}\frac{\partial u}{\partial r} = \frac{kT}{m_{\mathrm{H}} r} + \frac{A^2}{r^3} + g_r(r) . \tag{11}$$

b) The Nature of the Flow

We must now decide what physical quantities have to be specified to make the solution of equation (1) unique. Suppose u, v, and $\rho^{\dagger}$ have prescribed values u_0, v_0, and $\rho_0^{\dagger}$ at some initial radius $r = r_0$. Then $A = r_0 v_0$, and equation (11) contains only known constants. Furthermore, since it is first order, the condition $u = u_0$ at $r = r_0$ enables us to integrate forward in r to obtain a unique solution, as long as the equation remains regular. Now $Q = r_0 \rho_0^{\dagger} u_0$, and from equation (9) the density in the galactic plane follows.

Next we use equation (6) to determine the density at points out of the galactic plane; finally, the pressure follows from the equation of state. We shall in fact identify r_0 with the radius 800 pc at which there is a large observed v_0 of 270 km sec^{-1}. We will adopt small but slightly supersonic values for u_0 and take a value of $\rho_0^{\dagger}$ corresponding to 1 cm^{-3}, which values are consistent with a mass flux $\sim$0.1 $M_{\odot}$/yr. Before proceeding to the solution, we wish to stress that once u_0, v_0, and $\rho_0^{\dagger}$ are chosen, every feature of the flow for $r > r_0$ is uniquely determined. It might well be objected that we ought really to match our flow to conditions at the outer edge of the Galaxy, but we shall see below why this is not a suitable procedure. We assume now that

$$g_r(r) = -G\mathfrak{M}/r^2 , \tag{12}$$

so that we are attributing all the gravitational field to the galactic nucleus, of mass $\mathfrak{M}$. This is clearly not reasonable in the outer parts of the Galaxy, and Schmidt (1965) has given a more realistic form for the gravitational field as a sum of terms of the form $-g_n r^{-n}$. However, Schmidt's values for the constants g_n are calculated on the assumption that the dynamical effect of the radial motion is negligible, and we believe that this is not the case in the range 800 pc $< r <$ 4 kpc. Thus it does not appear possible to give a precise specification of g_r in the range of greatest interest to us, and the simple form we adopt must serve for the present.

We now introduce dimensionless variables

$$m = u/(RT_0)^{1/2} , \quad x = r/r_0 . \tag{13}$$

Then equation (11) for u takes the form

$$\frac{m^2 - 1}{m^2}\frac{dm^2}{dx} = 2\left(\frac{1}{x} - \frac{g}{x^2} + \frac{a}{x^3}\right) , \tag{14}$$

where

$$g = G\mathfrak{M}/(r_0 RT_0) , \quad a = A^2/(r_0 RT_0) , \tag{15}$$

and R is the gas constant. The equation of motion (14) is to be solved with the initial condition $m = m_0$ at $x = 1$, where $m_0 > 1$. This type of equation is well known in classical gas dynamics and in stellar-wind theory (Limber 1967), and we have previously discussed its solutions (reported in Spiegel 1966), so it is not necessary for us to elaborate on its properties.

With the values we have adopted for r_0 and v_0 we have, roughly, $a \sim 7 \times 10^4$, and if there are $n \times 10^9 \mathfrak{M}_\odot$ in the galactic nucleus, $g = n \times 10^4$. Thus g and a are very large numbers. This means that the first term on the right in equation (14) has a negligible effect on the flow, except at very large distances. In fact, for $x \ll 10^4$, i.e., $r \ll 10^3$ kpc, it can be neglected altogether, and a very simple discussion of the equation results. As we are interested in $r = O(10$ kpc) at most, we will make this approximation. Then we find that there are two cases: (1) If $g > a$, the flow decelerates rapidly. Thus there will be a sonic line near $x = 1$. This means that the gas has not enough energy to escape, and in this case the actual flow pattern must either contain a shock or depart from the galactic plane. At all events, it cannot possess large radial velocities at $r \sim 4$ kpc in contradiction to observation, and we will not consider this case further. (2) If $g < a$, the flow accelerates and rapidly achieves large velocities with $m = O(10^2)$. The velocity reaches a maximum at $x = a/g$, and for larger x the flow is decelerated. The velocity at any position is practically independent of the initial Mach number m_0 provided it is $O(1)$, as can be seen from the integral of equation (14):

