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Variable Stars with Period Greater Than 20 Days in Globular Clusters

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Analysis of irregular, semiregular, and long-period variables in globular clusters is made from published data. Eighty probable cluster members are selected in 31 clusters; the most frequent period is 100 days. Sixteen long-period variables show a period vs maximum luminosity relation similar to that for variables in the field with a peak around 175 days. The variables in globular clusters are brighter, however, by at least $M_{pg}=1.5$ mag. Their mean luminosity decreases with period. Their light amplitude increases with maximum luminosity and decreases with mean luminosity. Both quantities show much scatter for periods greater than 200 days.

Physical properties of the semiregular and irregular variables show little correlation and resemble both red and yellow types found in the field. For the nine RV Tauri stars among this group, however, the luminosity is maximum at a double period of 50 days and decreases linearly with increasing period. Amplitude and luminosity decrease with later spectrum. The correlation between amplitude and luminosity is closer for the RV Tauri variables than for the irregulars and semiregulars.

Long-period variables with periods greater than around 100 days and RV Tauri stars with long periods seem to occur in clusters where RR Lyrae stars are few or absent. These RR Lyrae poor globular clusters appear to have properties which are slightly more like those of old galactic clusters than of RR Lyrae rich globular clusters, in agreement with previous work. In galactic clusters, search shows that there are probably no known variables with $P>20$ days.

I. INTRODUCTION

IN recent years much information has accumulated on variable stars in globular clusters. The literature contains extensive treatment of the RR Lyrae stars in particular, because their magnitudes can presumably give accurate distance moduli. Arp (1955a) has discussed the cluster Cepheids, but very little analysis has been made of the more unusual variables of irregular, semiregular, and long-period classes. This lack has been due partly to insufficient statistics and partly to the difficulty of pursuing lengthy observations.

Apparently the first period–luminosity relation for the variables in globular clusters was obtained by Bailey, Leland, and Woods (1919), in whose list of cluster Cepheids four longer period variables were mistakenly included as long-period Cepheids. Sawyer (1931) plotted 12 variables with period greater than 20 days along with the cluster Cepheids and obtained a monotonic relation. Rosino (1951b), using eight W Virginis stars and ten RV Tauri and semiregular variables, showed the true peaked behavior of the period–luminosity law in this period range. Indeed, Arp (1955a) showed that five RV Tauri stars lie at brighter magnitudes than the type II Cepheids. In more extended plots, Arp (Payne-Gaposchkin 1954; Arp 1958c) showed the same sequence with a continuation into the fainter long-period domain. At present, data are available for still more stars. We wish to investigate in this paper such important relationships as the period–luminosity law, and to conclude with some comment on the presence of these variables in globular clusters.

II. DATA

Numerous lists of the longer period variables in globular clusters have been published in the literature during the past half-century (especially Ludendorff 1928; Shapley 1930; Sawyer 1931, 1939, 1955; Joy 1949; Hogg 1959a). For the sake of completeness, a new search through the literature since 1890 was made, and a bibliography of 179 references to the longer period variables is included as an appendix to this paper. About one-third of these references do not appear in Sawyer's catalogue (1955) or bibliography of globular clusters (1947).

Table I presents the most recent or most reliable data extracted from these references. From a collection of more than 120 variables with "long" periods that have been classified at one time or another as possible members of globular clusters, we have listed only those stars with periods determined or strongly suspected to be greater than 20 days; 105 stars in 38 globular clusters are included. Except for the clusters in the Magellanic Clouds and two intergalactic clusters, NGC 2419 and Abell 4, all of the clusters listed are in the Galaxy. Consecutive columns of the table give the following: NGC number; Messier or other designation; the number in Sawyer's catalogue (1955) or adopted number in order of discovery as published by the discoverer; distance from the cluster center in terms of radii listed by Mrs. Hogg (1959a); apparent photographic magnitude at maximum light; apparent photographic magnitude at minimum light; period in days; type of variable (LP=long-period, SR=semiregular, RV=RV Tauri type, I=irregular); spectral class at maximum (where parentheses indicate a value at an unspecified point in the spectral range); spectral class at minimum; luminosity class; name of star or other designation; discoverer and date; reference numbers in the bibliography for (1) magnitudes, (2) period (or type), (3) spectral and luminosity classes, and (4) discoverer and date. These references are the

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TABLE I. Catalogue of variable stars with period greater than 20 days in globular clusters.

