Of the Diurnal Variation of the Upper Atmosphere

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ABSTRACT

The diurnal variation of the upper atmosphere as revealed from satellite drag measurements has been further investigated on the basis of a simultaneous integration of the heat conduction equation and the hydrostatic law. In addition to the heat source due to absorption of solar extreme ultraviolet radiation and the hypothetical "second heat source," the heating due to absorption of solar radiation in the Schumann-Runge range by oxygen molecules has been included. Furthermore, the effects of time-dependent variations in the boundary conditions on the phase and amplitude of the diurnal variation in the upper thermosphere and exosphere have been investigated. Also the effects of lateral heat conduction and lateral convective heat transport on the diurnal variation of density and temperature are discussed.

The main purpose of the paper is to investigate several possibilities which could be thought to eliminate the requirement for the "second heat source." It is shown that neither the inclusion of absorption of solar radiation in the Schumann-Runge band by O_2 molecules in our heat source nor diurnal variations of the boundary conditions at 120 km can be invoked in order to explain the diurnal variation on the basis of an EUV heat source exclusively. Further the effect of horizontal conduction is found in a simplified analysis to be quantitatively insufficient to account for an energy transport toward the west large enough to explain the observed diurnal variation under the presumption that all heating comes from the solar EUV radiation.

1. Introduction

The diurnal variation of the atmospheric density at heights above 200 km has been determined from satellite drag measurements by several investigators. An extensive analysis has been given by Jacchia and Slowey (1962). The diurnal density variation can be described as follows: During the morning the density increases until it reaches a maximum at about 14:00 hours local time. Then it decreases rather rapidly in the afternoon and evening, followed by a less steep decrease during the night. The density minimum can be placed at around 04:00 hours local time. This holds true for at least the altitude range from 350 to 660 km and for the time interval from 1958 through 1963, during which sufficiently accurate satellite drag data were available. It is particularly remarkable that almost for the entire decreasing phase of solar activity, the diurnal maximum is always found very close to 14:00 hours local time (Jacchia, 1964). It furthermore should be noted that the density decrease is considerably less rapid during the hours around midnight than in the late afternoon. The description of the diurnal density variation is also applicable to the diurnal temperature variation to a good approximation. By using Nicolet's (1961) set of "static" models, Jacchia obtained temperature maxima and minima exactly at the same local times as for the densities. This results because Nicolet's models furnish a monotonic relationship between densities and temperatures independent of local time. Since the main physical processes which determine the time-dependent variation of the upper atmosphere (heating by absorption of solar energy and heat conduction) have different characteristics, some caution must be exercised when relying on this kind of transformation (for further details, see Harris and Priester, 1963b). For this reason, we shall always use the observed density variations rather than the inferred temperatures, for the analysis in this paper.

Two years ago we investigated how the observed diurnal variation could be understood by assuming hydrostatic equilibrium and integrating the time-dependent heat conduction equation (Harris and Priester, 1962a). We included an expression which represented the convective heat transfer due to the diurnal expansion and contraction of the atmospheric bulge. From this analysis we concluded that heating of the thermosphere due to absorption of the solar extreme ultraviolet (EUV) radiation alone cannot explain the observed diurnal variation of density and temperature, as extreme ultraviolet heating alone would yield the

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maximum value of the density at about 17 hours local time instead of 14 hours. Furthermore, if the EUV flux is adjusted to represent the observed average density of the diurnal variation, then the amplitude of the diurnal variation would greatly exceed the observed amplitude.

Also, comparison of the required flux with Hinteregger's (1961) measurements of the EUV flux would require a very high efficiency for the conversion of EUV radiation into heat. Recent improved EUV measurements by Hall, Schweizer and Hinteregger (1964) yielded considerably higher fluxes than one would have expected for low levels of solar activity from the previous measurements. This might indicate that the solar spectrum in the EUV range does contain sufficient energy to provide the required heat exclusively. If so, it would eliminate the requirements for an extremely high efficiency, but does not affect the wrong phase and too large amplitude of the diurnal variation when calculated with an EUV heat source only by the aforementioned method.