$$m^2 - \log m^2 = m_0^2 - \log m_0^2 + 2g/x - a/x^2 + a - 2g . \tag{16}$$

Moreover, neglecting the terms in m_0^2, we see that if $g < a/2$ the flow has $m > 1$ everywhere, whereas if $g > a/2$ there will be a sonic point at $x = 2a/(g - 2a)$. Thus if $a/2 < g < a$, the flow cannot everywhere be of the form we have postulated, and it is likely that a shock will form. Indeed, even when $g < a/2$ and our assumed form is self-consistent, the flow is likely to contain a shock to enable the flow to adjust to conditions far from the galactic center. We can see from equation (16) that for $x \gg 1$ we have $m \sim (a -$

$2g)^{1/2}$, so that the flow would have radial velocities $\sim 10^2$ km sec^{-1} even in the solar neighborhood if there were no shock. This is not observed, and we conclude that a shock must occur and discuss its nature in § III.

III. A GALACTIC BORE

We have just seen that there are reasons to expect a shock to form in the outflow of gas from the galactic center. Let us consider what happens if we insert a normal shock at some radius using the usual strong shock conditions. If subscripts 1 and 2 denote values just ahead of and behind the shock, the usual jump conditions are

$$p_2(z) = p_1(z) 2\gamma M_1^2 , \qquad \rho_2(z) = \rho_1(z)\left(\frac{\gamma+1}{\gamma-1}\right) ,$$
$$T_2(z) = T_1(z)\,\frac{2\gamma(\gamma-1)M_1^2}{\gamma+1} , \tag{17}$$

where $M_1 = U_1(\gamma R T_1)^{-1/2}$.

In the gas ahead of the shock ρ_1 and p_1 satisfy equation (3), and if we introduce conditions (17) this becomes

$$\frac{\partial p_2}{\partial z} = -\beta z \rho_2\left[\frac{2\gamma(\gamma-1)M_1^2}{\gamma+1}\right] . \tag{18}$$

This indicates that behind the shock the medium is out of hydrostatic equilibrium by a factor T_2/T_1. Now this breakdown of hydrostatics can be modified by three effects. First, radiative losses will tend to reduce the temperature jump; however, for strong shocks, one-dimensional theory, assuming LTE and a gray medium, indicates that the main effect of radiation is to smear out the jump in temperature rather than to diminish its amplitude. Second, ionization can absorb energy and thus reduce the temperature jump appreciably for $M_1 \sim 50$. Finally, the shock may be curved, and this conceivably could help preserve hydrostatic equilibrium. But as we are contemplating $M_1 = O(10^2)$, it seems unlikely that these effects can alter the qualitative conclusion that the shock wave causes a breakdown of hydrostatic equilibrium behind the shock. The excessive vertical pressure gradient will then drive the gas behind the shock away from the galactic plane with violent acceleration, producing turbulence and distorting the shock.

If we accept these qualitative conclusions, we are led to a picture in which the transition from supersonic to subsonic flow more closely resembles a hydraulic jump, or bore, than a plane shock wave. Though we have indicated how the bore may be initiated by a breakdown of hydrostatic equilibrium through a shock, it may well be that the bore develops more along the lines seen in the laboratory for the incompressible case. There, the bore develops through the steepening and breaking of gravity waves. In the case of a strong bore, the wave has large amplitude in the upper regions of the fluid; it breaks there first and becomes turbulent. The turbulence then entrains the flow at lower levels, and the result is a turbulent wall across which the flow changes from shooting to streaming. (Shooting flows have speeds greater than the gravity-wave speed; streaming flows have speeds which are less. In a perfect gas in hydrostatic equilibrium the sound speed and gravity-wave speed are comparable, and the transition from shooting to streaming flow can normally be accomplished simultaneously with the transition from supersonic to subsonic flow.)

