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INTERPLANETARY GAS. XIV. ON THE EXTENT OF EFFECTIVE COROTATION

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ABSTRACT

The super-Alfvénic point or distance of effective corotation for the loss of angular momentum in the solar wind plasma is found by extrapolating observed plasma densities and velocities, and magnetic fields back toward the Sun. The result is approximately 20–40 $R_\odot$ for quiet conditions, and possibly a larger value for disturbed conditions. This value is somewhat larger than the usual estimates made by extrapolating the solar surface field outward The theoretical treatment of Weber and Davis permits an estimate of the angular momentum carried by the plasma, and these estimates are consistent with the values of the tangential velocity of the solar-wind plasma at Earth inferred by Brandt from studies of Type I comet tails. Such effective corotation distances change the interpretation of Sun-Earth delay times, complicate the search for the solar M-regions, and imply an *e*-folding time for braking of solar rotation comparable to the age of the Sun.

I. INTRODUCTION

Estimates of the distance for effective corotation of the solar plasma, R_c, or the equivalent super-Alfvénic point, R_A, have generally been made by extrapolating an assumed solar surface magnetic field and seeking the point where the Alfvén velocity, $V_A = B/(4\pi\rho)^{1/2}$, is equal to the expansion velocity, w, of the plasma. Previous calculations usually gave $R_A \approx 10\ R_\odot$ (e.g., Axford, Dessler, and Gottlieb 1963).

Weber and Davis (1967) have shown that the total loss of angular momentum in the solar wind is equal to the amount given by assuming strict corotation out to R_A although strict rotation by itself is not approached; this latter result was previously obtained by Pneuman (1966). This result, namely, considerable *effective* corotation but little *real* corotation, resolves a possible conflict because Bohlin, Hansen, and Newkirk (1966) have determined the three-dimensional structure of a particular coronal streamer which showed little evidence of corotation. Hence, strict corotation near the Sun is not found and, in fact, is not required for a substantial loss of angular momentum.

Weber and Davis (1967) also showed that the angular momentum in the solar wind is carried both in azimuthal particle motions and in magnetic stresses. At large distances, the relative contributions are $[1 - (w_A/w_\infty)]$ and $[w_A/w_\infty]$, respectively; here w_A is the solar-wind velocity at the point R_A and w_∞ is the velocity at large distances from the Sun. In the following discussion, I set w_∞ equal to the solar wind velocity at Earth, w_E.

The assumptions outlined above give the tangential velocity at Earth, which is

$$w_{t,E} = \left(\frac{R_A}{10}\right)^2 \left(1 - \frac{w_A}{w_E}\right) \text{km/sec}\,, \tag{1}$$

where R_A is in solar radii.

Estimates of R_A made by working outward from the solar surface are complicated by our lack of detailed knowledge of the field configuration near the Sun. It appears that these difficulties can be avoided by utilizing measured properties near Earth and extrapolating inward. No such calculations are in the literature; the only possible exception is a remark by L. Davis (1966) made at the 1964 conference on the solar wind, in which he quoted 10–20 $R_\odot$ as the result of such an inward extrapolation.

Thus, R_A is determined for several models based on observations, and $w_{t,E}$ is computed

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from equation (1) for two values of $[1 - (w_A/w_E)]$. These computed values of $w_{t,E}$ are then compared with the comet observations of 4 ± 2 km/sec for quiet conditions and 9 ± 3 km/sec for the entire sample, which was thought to be indicative of average conditions (Brandt 1967).

The comparison of observed and computed values of $w_{t,E}$ is satisfactory, and lends support to Brandt's (1966) contention that loss of angular momentum in the solar wind may be important in the structure and evolution of the Sun; typical *e*-folding times for braking of the solar rotation are numerically close to the age of the Sun (Table 1).

II. THE CALCULATION

a) Quiet Conditions

We begin with a discussion of quiet conditions because of available information and because such calculations should be more accurate since they would be relatively uncomplicated by emission from discrete solar regions.