Fig. 1 illustrates the discrepancy between the diurnal density variation derived from observations (Martin *et al.*, 1961) for an altitude of 600 km, and the calculated variation when only an EUV heat source is used (dotted line). In order to overcome this discrepancy we assumed the existence of a second heat source which has a maximum in the morning (at about 9 or 10 hours local time), a rather low value in the afternoon, and a small contribution during the night. With this additional heat source one achieves a good agreement between observed and calculated densities. The calculated values are given in Fig. 1 by the solid line. The line represents our model S=200, wherein a peak flux of 0.93 erg cm^{-2} sec^{-1} for the EUV heat source and of 1.03 erg cm^{-2} sec^{-1} for the "second heat source" were used. These values correspond to the average level of solar activity in the fall of 1959. If one uses an EUV heat source only, it would be necessary to employ a peak value of $\sim$2 erg cm^{-2} sec^{-1} of the EUV flux, in order to obtain a diurnal average density in close agreement with the observed average density. If the efficiency for conversion of solar EUV radiation into heat in the thermosphere is 40 per cent (Lazarev, 1963), the total flux in the EUV range below 1000 Å would have to be as high as 5 erg cm^{-2} sec^{-1} for a level of solar activity that occurred in the fall of 1959. But even with this rather large flux the problem of a wrongly phased variation remains. It is the purpose of this paper to investigate what the effects are on the diurnal variation if some of the simplifications made in the previous calculations are removed.

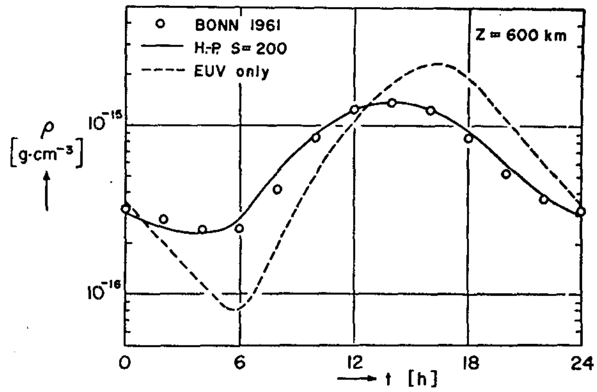

FIG. 1. Diurnal variation of the density at a height of 600 km. The abscissa is the local time. The circles represent the observations as given in the Bonn model 1961 (Martin *et al.*, 1961) which represents atmospheric conditions during the fall 1959. The dotted line is the calculated density variation if the absorption of solar extreme ultraviolet radiation is used as the only heat source. The solid line is the variation which is obtained if a "second heat source" is included in the calculations.

2. Discussion of the basic equations

In the attempt to understand the diurnal behavior of the upper atmosphere, it is important to determine to what extent the requirements for the "second heat source" are influenced by the simplifications employed in the basic equations.

The time dependent heat conduction equation is:

$$\frac{\partial}{\partial z}\left(K(T)\frac{\partial T}{\partial z}\right)-\rho C_p\frac{\partial T}{\partial z}T\int_{z_0}^{z}\frac{1}{T^2}\frac{\partial T}{\partial z}dz' +\sum_j Q_j=\rho C_p\frac{\partial T}{\partial t}. \quad (1)$$

Since we use, in this paper, the same notations as in the previous ones, we shall not repeat all the details here. The equation includes a convective term for the vertical heat transport during the diurnal expansion-contraction of the atmospheric bulge. This is represented by the second term on the left side of Eq. (1) and the appearance of C_p instead of C_v on the right side. A detailed derivation has been given earlier (Harris and Priester, 1962a). The second term in (1) has only a minor influence on the energy balance, as calculations with and without this term have shown. It, therefore, is unimportant in our discussion of the effects of simplifications in the theory on the calculated diurnal variation.

The first term in Eq. (1) accounts for the heat conduction in the vertical direction. $K(T)$ is the coefficient of heat conduction, taken as the weighted average of the coefficients. The weighting factor is the number density. The coefficient depends, furthermore, on the square root of the temperature (see Chapman and Cowling, 1952).