In the galactic case we can envision a similar behavior. The gravity wave breaks above the plane, giving rise to supersonic turbulence which rapidly heats the gas. Even with radiative losses, the kinetic energy available should cause this gas to move away from the plane, causing the wave closer to the plane to steepen. In this way the breakdown of the mean flow spreads toward the plane, the turbulent stresses slowing the outward flow and causing the local density to rise. But the turbulence rapidly heats the gas, causing it both to radiate and to leave the plane, thus lowering its density to a value possibly lower than its upstream value, depending on the relative importance of radiative losses.

Unfortunately, we cannot compute these processes in detail because we have no way of knowing the rates of turbulent instability and its consequent heating of the gas. Indeed, no theory of the structure of strong bores exists even for the laboratory case with incompressible, non-radiating flows. Nevertheless, some interesting results are suggested by the jump conditions, which are independent of the detailed structure of the bore. We shall therefore derive these, leaving out of account radiative losses.

We treat the bore as a sharp, almost discontinuous transition. We imagine that the turbulence in the bore dissipates its energy quickly, so that the emerging flow, though heated, is laminar and can be reasonably expected to satisfy the general assumptions applied in § II, though we shall admit the possibility of ionization behind the bore. We stress that these assumptions will break down almost immediately because of radiation and recombination, but we apply them just to the downstream edge of the bore to get some idea of its effects. We may add that, since the bore is thin in the radial direction, we can neglect curvature and treat g as constant through the bore.

Now, denoting conditions ahead of and behind the bore with subscripts 1 and 2 as before, we find the conservation equations

$$\int_{-\infty}^{+\infty} \rho_1 u_1 dz = \int_{-\infty}^{+\infty} \rho_2 u_2 dz \tag{19}$$

for mass;

$$\int_{-\infty}^{+\infty} (p_1 + \rho_1 u_1^2) dz = \int_{-\infty}^{+\infty} (p_2 + \rho_2 u_2^2) dz \tag{20}$$

for momentum; and, for energy,

$$\int_{-\infty}^{+\infty} \left(h_1 + \tfrac{1}{2}u_1^2 + \frac{\beta z^2}{2}\right) \rho_1 u_1 dz = \int_{-\infty}^{+\infty} \left(h_1 + \tfrac{1}{2}u_1^2 + \frac{\beta z^2}{2}\right) \rho_2 u_2 dz \, . \tag{21}$$

where h is the specific enthalpy. These conditions differ from those for a plane shock wave in two evident respects. First, because of the turbulence in the bore, the conservation conditions apply, not to individual parcels of fluid, but globally to the fluid passing through a cylindrical surface. Second, as in the normal hydraulic jump theory, there is a gravitational contribution to the energy which here is quadratic in z and therefore plays the role of a classical degree of freedom.

If the gas is perfect, monatomic, and of pure hydrogen, we have, on neglecting the excitation energy,

$$h = \tfrac{5}{2} R (1 + \alpha) T + \alpha I / m_{\mathrm{H}} \, , \tag{22}$$

where α is the fraction of hydrogen that is ionized. We have already assumed that ahead of the bore there is no ionization ($\alpha_1 = 0$), but the degree of ionization behind the bore, α_2, is a very important parameter.