TABLE 1

ANGULAR MOMENTUM PARAMETERS IN THE SOLAR WIND

Model $B_{E,r}$ (γ)	$V_{A,E}$* (km/sec)	R_A (min) ($R\odot$)	R_A (cal) ($R\odot$)	$w_{t,E}$ (min) (km/sec) $[1-(w_A/w_E)]=\frac{1}{3}$	$w_{t,E}$ (min) (km/sec) $[1-(w_A/w_E)]=\frac{1}{2}$	$w_{t,E}$ (km/sec) $[1-(w_A/w_E)]=\frac{1}{3}$	T_1 (min) (10^9 yrs)	T_2 (min) (10^9 yrs)	T_2 (10^9 yrs)
Model 1:									
3 5	30	18	21 5	0 6	1 1	1 5	20	13	9
5 0. .	42	26	.	1 4	2 3	3 2	10	6	4
Model 2:									
3 5 .	54	24		1 2	1 9	2 7	28	18	13
5 0.	77	35		2 4	4 1	5 7	13	8	6
Model 3:									
3 5	54	31		1 9	3 2	4 5	21	13	9
5 0.	77	44	.	3 9	6 5	9 0	10	7	5
Model 4:									
12 5	136	49		4 8	8 0	11 2	3	2	1

We fix the total interplanetary magnetic field at the Earth at 5 γ (1 $\gamma = 10^{-5}$ gauss) because it is generally observed (Greenstadt 1966, Table 5). The gardenhose angle at Earth is about 45° (Ness and Wilcox 1966), and hence the radial component is 3. 5 γ. However, the radial field at Earth is sometimes taken to be 5 γ, and it is included to show the effect of such a change and because such a field may be typical of mean conditions over a solar cycle. The radial component varies with distance as $B_r/B_0 = (r_0/r)^2$ in the region beyond R_A where the wind dominates (Parker 1958, 1963). We denote the Alfvén velocity computed with only the radial component as $V_A{}^*$; the field is expected to be essentially radial at R_A.

The expression for $V_A{}^*$, the equation of continuity, and the radial field variation can be combined to give

$$\frac{R_A}{R_E} = \frac{V_{A,E}{}^*}{(w_A w_E)^{1/2}}, \tag{2}$$

where R_E is 1 a.u. Thus, if we know $w(r)$, we can compute R_A; a lower limit, R_A (min), is obtained even without a model by setting $w_A = w_E$.

We also need some estimate of $[1 - (w_A/w_E)]$; Weber and Davis (1967) used models

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with $[1 - (w_A/w_E)]$ of approximately $\frac{1}{5}$ to $\frac{1}{4}$, and they noted that these values were probably low. Keeping the velocity high near the Sun (as implied by these low values of $[1 - (w_A/w_E)]$) leads to low coronal densities. The model of Noble and Scarf (1963, Table 1) has $[1 - (w_A/w_E)] = 0.3$, a reasonable value (see Model 1 below). Also, this model assumes that heating of the lower corona is by thermal conduction rather than by energy deposition; this is appropriate because Noble and Scarf (1963) and Brandt, Michie, and Cassinelli (1965*b*) have shown that mechanical energy deposition does not extend beyond 2 $R_\odot$. Noble and Scarf can reproduce the coronal densities and they also use a coronal temperature of 2×10^6 ° K which is now thought appropriate (Brandt *et al.* 1965*a*). Hence, values for $[1 - w_A/w_E)]$ of $\frac{1}{5}$ and $\frac{1}{3}$ are taken as representative (Table 1).