NGC	Cluster name	No.	r	max m_{pg}	min m_{pg}	P (days)	Type	Sp max-Sp min	Lum	Variable name	Discoverer	Date	References
104	47 Tuc	1	0.1	11.3	[16.0	212.40	LP	M2.5e	II–Ib		Bailey	1894	159, 159, 87, 101
		2	0.2	11.55	15.3	202.84	LP	M2.5e	II–Ib		Bailey	1894	159, 159, 87, 101
		3	0.3	11.35	16.1	192.34	LP	M2.5e	II–Ib		Bailey	1894	159, 159, 87, 101
		4	0.1	12.0	14.0	150:	SR or I				Fleming	1894	159, 159, ···, 101
		5	0.3	13.0	14.0		I or SR	M2.5	II–III		Fleming	1894	159, 159, 87, 101
		7	0.3	13.3	13.8		I or SR	M2.5	II–III		Bailey	1902	159, 159, 87, 18
		8	0.0	12.7	14.7	150:	SR or I	M2.5e	II–III		Bailey	1902	159, 159, 87, 18
		10	0.5	13.1	13.6		I			HV 811	Leavitt	1904	85, 85, ···, 70
		11	0.3	13.2	14.0		I			HV 813	Leavitt	1904	85, 85, ···, 70
121	(SMC)	1	0.4	17.5		140.2	LP				Thackeray	1958	69, 166, ···, 166
		8	0.4			112.4:	LP				Thackeray	1958	···, 166, ···, 166
288		1	0.3	13.5	14.1:	103	LP or SR				Shapley	1930	90, 90, ···, 156
362		2	0.4	13.0	14.5	105.22	LP or SR	(M0)		HV 206	Bailey	1895	126, 126, 126, 14
1866	(LMC)		]2.0	12.8	14.25		I			HV 2886	Leavitt	1906	160, 160, ···, 71
			]2.0	14.0	15.15		I			HV 12185	Nail	1950	160, 160, ···, 160
1904	M79	2	0.4	14.2	14.80		I or SR				Bailey	1898	112, 112, ···, 16
1978	(LMC)		[1.5			186	LP				Thackeray	1958	···, 87, ···, 87
2419		1	0.4	17.59	18.32		I				Baade	1935	11, 11, ···, 11
		8	0.2	17.50	18.10		I				Baade	1935	11, 11, ···, 11
		10	0.3	17.31	17.93		I				Baade	1935	11, 11, ···, 11
		18	0.6	17.84	18.53		I				Baade	1935	11, 11, ···, 11
		20	0.3	17.65	18.16		I				Baade	1935	11, 11, ···, 11
3201		68	1.0				LP?			HV 10334	Dowse	1940	···, 174, ···, 43
Abell 4		1	0.2	17.7	20	166.7	LP				Rosino	1957	116, 35, ···, 116
		2	0.6	17.6	19.3	131.6	LP				Rosino	1957	116, 35, ···, 116
4590	M68	27	1.6	10.2	17.4	324.1	LP	M4e		FI Hya, HV 8460	Ritchie	1919	120, 176, 52, 148
4833		2	1.4	13.0	16.2:	333.7	LP			RZ Mus, HV 3693	Bailey	1923	175, 175, ···, 22
		9	0.1	14.5	15.16	87.7:	SR			HV 10781	Wright	1942	175, 175, ···, 175
		11	2.3	14.5	16.0:	303.8	LP			HV 10783	Wright	1942	175, 175, ···, 175
5139	ω Cen	1	0.3	10.7	12.6	58.7027	RV	F6			Bailey	1897	80, 78, 63, 15
		2	0.2	]13.06	16.12	484	LP				Bailey	1897	79, 18, ···, 15
		6	0.2	13.84	15.24		I				Bailey	1897	79, 79, ···, 15
		17	0.3	14.18	14.61	60:	SR or I				Bailey	1897	79, 79, ···, 15
		42	0.0	12.5	14.9	149.4	LP				Bailey	1897	79, 79, ···, 15
		53	0.3	13.30	13.87	87:	SR or I				Bailey	1897	79, 79, ···, 15
		129	0.1	14.18	14.74		I or SR				Bailey	1902	144, 144, ···, 18
		138	0.1	12.5	13.6	74.6:	SR or I				Martin	1937	79, 79, ···, 79
		148	0.2	12.9	13.8	90:	SR or I				Martin	1937	79, 79, ···, 79
		152	0.0	12.8	13.7	124:	SR or I				Martin	1937	79, 79, ···, 79
		161	0.1	13.3	13.8		I				Martin	1937	79, 79, ···, 79
		162	0.2	12.9	13.6		I				Martin	1937	79, 79, ···, 79
		164	0.3	13.7	14.0		I				van Gent	1935	65, 79, ···, 79
							I			Eggen G	Eggen	1961	···, 44, ···, 44
5272	M3	95	0.3	13.73	14.42	103.19	LP or SR	G4e–M2e		von Zeipel 318	Bailey	1897	50, 50, 63, 15
			0.8				slow?	(G5)		von Zeipel 297	Baum	1952	···, 32, 32, 32
			0.8				slow?	(K0)	Ib	von Zeipel 1397	Baum	1952	···, 32, 32, 32
5904	M5	42	0.3	11.20	12.24	51.476	RV	F4–G3e			Packer	1890	144, 63, 63, 94
		50	0.2	14.00:	14.54:	106.0	LP or SR	G4–M0	Ib		Bailey	1897	89, 20, 63, 15
		84	0.1	11.54	12.61	53.0	RV	F5e–G5			Packer	1890	144, 63, 63, 94
6093	M80	6	3.9	9.3	15.8	177.41	LP	M3e		S Sco			144, 176, 82, ···
		7	3.4	9.5	15.5	222.61	LP	M3e		R Sco			144, 176, 82, ···
			2.7	12.0	16	199.4	LP			TW Sco, HV 1048	Leavitt	1904	176, 24, ···, 70
			6.9	12.9	[14.9	151.3	LP			UY Sco, HV 1051	Leavitt	1904	176, 24, ···, 70
			3.9	14	[16	257.6	LP			VW Sco, HV 1057	Leavitt	1904	176, 24, ···, 70