The third term accounts for the heat sources and losses. The heat source due to absorption of solar EUV radiation is given by:

$$Q_{\mathrm{EUV}}=\sum_i \epsilon_i n_i(z,t)\int_0^\infty F_\lambda \sigma_i(\lambda)\exp[-\sum_i \tau_i(z,t,\lambda)]d\lambda, \quad (2)$$

where

$$\tau_i(z,t,\lambda)=\int_z^\infty \sigma_i(\lambda)\frac{n_i(z,t)}{\cos\theta(t)}dz. \quad (3)$$

$\sigma_i(\lambda)$ is the cross section for absorption by the ith constituent of radiation of wavelength λ in the region $d\lambda$, F_λ is the incident flux of wavelength λ in the region $d\lambda$ at the top of the atmosphere and ϵ_i is an efficiency factor for the conversion to thermospheric heat of energy in the extreme ultraviolet absorbed by the ith consituent. θ is the zenith angle of the sun.

In our previous paper (1962a), the summation sign in the exponential function of Eq. (2) was accidentally left out in the printing. The correct formula, however, was used in the computer program. The correct formula is also given in our paper which was printed in the proceedings of the Third International Space Science Symposium (Harris and Priester, 1963a).

We summed Hinteregger's measurements of the solar EUV flux from 40 to 1000 Å and used appropriately averaged cross sections for the absorption by the different constituents. This simplification was made after calculations had shown that the temperature profiles of the thermosphere were only slightly affected whether the EUV region was divided into five different regions with appropriate mean cross sections or a proper average over the entire EUV region was used.

The efficiency for the conversion of solar EUV energy into heat was taken to be 37 per cent in close agreement with the result of Lazarev's (1963) recent paper (40 per cent). One should, however, be aware that the uncertainty of the above value is still quite large. Smaller values (15 to 30 per cent) have been recommended by Hanson and Johnson (1961) and Chamberlain (1961). Therefore all arguments about the heating of the thermosphere which are based on the absolute values of solar EUV fluxes must be considered with caution.

In this paper we have investigated the effects of additional heating caused by the absorption of solar radiation in the Schumann-Runge range by oxygen molecules. A detailed discussion of this process and its heating efficiency is given in the next section.

F. S. Johnson argues in a recent paper (1964) that it would not be entirely unreasonable that the generally used values for the heat conductivity might be too large by a factor of three or even ten. We have investigated the influence of different values for the conductivity. A smaller value would decrease the discrepancy in the thermospheric heat budget, but it would increase the discrepancy between the observed and calculated time of the diurnal maximum. This is an argument against considerably smaller values for the conductivity.

A shortcoming of our Eq. (1) is the neglect of horizontal conduction as well as horizontal convection. This will be discussed later. The horizontal conduction depends, of course, on the horizontal temperature gradients. Due to the large distances involved, these gradients are rather small. MacDonald estimates in his recent review (1963) that the average temperature gradient at an altitude of 1000 km is 3×10^{-7} K cm^{-1} for a temperature difference of 600K between the dark and the sunlit side of the earth. The corresponding heat flow is then only of the order of 10^{-2} erg cm^{-2} sec^{-1}.

3. Schumann-Runge absorption

In our previous integrations of the time dependent heat conduction equation we included electromagnetic radiation in the extreme ultraviolet range only (40 to 1000 Å). As the number density of molecular oxygen at the 120-km level is still rather large and the flux in the Schumann-Runge region is large compared to that in the extreme ultraviolet region, such a neglect has been considered as an oversimplification. However, integrations of the time dependent heat conduction equation which includes heating from solar flux in the Schumann-Runge region show that our previous conclusions are not affected.

Walker (1964) has estimated the amount of flux in the Schumann-Runge region that can be optimistically converted into local heating of the atmosphere above 120 km. His conclusion is that at most a flux of 0.5 erg cm^{-2} sec^{-1} can be converted into local heating. We have performed integrations of the time dependent heat conduction Eq. (1) where we have included the absorption of Schumann-Runge radiation by molecular oxygen in addition to the heating due to extreme ultraviolet radiation. We used a cross section for absorption of 1.5×10^{-17} cm^2 (Hinteregger, 1961).