Consider first the case where we admit no ionization behind the bore, $\alpha_2 = 0$. Then, since u and T are assumed independent of z, outside the bore, the conservation equations are readily integrated. It is easy to show that they reduce to the jump conditions for a plane shock wave in a "gas" with $\gamma = \frac{3}{2}$, velocity u, a temperature T, and a density

$\alpha \rho_{\dagger} T^{1/2}$. This value of γ is to be expected from the usual relation between γ and the number of degrees of freedom, and from the fact that potential energy is a quadratic term in z. We need not write down these jump conditions as they are well known, but we note that in the strong shock limit ($M_1 \gg 1$) they become

$$u_2/u_1 = \tfrac{1}{5}\,, \quad T_2/T_1 = \tfrac{3}{5}M_1^2\,, \quad \rho_2{}^{\dagger}/\rho_1{}^{\dagger} = 5\sqrt{(\tfrac{5}{3})}/M_1\,. \tag{23}$$

Thus the density in the galactic plane, $\rho_{\dagger}$, is greatly reduced behind the bore since the heating of the gas raises the scale height and, for strong bores, this effect outweighs the compression due to the decrease in u. It is the change in scale height which is most reminiscent of the classical bore.

For the galactic case with $T_1 \sim 100°$ K and $M_1 \sim 10^2$ these conditions imply $T_2 \sim 10^6$ ° K, which is inconsistent with the neglect of ionization and radiation. If we then turn to the other extreme, that the gas is completely ionized behind the bore, $\alpha_2 = 1$, the picture changes completely. The jump conditions are still simple to work out, and we can solve them to find

$$u_2 = \frac{1}{5u_1}\{3(u_1^2 + RT) - 2[(u_1^2 - \tfrac{3}{2}RT_1)^2 + \tfrac{5}{2}u_1^2 I/m_{\rm H}]^{1/2}\}\,, \tag{24}$$

where I is the ionization energy per atom. Here we have selected that root of the quadratic equation arising for u_2 which reduces to the solution for the non-ionized case where $I \to 0$.

In cases of interest here, $u_1^2 \gg RT_1$, and equation (24) can be replaced by

$$u_2 \sim \tfrac{3}{5}u_1 - \tfrac{2}{5}u_1\left(1 + \frac{5I_1}{2m_{\rm H}u_1^2}\right)^{1/2}, \tag{25}$$

while the jump condition in temperature becomes

$$\frac{T_2}{T_1} \sim \frac{1}{6RT_1}\left(u_1^2 - u_2^2 - \frac{2I}{m_{\rm H}}\right). \tag{26}$$

Thus the absorption of energy by ionization can result in smaller velocities and temperature behind the bore, and hence the bore can raise the density of the gas. If

$$u_1 \gg u_c \equiv (2I/m_{\rm H})^{1/2} = 51 \text{ km sec}^{-1}\,, \tag{27}$$

the effect is small, but, at the other extreme, when $u_1 = u_c$, u_2 and T_2 are both zero and $\rho_{\dagger 2}$ must be infinite. The gas would then be brought to rest and completely refrigerated as a result of the conversion of kinetic energy into ionization energy. Of course, this state is inconsistent with the assumption of complete ionization behind the bore, and what is really called for is a consistent calculation of α_2 rather than its treatment as a free parameter. Indeed, in the absence of radiative losses and at the low densities we contemplate, large values of T_2 result when u_1 is only moderately in excess of u_c. Nevertheless, the singular behavior at u_c suggests that when the gas flows into the bore near the critical velocity u_c it will emerge slowly and with high density, especially if radiative losses play a role.

To get a qualitative idea of the intermediate case where α_2 need not be 0 or 1, we study the jump conditions using the Saha formula for α. Clearly this cannot give precise information for the interstellar gas, but it serves as a simple interpolation device in the absence of a more precise theory. In the Appendix we calculate the jump conditions using the Saha formula, taking into account the consequent variation of α with z. We need not go into the algebra here but rather will display some of the main results.

The nature of the jump across the bore depends sensitively on u_1, the speed with which the gas flows into the bore, but only weakly on the incoming density, $\rho_1{}^\dagger$. Accordingly, we show results for a typical density, $\rho_1{}^\dagger = 10^{-24}$ g cm^{-3} for different u_1; we adopt $T_1 = 10^2$ ° K as the upstream temperature.