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TABLE I (*continued*)

NGC	Cluster name	No.	r	max m_{pg}	min m_{pg}	P (days)	Type	Sp max-Sp min	Lum	Variable name	Discoverer	Date	References
6121	M4	4	0.6	11.0	12.5	65:	SR	G2–G8			Leavitt	1904	144, 51, 63, 70
		13	0.4	12.37	13.08		I or RV	G3–K0			Leavitt	1904	144, 63, 63, 70
6171		1	2.3	12.5	[16.75	197:	LP			V720 Oph	Oosterhoff	1938	177, 177, ···, 88
6205	M13	10	0.1	13.1	14.0		SR or RV	G2–K0		Ludendorff 487	Sawyer	1940	132, 63, 63, 132
		11	0.1	12.9	13.8	92.5	RV	G2–K0	Ib	Ludendorff 324	Sawyer	1940	132, 144, 63, 132
		15	0.2	13.32	13.67		I			Ludendorff 835	Arp	1953	144, 144, ···, 144
6229		22	0.1	15.2	16.3		slow?				Sawyer	1953	143, 143, ···, 143
6254	M10	1	0.1	13.2	13.8		I or RV	G0–G4			Sawyer	1933	130, 63, 63, 127
6341	M92		0.9				LP?	(F5)		Walker VII–18	Baum	1952	···, 171, 32, 32
			0.9				LP?			Walker III–13	Walker	1955	···, 171, ···, 171
							LP?			Walker X–49	Walker	1955	···, 171, ···, 171
6356		1	0.3	16.3	17.2		I or LP				Sawyer	1953	143, 124, ···, 143
		2	1.4	16.8	17.1		I or LP				Sawyer	1953	143, 124, ···, 143
		3	0.5	16.0	[17.5		I or LP				Sawyer	1953	143, 124, ···, 143
		4	1.8	15.9	[17.5		I or LP				Sawyer	1953	143, 124, ···, 143
		5	2.8	15.7	[17.5		I or LP				Sawyer	1953	143, 124, ···, 143
6397		1	0.9	11.2	16.0	314.6	LP				Bailey	1898	126, 126, ···, 16
		2	0.9	13.8	14.8	52:	RV				Bailey	1898	126, 126, ···, 16
6426		14	6.8	15.15	15.85		SR				Grubissich	1958	53, 53, ···, 53
6522		7	1.8	17.02	17.61		I	(M6)		Gaposchkin 172	Baade	1953	144, 144, 86, 144
				17.02	17.80	196.5	LP	(M6)		Gaposchkin 214	Gaposchkin	1955	48, 48, 86, 48
6528				16.76	[18.62	360.0	LP			Gaposchkin 88	Gaposchkin	1955	48, 48, ···, 48
				17.78	18.57	290.0	LP or SR	(M8:)		Gaposchkin 147	Gaposchkin	1955	48, 48, 86, 48
6541		1	0.2	12.5	[16		LP?				Woods	1922	173, 173, ···, 173
6553		4	0.2			]100	LP or SR				Thackeray	1951	···, 165, ···, 165
		5	0.8			]100	LP or SR				Thackeray	1951	···, 165, ···, 165
6626	M28	6	0.1	14.3	15.2	] 40	I or SR				Bailey	1898	140, 140, ···, 16
		10	0.3	13.5	14.6		slow				Sawyer	1949	140, 140, ···, 140
6656	M22	5	0.4	12.0	12.8		I or RV	G0–G6e		Chevalier 158	Bailey	1897	137, 63, 63, 15
		8	0.1	12.0	12.7	67:	RV	G2–G5e		Chevalier 382	Bailey	1898	137, 137, 63, 16