In these calculations we obtain a behavior of the upper atmosphere which is qualitatively similar to our previous results. That is: The temperature still peaks at 17 hours local time and the amplitude of the diurnal variation is much too great.

The inclusion of Schumann-Runge absorption has the effect of steeping the temperature gradient at the lower boundary. This makes it possible to obtain better agreement with the densities measured at the lower boundary altitude by the falling sphere method (Faucher *et al.*, 1963) and still to maintain good agreement with the satellite drag data. Fig. 2 is the comparison of the density profiles at the diurnal maximum (14^h) and minimum (4^h) with and without inclusion of absorption in the Schumann-Runge range.

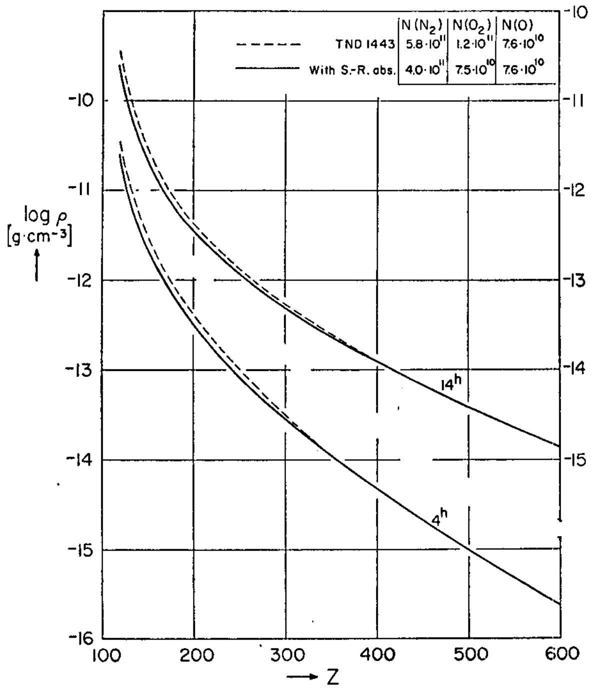

FIG. 2. Density versus height from 120 to 600 km for 14 and 4 hours local time. The dotted line represents our model S=200 (1962a) for mean atmospheric conditions during the fall 1959. The solid lines give the densities when, in addition, the absorption by oxygen molecules in the Schumann-Runge range is included in the calculations. In the legend the boundary number densities of N_2, O_2 and O at 120 km are given.

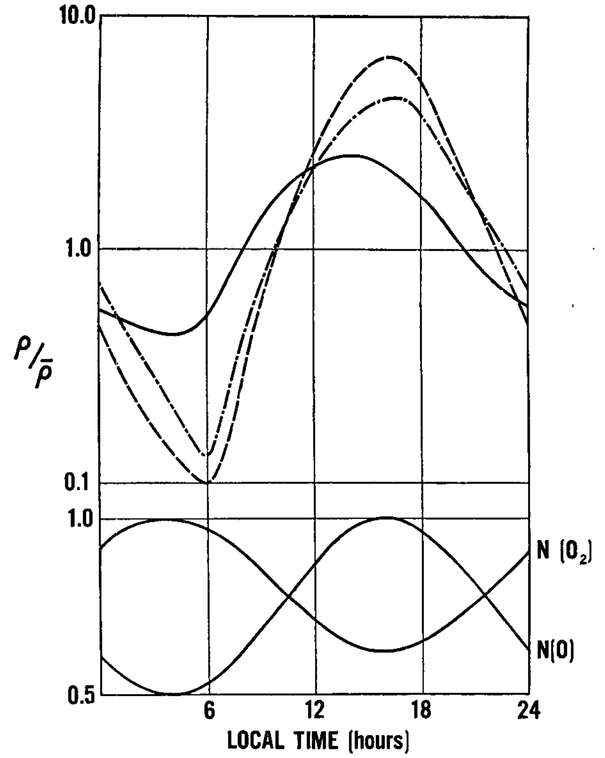

FIG. 3. Diurnal density variation at 600 km calculated with time-dependent boundary conditions. The number densities of O_2 and O at 120 km were forced to vary according to formulae (4) and (5) (see lower part of the figure). The resulting density variation at 600 km is represented by the dashed curve in the upper section of this figure. As heat sources, only the absorption of solar radiation in the EUV and Schumann-Runge range are used. For comparison, the dashed-dotted curve gives the densities when the number densities at 120 km are kept constant at the mean values. The solid line again shows our model S=200, which represents the observed variation in the fall 1959.