The results are shown in the figures. Figure 1 gives the temperature on the downstream side of the bore, T_2, as a function of the velocity on the upstream edge, u_1, for $T_1 = 100°$ and $\rho_1{}^\dagger = 10^{-24}$ g cm^{-3}. The plateau occurs when the ionization soaks up the bulk of the kinetic energy. For u_1 in excess of 53 km sec^{-1}, the downstream temperature rises steeply as a function of u_1. Of course, this rise would not occur if the radiative losses had been included; rather, it is likely that the plateau would just continue beyond $u_1 = 53$ km sec^{-1}. It is, however, difficult to calculate this temperature precisely,

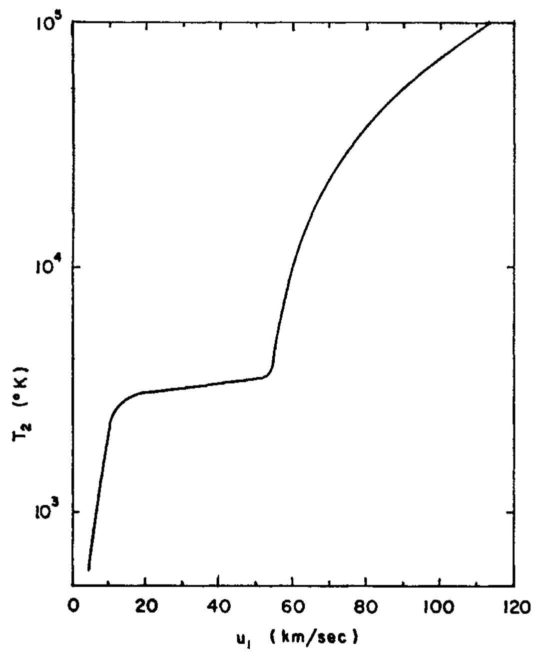

FIG. 1.—Temperature on the downstream edge of the bore as a function of velocity component into the bore on the upstream edge. Results for $\rho_1{}^\dagger = 10^{-24}$ g cm^{-3}, $T_1 = 100°$ K.

since the kinetic energy is dissipated gradually throughout the bore. But if the assumption is made that the kinetic energy is converted discontinuously into thermal energy, it is easy to see that for these high temperatures the gas would be cooled before it travels a parsec. It does seem reasonable, though, to anticipate an ionized region just outside the bore with $T \sim 3000°$ K.

In Figure 2 we show the density jump across the bore. The lower curve gives $\rho_2{}^\dagger/\rho_1{}^\dagger$, which is the ratio of the actual densities in the plane $z = 0$. The initial conditions are, again, $T_1 = 100°$ K and $\rho_1{}^\dagger = 10^{-24}$ g cm^{-3}. The density enhancement has a broad maximum at $u_1 \sim 50$ km sec^{-1}, with a maximum enhancement factor $\sim$6. For $u_1 > 53$ km sec^{-1}, the density actually diminishes across the bore, because the scale height behind the bore increases. We stress that this particular feature is exaggerated because of the neglect of radiative losses. The upper curve in Figure 2 shows the ratio ρ_{*2}/ρ_{*1}, where

$$\rho_* = \int_{-\infty}^{\infty} \rho dz = (2\pi)^{1/2} H \rho^\dagger \tag{28}$$

is the integrated density. Mass conservation requires that $\rho_{*2}/\rho_{*1} = u_1/u_2$, so that this curve also indicates the slowing down of the flow by the bore. For $u_1 \gg u_c$ the result is close to that obtained on the assumption of complete ionization. But in the neighborhood of 53 km sec^{-1} where partial ionization occurs, a maximum enhancement of matter accumulation is found, corresponding to a slowing down of the outflow by a factor 50. Radiation will cause this enhancement factor to increase, but the qualitative nature of the curve should remain unaltered. Also, we have found that these curves are reasonably insensitive to T_1 and $\rho_1{}^\dagger$.