		9	0.4	12.7	13.3	87.71	RV	G2–G5		Chevalier 135	Bailey	1898	137, 137, 63, 16
		14	1.1	13.8	[15.5	200.2	LP	M3.5e			Bailey	1898	153, 137, 33, 16
		17	0.9	14.6	[15		I?			HV 4005	Swope	1927	153, 137, ···, 153
6712		2	0.6	14.0	14.9	105:	RV			AP Sct, HV 3832	Cannon	1924	143, 91, ···, 36
		7	1.0	14.2	[17.0		LP?				Sawyer	1953	143, 144, ···, 143
6715	M54	25	0.6	15.4	17.2	101:	SR				Rosino	1952	119, 119, ···, 111
		64	3.4	16.7	17.5		I or SR				Rosino, Nobili	1959	119, 119, ···, 119
		66	1.9	15.6	17.1		slow				Rosino, Nobili	1959	119, 119, ···, 119
		71	0.7	14.8	16.2	77:	SR				Rosino, Nobili	1959	119, 119, ···, 119
6779	M56	3	0.6	14.4	15.1	101.67	SR	F8–G5		Küstner 337	Davis	1917	108, 63, 63, 41
		5	0.9	14.4	15.2	43:	SR	F6–G4e		Küstner 305	Sawyer	1940	108, 108, ···, 132
		6	0.2	12.9	14.8	90.02	RV	F6–G4e		Küstner 284	Sawyer	1940	108, 141, 63, 132
		7	2.4	15.6	16.3	45:	SR			Küstner 504	Sawyer	1940	108, 108, ···, 132
		8	2.3	15.9	16.7	68:				Küstner 150	Sawyer	1940	108, 108, ···, 132
		9	3.7	15.6	16.1		SR				Sawyer	1940	144, 108, ···, 132
6838	M71	1	0.8	13.5	14.9	]100	LP or SR	(M4)		Z Sge		1912	143, 142, 161, 176
		2	0.8	13.8	14.7	]100	LP or SR			Baade 48	Baade	1928	143, 142, ···, 10
6981	M72	26	0.7	16.90	17.20		I				Rosino	1953	114, 114, ···, 114
		33	0.4	16.95	17.25		I				Rosino	1953	114, 114, ···, 114
7006		19	0.4	]16	18.5:	252:	LP				Hubble	1954	118, 123, ···, 123
7089	M2	11	0.2	12.5	13.8:	67.1:	RV	F5–G3e			Chèvremont	1897	141, 141, 63, 38
7789			1.5	10.8	[16	477.4	LP	(Se)		WY Cas		1918	81, 81, 81, 176

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original sources if the data have not been improved since then. Unless otherwise specified, the discoverer was assumed to be the writer of the paper in which the variable is first mentioned. In general, the reference that gave the magnitudes or discoverer gave also the coordinates from which r was computed; otherwise, Sawyer (1955) was consulted. Since we used the cluster radius only as a rough guide to membership of the variables, Hogg's composite list of cluster radii was sufficiently accurate. Kron and Mayall's values (1960) were not numerous enough for this purpose, but were used for the Magellanic clusters NGC 121 and 1978, neither listed by Hogg.