4. Variation of the ratio of atomic to molecular oxygen

In all of our previous calculations we have assumed that the number densities of the various constituents at our boundary level (120 km) do not vary within one day. This is reasonable for an example, as the lifetime for recombination of atomic oxygen at this altitude is of the order of years (Nicolet, 1960). Also the lifetime for molecular oxygen due to dissociation is of the order of several days.

Since it has been suggested that the ratio of atomic to molecular oxygen might vary by a factor of three to four over a day-night cycle (MacDonald, 1963; Kallmann-Bijl and Sibley, 1964), calculations have been made to ascertain how this affects the calculated diurnal variation. However, it is difficult to see how such a variation at 120 km could influence the density variations of higher altitudes within one day as the diffusion time at 120 km is of the order of one day. Nevertheless, despite the above objections, we have performed calculations where we have varied the amount of atomic and molecular oxygen sinusoidally and in such a manner that the variation would correspond to photodissociation of molecular oxygen (or recombination of atomic oxygen) and maintain a constant density at 120 km. Thus the variation at 120 km chosen was the following:

$$N(\mathrm{O})=N_0(\mathrm{O})\left[1+\frac{1}{3}\sin\frac{2\pi}{24}(t-10)\right], \qquad (4)$$

$$N(\mathrm{O_2})=N_0(\mathrm{O_2})\left[1-\frac{1}{6}\frac{N_0(\mathrm{O})}{N_0(\mathrm{O_2})}\sin\frac{2\pi}{24}(t-10)\right], \qquad (5)$$

where $N_0(\mathrm{O})$, $N_0(\mathrm{O_2})$ are the diurnal average values of the number density of atomic and molecular oxygen, t is the local time in hours. The ten-hour phase was chosen so as to have the number density of atomic oxygen increasing until 16:00 hours local time. Eqs. (4) and (5) yield a variation of the ratio of atomic to molecular oxygen of about a factor of 3. In the calculations only heating due to extreme ultraviolet and Schumann-Runge radiation corresponding to an average level of solar activity of autumn 1959 was included. Fig. 3 presents the relative variation of the density at 600 km together with the observed variation. For comparison the calculations with constant boundary conditions are also shown. It is noticed that such a variation hardly affects the behavior of the diurnal variation, that is, the calculated variation still peaks at about 17 hours local time with a too large amplitude. Thus such a variation at 120 km cannot account for the diurnal variation observed in the height range from 300 to 700 km. On the basis of these calculations it is also difficult to interpret the rocket measurements of the number densities of atomic and molecular oxygen at 190 km by Hall, Schweizer and Hinteregger (1963) as a diurnal variation.

5. Variation in the turbopause height

The level at which diffusive equilibrium begins is not well known. Furthermore it might be that the height of the diffusive equilibrium level changes from day to night. For the various atmospheric constituents it may vary from about 100 to 120 km. For example if the level for the diffusive separation of atomic oxygen changes by 28 km from 90 to 118 km, then the number density of atomic oxygen would change by a factor of nine at 120 km. We have performed calculations in which we forced the number density of atomic oxygen at 120 km to vary diurnally so as to give good agreement with the observed densities at 600 km. The diurnal variation we used is given in Fig. 4. Again only extreme ultraviolet and the radiation in the Schumann-Runge region were used as heat sources. Fig. 4 shows that a fairly good agreement can be obtained due to the manner in which the number density of atomic oxygen is varied at 120 km. Even perfect agreement could be obtained by an additional slight variation of the boundary conditions. But Fig. 4 also shows that one cannot obtain good agreement simultaneously at lower altitudes between 200 and 400 km. Thus such an assumed diurnal variation of the height of the turbopause cannot account for the observed diurnal density variation in the 300 to 700 km range. Furthermore, since the diffusion time for atomic oxygen at 120 km is of the order of one day, it is difficult to understand how such an effect can be propagated upward within a day without smoothing out the variation at greater heights.