IV. CONCLUDING REMARKS

In § II we have seen how the initially supersonic outflow should give rise to a rapid transition to subsonic outflow. In § III we have argued that the transition takes place in a strong bore which produces intense turbulence, ionization, and radiation. What remains to be specified is the location of the bore.

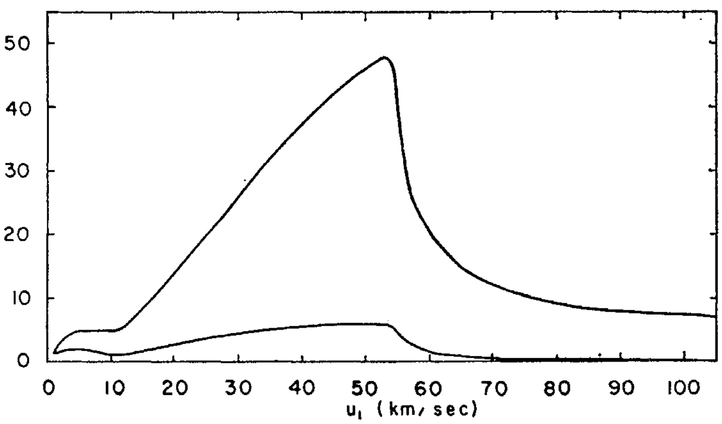

FIG. 2.—Density enhancement across the bore as a function of incoming velocity. *Upper curve,* ratio of integrated densities ρ_{*2}/ρ_{*1}; *lower curve,* ratio of densities in the plane $\rho_2{}^\dagger/\rho_1{}^\dagger$.

The normal procedure would be to place the bore so that the flow matches onto some specified downstream conditions, but this cannot be done here. For example, if the bore occurred at 4 kpc from the galactic center and the mass in the galactic center is 7×10^9 $\mathfrak{M}_\odot$, the escape velocity is $\sim 10^2$ km sec^{-1}. Reference to the curve for $u_1/u_2 = \rho_{*2}/\rho_{*1}$ in Figure 2 then shows that no reasonable upstream conditions will permit the gas to pass appreciably beyond 4 kpc once it has been decelerated in the bore. Thus, it is not possible to locate the bore by using conditions for downstream flow, and in our present model its position remains undetermined. The question of what physics determines the location of the bore remains open. But it appears that wherever the bore occurs, the matter will likely be trapped and the gas density will build up. If star formation occurs in the high-density region, a steady state can nevertheless be achieved. We can also consider the possibility that some of the gas departs sufficiently from the plane to be able to flow back into the center and continue circulating.

These qualitative considerations suggest that we associate the bore with the expanding arm. This is in line with the idea that spiral arms represent some kind of wave phenomenon, and indeed we suggest that the non-linear form of the waves thought to be associated with spiral arms are bores. At any rate, we wish to explain the expanding arm in this way, and we postulate that the bore occurs at a velocity which gives rise to the largest possible density in the bore. We have seen that this occurs at $u_1 \sim 53$ km sec^{-1}, which accords with the observed outward velocity in the expanding arm.

Clearly, the model discussed here is far too schematic to be of value in predicting details of the flow, but we believe that the following general properties may be of relevance:

1. If the outflow is axisymmetric and steady and if no turbulent or magnetic stresses exist, then, irrespective of the mass distribution in the central regions, the circular velocity is given by equation (10). Thus, it does not seem possible to use observed velocities of rotation near the galactic center to infer the mass distribution there, though this may become possible if account is taken of non-radial forces.

2. The occurrence of a compressible hydraulic jump (galactic bore) in the outflow may be the mechanism that forms the expanding arm. This is consistent with the occurrence of H II regions only outside the arm.

3. We believe that the discussion indicates that waves in the interstellar gas in their non-linear form generally take the form of bores. If the agreement between the speed of outflow in the expanding arm and the preferred velocity in the jump conditions, $u_c \sim 51$ km sec^{-1}, is not coincidental, we may expect u_c to play some role wherever bores occur in the Galaxy. The criterion that seems indicated is that the velocity component normal to the bore is u_c, and for a given rotation velocity (and radial velocity) this fixes the pitch angle of the bore; however, this last possibility is highly tentative.