To eliminate probable field variables, we neglected (according to frequent procedure) all stars in Table I that had $r>1.5$. However, very few of the stars were found to lie in the range $1<r<2$. In fact, only two stars, No. 27 in NGC 4590 and No. 14 in NGC 6656, lying near the critical distance had to be eliminated [according to Sawyer (1955) these two stars definitely belong to the field]. Otherwise, we might extend our limit to $r<2$ without affecting the conclusions in this paper. Sawyer (1955) also considers Nos. 1 and 2 in NGC 6397 possible field stars and a glance at the absolute magnitude of No. 2 ($M_{pg}=+0.8$; cf. Fig. 7) is almost certainly sufficient to place this star in the field.

As a curiosity, all the variables with period greater than 20 days listed in the Russian catalogue of variable stars in galactic clusters (Kholopov 1956) were checked for membership. Distance moduli given by Johnson *et al.* (1961), Trumpler (1930), or Barkhatova (1950) yielded absolute magnitudes at maximum much fainter than $M_{pg}=+1.0$ for all the variables except three, which had $M_{pg}\approx+0.1$, -3.7, and -4.8, respectively. Moreover, all except two (not among the above three) lay at $r>1$—far, if we consider the looseness of galactic clusters. We can say that probably all 28 variables with period greater than 20 days listed in the Russian catalogue are field stars.

The intermediate-age cluster NGC 7789 has recently been suspected of containing a long-period variable (Mavridis 1961). This cluster has been included in Table I because it appears to represent a transition case between a galactic and globular cluster (Arp and Cuffey 1962; Arp 1962). The variable is of spectral class S. Thus it is unique and important from the standpoint of cluster evolution.

Altogether, 80 variables, almost certain cluster members, have been selected from 31 clusters and used in the present investigation. Those for which periods are available have been summed in a histogram at 10-day intervals of period (Fig. 1). A strong tendency exists to cluster at integral multiples of 50, at least up to 250 days, with a peak occurring around 100 days. One hundred days appears to be the approximate period at which we cease to find RV Tauri and semi-regular variables, but at which long-period variables may undergo incipient pulsations, as is seen in Sec. IV.

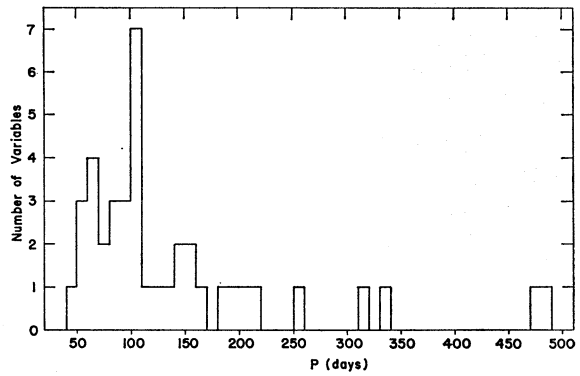

FIG. 1. Frequency distribution of variables with period greater than 20 days in globular clusters. (RV Tauri stars are included with their double period.)

III. DISTANCE MODULI

The best method to determine the apparent distance modulus of a cluster is still by averaging the magnitudes of its RR Lyrae stars at mean light. Whenever possible, this method was used and $M_{pg}^*=0.0$ was consistently chosen for the absolute magnitudes of these stars. (In this paper we shall denote by an asterisk the average of some mean stellar quantity for a group of stars. This mean quantity alone will be denoted by a bracket $\langle\ \rangle$ and the average for a group of stars by a superscript bar; thus * is equivalent to $\overline{\langle\ \rangle}$. Furthermore, all magnitudes refer to maximum light unless they have brackets or asterisks.) All the values of $(m-M)_{pg}$ were taken from Hogg (1959a) whenever they were well determined by the RR Lyrae stars compiled in her catalogue (Sawyer 1955); she assumed that $M_{pg}^*=0.0$. Otherwise, they were obtained in the manner indicated in Table II.