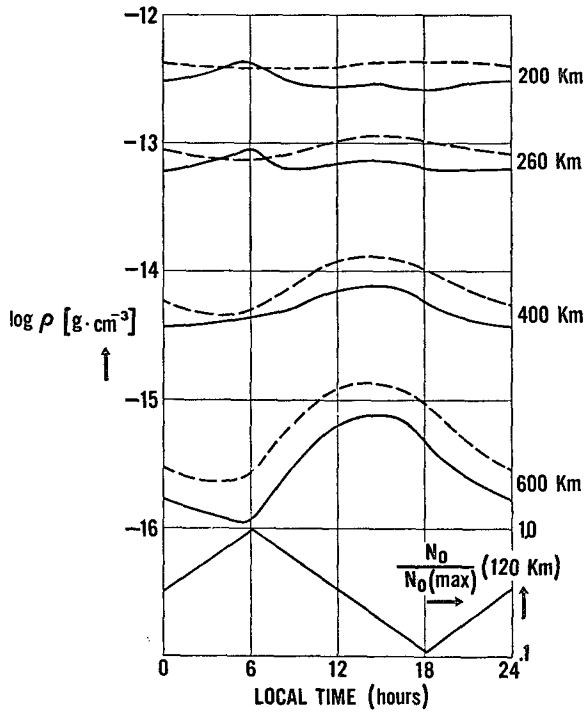

FIG. 4. Diurnal density variations at 200, 260, 400 and 600 km calculated under the assumption that the height of the turbopause (diffusion level) changes diurnally from 90 km at 6 hours local time to 118 km at 18 hours local time. The corresponding variation of the number density of atomic oxygen at 120 km is given in the lower part of the figure. The solid lines in the upper part represent the calculated densities. For comparison again the densities of our model S=200 are given (dashed curves). The agreement at 600 km is good, but agreement at the lower altitudes cannot be achieved simultaneously with height variations of the turbopause.

6. Variation of the boundary temperature

The third type of variation one may attempt to use to explain the observed diurnal variation is the variation of the boundary temperature. As the characteristic conduction time (MacDonald, 1963) at 120 km is large compared to one day, one does not expect any appreciable diurnal variation in the temperature at 120 km.

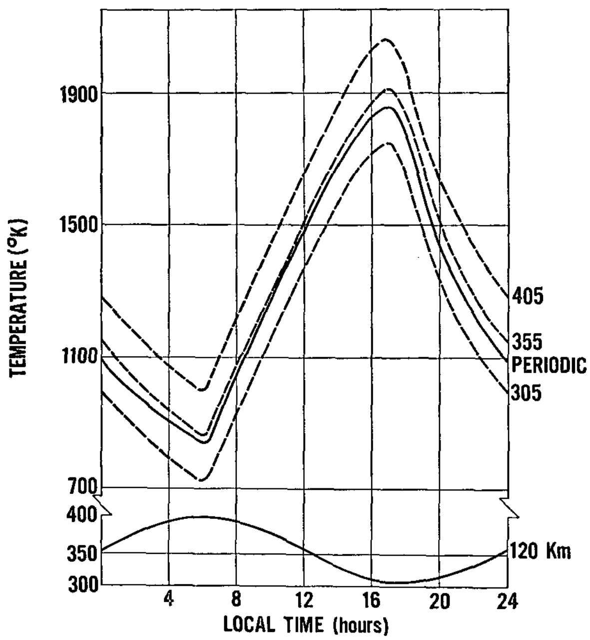

FIG. 5. Diurnal variation of the exospheric temperature for different boundary temperatures at 120 km. As heat sources, only the absorption of solar radiation in the EUV and Schumann-Runge range are used with the same fluxes in all cases. The dashed curves give the temperature variations when the boundary temperature is kept constant at 305, 355 and 405K, respectively. The solid line represents the variation of the exospheric temperature when the boundary temperature at 120 km is forced to vary diurnally as given in the lower part of the figure.