In conclusion, we express our thanks to Professors P. Sweet and L. Woltjer for their suggestions.

APPENDIX

The equation of state of partially ionized hydrogen is

$$p = \rho RT\left(1 + \frac{K(T)}{2\rho}\left\{\left[1 + \frac{4\rho}{K(T)}\right]^{1/2} - 1\right\}\right), \tag{A1}$$

where, in the usual notation,

$$K(T) = \frac{m_{\rm H}}{h^3}(2\pi m_e)^{3/2}(kT)^{3/2}\exp\left(-\frac{I}{kT}\right). \tag{A2}$$

Our first task is to find the distribution of density and pressure with height in an isothermal layer of gas obeying equation (A1) and in which the gravity varies linearly with height. The condition for hydrostatic balance is equation (3), so that by using equation (A1), we get

$$\frac{d\rho}{dz}\left[\frac{1}{\rho} + \frac{1}{\rho}\left(1 + \frac{4\rho}{K_0}\right)^{-1/2}\right] = -\frac{\beta z}{RT_0}, \tag{A3}$$

where

$$K_0 = K(T_0) \tag{A4}$$

is a constant depending only on the uniform temperature T_0 of the atmosphere. We can easily integrate equation (A3) to give

$$\frac{\rho}{\rho^{\dagger}} = \tfrac{1}{2}fq\exp\left(-\frac{\beta z^2}{4RT_0}\right) + \tfrac{1}{4}fq^2\exp\left(-\frac{\beta z^2}{2RT_0}\right), \tag{A5}$$

where

$$f = K_0/\rho^{\dagger} \tag{A6}$$

and

$$q = [(1 + 4/f)^{1/2} - 1]. \tag{A7}$$

It is interesting that the density distribution is just a linear combination of the distributions for a completely ionized and a completely non-ionized atmosphere, with weightings depending on the degree of ionization $\alpha^{\dagger}(=\frac{1}{2}fq)$ in the galactic plane.

With the distribution (A5) at hand, it is a straightforward matter to work out the integrals in the jump conditions (19)–(21), and we get

$$\int_{-\infty}^{\infty} \rho dz = 2\rho^\dagger (RT_0/\beta)^{1/2} P \, , \tag{A8}$$

$$\int_{-\infty}^{\infty} \rho a dz = 2\rho^\dagger (RT_0/\beta)^{1/2} Q \, , \tag{A9}$$

and

$$\int_{-\infty}^{\infty} \rho z^2 dz = 2\rho^\dagger (RT_0/\beta)^{3/2} S \, , \tag{A10}$$

where

$$P = \tfrac{1}{2}\sqrt{\pi}\left(fq + \frac{\sqrt{2}}{4} fq^2\right) , \tag{A11}$$

$$Q = \frac{\sqrt{\pi}}{2} fq \, , \tag{A12}$$

and

$$S = \frac{\sqrt{\pi}}{4}\left(4fq + \frac{1}{\sqrt{2}} fq^2\right) . \tag{A13}$$

The jump conditions are

$$[u\rho^\dagger (RT/\beta)^{1/2} P]_1^2 = 0 \, , \tag{A14}$$

$$\{\rho^\dagger (RT/\beta)^{1/2} P[u^2 + RT(1 + Q/P)]\}_1^2 = 0 \, , \tag{A15}$$

and

$$\left[\left(\tfrac{5}{2} + \tfrac{5}{2}\frac{Q}{P} + \tfrac{1}{2}\frac{S}{P}\right)RT + \tfrac{1}{2}u^2 + \frac{I}{m_{\rm H}}\frac{Q}{P}\right]_1^2 = 0 \, , \tag{A16}$$

where $[f]_1^2 = f_2 - f_1$. Clearly T and $\rho^\dagger$ change across the jump so that f and q and hence P, Q, and S change also. Thus equations (A14)–(A16) form a complicated transcendental system of equations for the conditions behind the jump.