Various attempts have been made in the literature to fit the color–magnitude diagrams of globular clusters to assumed "age zero" main sequences (e.g., Arp 1959). These and semiempirical formulas based on color and period of the RR Lyrae stars (e.g., Sandage in O'Connell 1958; Kinman 1959) have not been entirely successful and unambiguous in determining M^* for the RR Lyrae stars in a cluster. Therefore, our use of $M_{pg}^*=\text{const}$ is only reasonable at present. For simplicity we have assumed the constant to be zero; the systematic effect of changing this value is discussed in Sec. IV.

IV. LONG-PERIOD VARIABLES

The first long-period variables in globular clusters were discovered in NGC 104 (47 Tuc) by Bailey in 1894. It has been known for a long time that these variables and similar ones in several other clusters are brighter than corresponding field variables (e.g., Joy 1949), and they have been often ascribed to the field for just this reason. Now, however, radial velocities present contradictory evidence in some cases. Moreover, geometrical positions relative to the cluster center and

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TABLE II. Apparent distance moduli of globular clusters.

NGC	$(m-M)_{pg}$	Wt	Method	Reference
104	14.7	1	bright stars	Shapley 1941
	14.5	1	25 brightest stars	Hogg 1959a
	14.1:	2	horizontal branch	Wildey 1961
	14.0	1	2 bright RR	Feast & Thackeray 1960
	14.3	···	adopted	
121	19.1	···	RR	Thackeray & Wesselink 1955
288	15.7	···	25 brightest stars	Hogg 1959a
MC	19.2	4	RR in NGC 121, 1466, 1978	Thackeray & Wesselink 1955
	19.2	1	novae	Schmidt 1957
	18.9	1	globular clusters	Kron & Mayall 1960
	18.8	1	av lum. classif.	van den Bergh 1960
	18.6	2	av SMC Cepheids	Arp & Kraft 1961
	19.0	···	adopted	
1866	19.0	···	MC (adopted)	
1978	19.2	1	2 RR	Thackeray & Wesselink 1955
	19.0	1	MC (adopted)	
	19.1	···	adopted	
2419	19.4	···	RR (revised)	Kron & Mayall 1960
Abell 4	20.7:	1	horizontal branch	Burbidge & Sandage 1958
	20.3	1	bright stars	Rosino 1957
	20.5	···	adopted	
6229	17.5	···	RR	Mannino 1960
6254	15.2	···	horizontal branch	Arp 1955b
6397	13.0	···	horizontal branch	Swope & Greenbaum 1952
6426	17.8	···	RR	Grubissich 1958
6528	18:	···	integrated magnitude	Shapley & Sawyer 1929
6541	14.7	···	25 brightest stars	Hogg 1959a
6553	17.1	···	diameter	Shapley & Sawyer 1929
6715	17.1	···	RR	Rosino & Nobili 1959
6838	14.6	···	2 assumed RR	Hogg 1959a
7006	19.0	···	RR	Rosino & Mannino 1955
7789	12.5	···	photometric	Johnson *et al.* 1961

statistical counts of field variables also indicate the real cluster membership of many of these variables.

Few variables of very long period are found in globular clusters, however. In fact, the average period of cluster long-period variables is only about 200 days (see Table III). The most frequent period is about 100 days, whereas in the field the most frequent period is 300 days (Merrill 1960).

The shape and slope of the period–luminosity relation are very close to those for the field, however. Twelve long-period variables and four other suspects with periods around 100 days are plotted in Fig. 2 as filled circles; semiregular variables without known RV Tauri characteristics and with periods greater than 90 days are plotted as open circles. Two lines of opposite slope are indicated by the long-period variables. The one joining the stars of period around 200 days and greater seems to be clear, whereas those stars of period less than about 200 days present an apparent scatter. However, the stars around 100 days are of spectral class M0 at minimum and do not obey strict regularity; it is well known that the yellow semiregular variables which merge with the long-period variables around 100 days exhibit a wide range in magnitude (see Sec. V). Undoubtedly a small intrinsic difference of M_{pg}* of the RR Lyrae stars from cluster to cluster exists in addition to the small intrinsic differences that may exist for the other variables at a given period. These differences would be due to differences in cluster ages and chemical compositions.