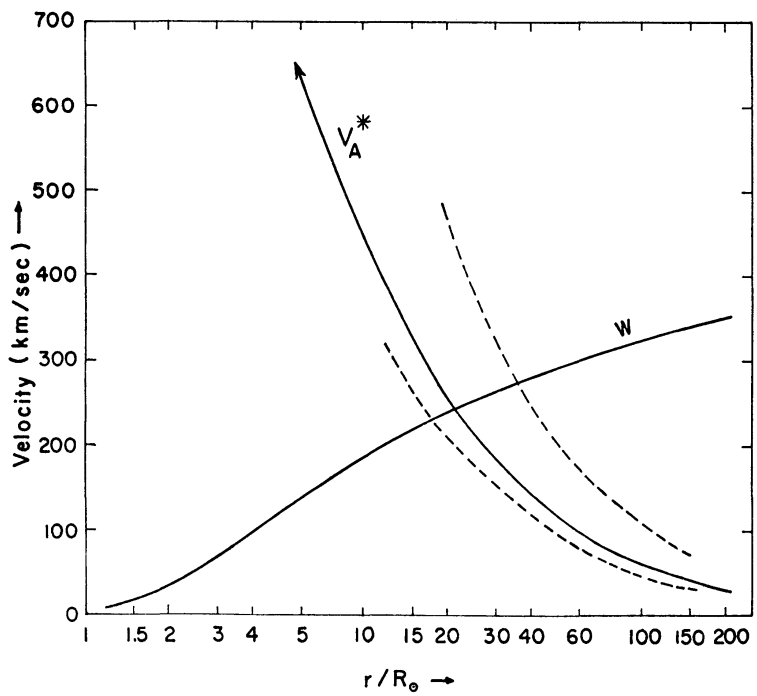

FIG 1.—The run of expansion velocity, w (from Noble and Scarf 1963), and $V_A{}^*$ (the Alfvén velocity computed with the radial component of the interplanetary magnetic field) showing the determination of R_A (cal) = 21 5 $R_\odot$ for Model 1 and $B_{E,r}$ = 3 5 γ.

The curves above and below the one labeled $V_A{}^*$ are the results using an electron density at the orbit of Earth of 2/cm³ and 10/cm³, respectively, for the same run of w.

The plasma models are as follows:

Model 1.—Here w_E = 353 km/sec and $N_{e,E}$ = 6.7/cm³ (Bridge, Egidi, Lazarus, Lyon, and Jacobsen 1965; Lyon 1965; Wilcox, Schatten, and Ness 1967, Fig. 4). The exact values quoted are in agreement with, and come from, the model of Noble and Scarf (1963).

Model 2.—Here w_E = 475 km/sec and $N_{e,E}$ = 2/cm³ as quoted by Neugebauer and Snyder (1962) for typical quiet conditions.

Model 3.—Here w_E = 378 km/sec and $N_{e,E}$ = 2/cm³ (Wolfe, Silva, and Myers 1966); the velocity is quoted by Wolfe *et al.* and the density follows from their statement that the electron density rarely exceeded 3/cm³.

The various parameters for a discussion of the angular momentum in the solar wind are tabulated in Table 1 and most of the symbols have been defined or are obvious. The R_A (cal) given in the fourth column follows from the detailed determination shown in Figure 1.

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This shows that the relatively large values of R_A presented here are rather well determined, and that R_A (cal) is about 20 per cent larger than R_A (min). The fifth and sixth columns give $w_{t,E}$ values calculated from R_A (min) while the seventh column contains the effect of a 20 per cent increase in R_A. Similarly, the e-folding times T_1 and T_2 in the eighth and ninth columns use R_A (min) while the tenth column uses the larger R_A (cal). The T_1 assumes that the angular momentum is lost only within $\pm 30°$ of the solar equator (Brandt 1967), while T_2 assumes a constant solar wind in all directions and reduces the angular momentum loss by the sine of the polar angle (Weber and Davis 1967). The ratio $T_1/T_2 = 2/\pi$, and the effective e-folding time is probably intermediate.

The results for Models 1, 2, and 3 as shown in the fifth, sixth, and seventh columns show that the theory and parameters used for the plasma and magnetic field near Earth are entirely compatible with the cometary measurements of 4 ± 2 km/sec for quiet conditions. Some of the values are close to or within the measurement of 9 ± 3 km/sec thought to apply to conditions averaged over a solar cycle (Brandt 1967).

b) Disturbed Conditions

The R_A appropriate to disturbed interplanetary conditions is more difficult to determine because both theory and observation are more uncertain. However, there are reasons to believe that R_A increases for active conditions. For the purpose of illustration, we adopt $w_E = 600$ km/sec, $B_{E,r} = 12.5\ \gamma$ and $N_{e,E} = 4/\text{cm}^3$, which is consistent with the pertinent evidence (Davis 1966; Wilcox *et al.* 1967; Brandt 1967). The increased magnetic field probably accounts for nearly all the change in R_A; however, the density appears to be lower in the faster streams (Snyder and Neugebauer 1964, Fig. 13). This effect can only increase R_A.