We have seen that the gas ahead of the jump is not ionized, so that $f_1 = 0$ and $Q_1/P_1 = 0$, $S_1/P_1 = 1$. Then solving the jump conditions in the usual way for u_3, as if Q_2 and P_2 were known constants, we get

$$u_2 = \frac{1}{5u_1}\left\{3(u_1^2 + RT_1) - 2\left[(u_1^2 - \tfrac{3}{2}RT_1)^2 + \tfrac{5}{2}\frac{Q_2}{P_2}u_1^2\frac{I}{m_{\rm H}}\right]^{1/2}\right\} . \tag{A17}$$

(The choice of the sign of the square root is discussed in the text.) Having found u_3, we can calculate T_2 and $\rho_2^\dagger$ in terms of the upstream conditions $\rho_1^\dagger > u_1$ and T_1 by means of the equations

$$\frac{T_2}{T_1} = \left(\frac{u_2}{u_1} + \frac{u_2 u_1}{RT_1} - \frac{u_2^2}{RT_1}\right)\left(1 + \frac{Q_2}{P_2}\right)^{-1} , \tag{A18}$$

$$\frac{\rho_2^\dagger}{\rho_1^\dagger} = \frac{u_1}{u_2}\left(\frac{T_1}{T_2}\right)^{1/2}\frac{\sqrt{(2\pi)}}{2P_2} . \tag{A19}$$

Suppose we guess a value of the degree of ionization, $(a_2^\dagger)_g$, say, in the galactic plane downstream of the jump. This fixes f_2 and q_2, and we can use equations (A17)–(A19) to calculate u_2, T_2, and $\rho_2^\dagger$. Then from equations (A2), (A6), and (A7) we can calculate a new downstream ionization, say $(a_2^\dagger)_i$, and compare it with $(a_2^\dagger)_g$. We can then improve our guess in some systematic way and carry on until the process converges. In practice we took as our improved guess $(a_2^\dagger)_i S + (1 - S)(a_2^\dagger)_g$. The iteration was stable with small values of the constant S, and convergence was quite rapid.

REFERENCES

Burbidge, G. R., and Hoyle, F. 1963, *Ap. J.*, **138**, 57.

Kerr, F. J. 1967, in *Radio Astronomy and the Galactic System*, ed. H. van Woerden (New York: Academic Press).

Kerr, F. J., and Westerhout, G. 1965, in *Stars and Stellar Systems*, Vol. **V**, ed. A. Blaauw and M. Schmidt (Chicago: University of Chicago Press).

Limber, D. N. 1967, *Ap. J.*, **148**, 141.

Mezger, P. G., and Högland, H. P. 1957, *Ap. J.*, **147**, 471 and 490.

Oort, J. H. 1966, *Trans. I.A.U.*, Vol. **XII**.

Rougoor, G. W. 1960, *B.A.N.*, **17**, 381.

Rougoor, G. W., and Oort, J. H. 1960, *Proc. Nat. Acad. Sci.*, **46**, 1.

Schmidt, M. 1965, in *Stars and Stellar Systems*, Vol. **V**, ed. A. Blaauw and M. Schmidt (Chicago: University of Chicago Press).

Spiegel, E. A. 1966, *Summer School on Interstellar Gas Dynamics*, ed. D. E Osterbrock (Department of Astronomy, University of Wisconsin).

Wentzel, D. 1961, *Nature*, **189**, 907.

Woerden, H. van, Rougoor, G. W., and Oort, J. H. 1967, *C.R. Acad. Sci. Paris*, **244**, 1691.

Woltjer, L. 1965, in *Stars and Stellar Systems*, Vol. **V**, ed. A. Blaauw and M. Schmidt (Chicago: University of Chicago Press).