The intersection of the two cluster lines at about 175 days is in good agreement with data based on statistical parallaxes of the long-period variables in the solar neighborhood. Wilson and Merrill (1942) wrote the classical paper on the space motions and absolute visual magnitudes of the long-period variables. Other investigations followed, but most recently Osvalds and Risley (1960) made a redetermination of these magnitudes and found not too great departures from the previous work except at the longer periods. Wilson and

TABLE III. Mean quantities for the variables with $P>20$ days.

Type		No.	P range	$\bar{P}$	$\overline{\text{Sp range}}$	Sp*	$\bar{M}_{pg}$	M_{pg}*	$\bar{A}_{pg}$
LP		18	103–484	210	···	···	−2.3	−1.0	2.8
	Mira	9	192–484	295	···	···	−2.4	−0.4	4.0
	Long-P	9	103–167	124	···	···	−2.2	−1.6	1.4
RV		9	51–105	75	F7.8–G5.0	G1.2	−3.0	−2.4	1.1

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Merrill, however, had pointed out that the largest scatter in magnitudes lay at the shorter periods, as we have found for the variables in the globular clusters. Both of the determinations are therefore used in the following discussion.

To compare the cluster variables with the field variables, we need the color index as a function of period (or of spectrum, since the above two papers give a period–spectrum relation). However, this function is not exactly known; following Arp (1958c), we have assumed a constant value of $C.I.=+2.0$ to illustrate diagrammatically the evident *systematic* luminosity excess in the clusters. In Fig. 2, Wilson and Merrill's data are represented by the dashed line, and Osvalds and Risley's data by the solid line.

Using the observational quantities at present available, we can write an equation expressing more exactly this luminosity difference. If we let f refer to a field variable and c to a cluster variable, then the absolute

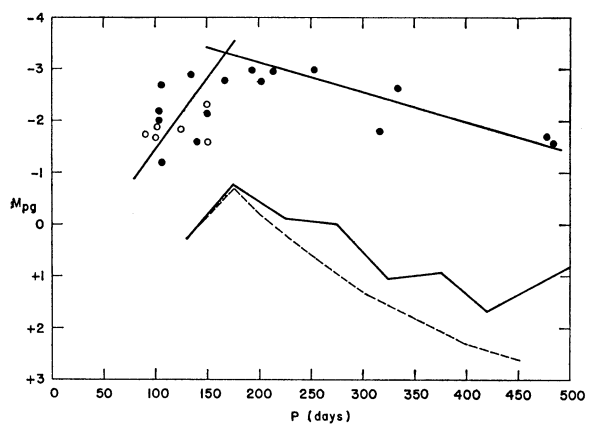

FIG. 2. Period is plotted against maximum magnitude for LP variables (solid circles) and long-period SR variables (open circles). Depressed dashed and solid lines refer to the mean relation determined for field variables by Wilson and Merrill (1942) and Osvalds and Risley (1960), respectively.

magnitude excess, taken at a given period, is

$$\begin{aligned}(\Delta M_{\mathrm{pg}})_{\mathrm{LP}} &= (M_{\mathrm{pg}})_{\mathrm{LPf}} - (M_{\mathrm{pg}})_{\mathrm{LPc}} \\ &\approx [(M_{\mathrm{v}})_{\mathrm{LP}} + (C.I.)_{\mathrm{LP}}]_{\mathrm{f}} \\ &\quad - [(m_{\mathrm{pg}})_{\mathrm{LP}} - (m_{\mathrm{pg}}{}^{*})_{\mathrm{RR}} \\ &\quad + \{(M_{\mathrm{v}}) + (C.I.)\}_{\mathrm{RR}}{}^{*}]_{\mathrm{c}}.\end{aligned}$$

The use of this formula allows us to find the minimum ΔM_{pg} possible in view of the foregoing discussions. For example, we select variable No. 19 in NGC 7006, with a period of 252 days. According to Osvalds and Risley (1960), a star of this period should have $M_{\mathrm{v}}=-2.1$. Color indices are not well known for these stars, but we shall assume the unfavorably low value of $+1.7$. Let us also choose an unfavorably faint value for the absolute visual magnitude of the RR Lyrae stars, $M_{\mathrm{v}}=+0.8$. With a color index for them of $+0.3$, we eventually arrive at $(\Delta M_{\mathrm{pg}})_{\mathrm{LP}} \geq 1.5$. We emphasize that the formula for ΔM_{pg} applies only to comparison of an