This model (No. 4 in Table 1) is plausible but uncertain; it is included only to show the possible effects of M-region plasma streams. For what they are worth, the parameters associated with Model 4 indicate that it is not difficult for a relatively small fraction of M-region streams to bring the average $w_{t,E}$ into or near the range 9 ± 3 km/sec.

III. DISCUSSION

a) General

The values of R_A and $w_{t,E}$ found here are compatible with the requirements of the comet-deduced tangential velocities at Earth (Brandt 1967); the values of R_A are in the range 20–40 $R_\odot$ and are larger than most estimates in the literature.

Brandt (1966) has considered that the angular momentum loss in the solar wind could be important in the structure and evolution of the Sun; the calculations presented here definitely support his contentions. The values of the e-folding times, T, listed in Table 1 are comparable to the accepted age of the Sun. For a given $B_{E,r}$ the T's are approximately constant; this stems from the fact that higher densities near the Earth have smaller R_A's, etc., and hence, the angular momentum loss is approximately the same. The difference in the models is thus in the angular momentum loss per unit mass.

The field in the lower corona can be estimated by an additional inward extrapolation. Fields of 1.5 and 2.1 gauss are found for $B_{E,r}$ of 3.5 and 5.0 γ, respectively. A radial field is assumed down to the surface which is consistent with the existence of long, radial coronal streams (e.g., Chapman 1961).

An independent theoretical discussion by Modisette (1967) also shows that the net angular momentum loss corresponds to corotation out to R_A, but on his picture the tangential velocity at Earth is quite small. To reconcile his picture with the cometary results, Modisette (1967) suggests that the comet tails are responding to the total momentum flux. Calculations shown in Table 1 indicate that this postulate may not be necessary, but, in any event, continued observation of the solar wind parameters near Earth should settle the question.

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Measurement of the tangential plasma velocity near the Earth can also allow a probing of the velocity structure of the solar wind near the Sun. The value of R_A can be determined fairly well, and thus $w_{t,E}$ depends on $[1 - (w_A/w_E)]$; hence, accurate observations of $w_{t,E}$ permit a determination of w_A.

b) Delay Times and M-Regions

The Sun-Earth delay times and the identification of the solar M-regions are affected by these large values of R_A. A specific calculation by Weber and Davis (1967) is for $w_E = 400$ km/sec and has a delay time for encountering the beam of 3.5 days as compared to a Sun-Earth delay time of 4.38 days. Thus, the method of superimposed epochs (already subject to considerable uncertainty) must take into account still another complication; the added delay time because of the acceleration near the Sun must also be considered. Note that the plasma striking Earth is well away from the solar surface at central meridian passage of the beam base.

These developments can alter velocities found from the comparison of the photospheric field and the interplanetary field. A tangential motion and a starting distance of $\approx 10\ R_\odot$ for the plasma tend to reconcile the results of Ness and Wilcox (1966) with the directly observed plasma velocities (Lyon 1965).

IV. CONCLUSIONS

The values of $w_{t,E}$ found from an analysis of ionic comet tails (Brandt 1967) are compatible with the theory of Weber and Davis (1967). The large values of R_A found here have many implications in solar physics.

The observation of Type I comets is continuing. Investigations carried out during the years approaching solar maximum should permit R_A and the resulting consequences for the solar angular momentum loss to be determined for a variety of degrees of solar activity.

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Note added April 5, 1967. A paper by Strong, Asbridge, Bame, and Hundhausen (presented at the Symposium on the Zodiacal Light and the Interplanetary Medium, Honolulu, Hawaii, February 1967) reports a tangential velocity of 10 ± 4 km/sec as determined from observations made aboard Vela satellites. These results are independent confirmation of the value (9 ± 3 km/sec) obtained from Type I comets (Brandt 1967), and they provide additional support for the contentions of this paper. Professor R. H. Dicke notes that he and R. J. Peebles have previously announced (*Nature*, **202**, 432, 1964) the result that the total angular momentum loss is equal to that obtained by assuming corotation out to the super-Alfvénic point.

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