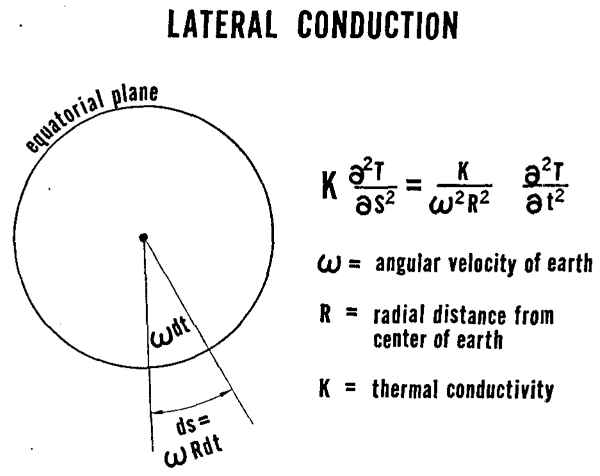

FIG. 6. This figure demonstrates how the lateral conduction in longitude can be included in our Eq. (1) if one takes advantage of a conversion of length units into time units.

Thus any forced variation at the 120-km level, even if the total amplitude would be as large as 100K, would be rapidly damped out with increasing altitude. This is borne out by actual calculations where we have varied the temperature at 120 km sinusoidally with a total variation of 100K from 305 to 405K, peaking at 6 hours local time. Again only extreme ultraviolet radiation and Schumann-Runge radiation were used. In Fig. 5 we compare the exospheric temperature obtained with the results for fixed boundary temperatures of 305K, 355K and 405K. The results for the periodic boundary temperature variation do not deviate appreciably from the results when a constant boundary temperature of 355K is used. This demonstrates that a periodic temperature variation at 120 km cannot propagate rapidly enough to affect the diurnal variation in the upper thermosphere.

Thus diurnal variations of the boundary conditions of the type illustrated above cannot account for the density peaking at 14 hours local time (instead of 17 hours) nor for the much lower observed ratio of daytime maximum temperature (or density) to minimum night-time temperature (or density). Thus the considerations in Sections 3 to 6 of this paper have no influence upon our requirement of an additional heat source which when included well represents the observations.

7. Lateral heat transport

The basic equation (1) depends on height and time only. A complete treatment of two or three dimensions in space is not quite feasible with the high speed computers readily available at the present time. However, lateral heat conduction parallel to the equator can be included in our one-dimensional treatment by taking advantage of the relationship between the longitude and the local time. Lateral conduction is expected to be small as the lateral temperature gradients are small (or the order of the difference between maximum daytime temperature and minimum nighttime temperature divided by the circumference of the earth). At high altitudes, however, the lateral temperature gradient can be comparable to the vertical temperature gradient—but at these altitudes the heat content of the thermosphere is small so that such a lateral heat flow hardly affects the total heat budget in a given column of air. If horizontal conduction would be sufficient to decrease the diurnal amplitude, which is obtained when heating by EUV radiation alone is used, towards the observed value and also sufficient to shift the diurnal maximum towards 14 hours local time, this might offer an immediate explanation of why the local time of the maximum remains close at the same local time (14 hours) for the whole solar cycle. If one calculates the local time of the maximum using only solar EUV radiation as the heat source for the entire solar cycle, by changing the flux parallel to the solar activity one finds that the maximum proceeds from 17 hours for high solar activity towards 15 hours for extreme low activity. Parallel to this, however, the temperature maximum and the diurnal amplitude of the temperature decrease. From the decrease of the latter it is plausible that the effectiveness of horizontal conduction also becomes gradually smaller. So it can only provide a smaller time-shift for the maximum at periods of low solar activity. This just might then result in having the maximum always at about 14 hours. Of course, this requires quantitative proof.

The method we employed to include lateral conduction along the equator is the following. It is illustrated in Fig. 6. A horizontal displacement ds is equivalent to a change in local time dt, thus the lateral temperature gradient dT/ds can be replaced by $[1/\omega(R_0+z)]\partial T/\partial t$, where ω is the angular velocity of the earth and R_0 the radius of the earth, and z the height. Thus the net heat input to a given amount of air $K(\partial^2 T/\partial s^2)$ can be included by adding the term

$$\frac{K(z)}{\omega^2(R+z)^2}\frac{\partial^2 T}{\partial t^2} \tag{6}$$

to Eq. (1). In this term we have ignored the explicit temperature dependence of the thermal conductivity as it is a small correction to a small term. Calculations have been made including this term as a perturbation and with extreme ultraviolet and Schumann-Runge radiation alone. The result is a change of the exospheric temperature by less than 25K, too small to be of any significance. A comparison of the term given in Eq. (6) with the net heat input due to vertical conduction (first term in Eq. (1)) showed that the former term only becomes comparable to the latter for heights above 500 km and times around 06 and 18 hours local time. Below 300 km it is always smaller by more than two orders of magnitude.

But we have, of course, to consider also the meridional component of the heat conduction. This can reasonably be expected to be of the same order as the longitudinal component. Thus lateral heat flow does not affect the gross properties of the upper atmosphere, that is, it does not shift the time of the diurnal maximum, nor does it decrease the ratio of the maximum to minimum temperature appreciably.

Another factor which can be considered to influence the diurnal behavior of the upper atmosphere is lateral convective energy transport. From the temperature gradient one expects speeds of convective flow much too small to furnish any appreciable horizontal energy transport. In order to have an effective horizontal energy transport, one has to require large range flow velocities of the order of 10^4 cm sec^{-1} (MacDonald, 1963).

At present it cannot be proved or disproved whether this effect can be, indeed, large enough to account for a temperature difference of 250K at the bulge maximum between the "observed" temperature and the higher value which has been calculated on the basis of a sufficiently strong EUV heat source (with no other source) and no horizontal convective energy transport. Quantitative proof is needed which is at present not yet feasible in order to see whether horizontal convection can provide the required time shift and flattening of the temperature maximum. As long as this cannot be done one will have to rely on a "second heat source." There seem to be two different ways to interpret this "source":

1) It may be taken as a heat source other than the absorption of solar EUV radiation. This is particularly suggested if one relates the semi-annual variation to the solar wind, since, in this case, one surely would need a "second source," providing about the same amount of heat as the EUV radiation. Furthermore it was recently found that much stronger additional heating of the thermosphere occurs parallel to slight changes of geomagnetic activity during quiet periods than was anticipated before (Jacchia and Slowey, 1964; Newton *et al.*, 1964). This favors the assumption of additional heating other than solar EUV radiation.

2) If one assumes that the solar EUV provides enough energy (a flux up to 6 erg cm^{-2} sec^{-1} for very high solar activity if the efficiency for conversion into heat is 40 per cent or up to 12 erg cm^{-2} sec^{-1} if the efficiency is only 20 per cent), then one might interpret the sum of the EUV heat source and the "correction term" which provides the agreement with the observed diurnal variation as the "effective heat source" of the upper atmosphere. This source then incorporates the horizontal energy transport in such a way that the simplified theory provides the agreement with the observations. This interpretation depends, however, strongly on a quantitative proof of whether horizontal convection is effective enough.

8. Conclusions

In this paper we have investigated several possibilities which could eliminate the requirement for a "second heat source" which we had to introduce in order to reproduce the observed diurnal variation by solving the time-dependent heat conduction equation as a function of height and time only. We were able to show that neither the inclusion of absorption of solar radiation in the Schumann-Runge band by O_2 molecules in our heat source nor diurnal variations of the boundary conditions can be invoked in order to explain the diurnal variation on the basis of an EUV heat source exclusively.

Further, the effect of horizontal conduction is insufficient to account for an energy transport toward the west large enough to explain the observed diurnal variation. A complete three-dimensional quantitative analysis which will eliminate the need of the artifice employed here would be useful. But this requires a much larger and faster computer than presently available.

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