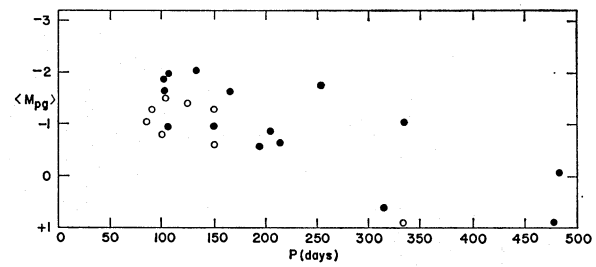

FIG. 3. Period is plotted against mean magnitude for LP variables (solid circles) and long-period SR variables (open circles).

individual variable with the mean of field variables of the same period. Figure 3, however, shows that ΔM_{pg} is roughly constant for $100<P<500$ days. Moreover, if a mixture of variables with properties similar to the globular cluster population occurs in the field investigations, a removal of these pure Population II objects could only serve to increase the luminosity excess.

In general, the giants in globular clusters are known to be more luminous than giants of the same mass but with more normal composition because of a metal-deficiency in the former (cf. top three rows of Table VI). Since the *TiO* bands are ordinarily weak, well-determined M-type spectra are rare in the globular clusters (except for the variables in NGC 104). Sandage (Thackeray 1959) pointed out that since the abundance of *TiO* is proportional to the square of the metal content, the low-metal clusters might not have *TiO* although the temperatures of the reddest giants are certainly low enough. In fact, Baum (1952) found that the reddest giants in NGC 5272 and 6341 have a color surplus, the spectral types being far earlier than those predicted by their colors. Probably metal deficiency in the globular clusters and in particular the apparent weakness of *TiO* in their red variables can together explain the color surplus and luminosity excess.

In Fig. 3 the well-known decrease of mean absolute magnitude with advancing period is plotted for the long-period variables. The peak occurs at 100 days (cf. Arp 1958c).

Photographic light amplitude has been plotted in Fig. 4 against period for the long-period variables

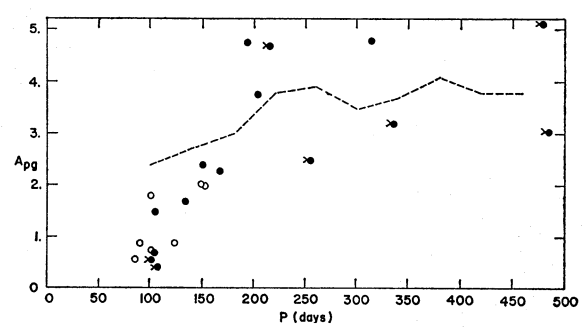

FIG. 4. Period is plotted against light amplitude for LP variables (solid circles) and long-period SR variables (open circles). Dashed line refers to mean of field variables, according to Merrill (1960). Arrows indicate minimum amplitudes.

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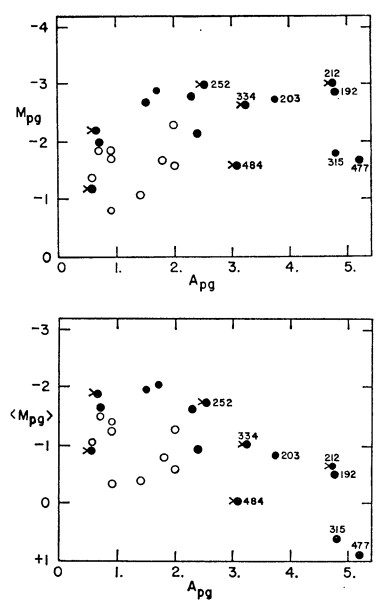
(attached arrows indicate that the amplitudes may be greater than as plotted). The rise of amplitude to 150–200 days and then an apparent, but roughly constant, scatter are similar to the results of Merrill's work (1960) on long-period variables in the field. Merrill averaged what we refer to as long-period variables (his Mira and SRa variables) to obtain extreme photographic light ranges, illustrated by dashed lines on our Fig. 4. He gives no mean light ranges for the photographic magnitudes, but since his visual means are roughly 1 mag. fainter than his visual extremes, the relation he would obtain might more nearly agree with our points up to 150–200 days.

Although Fig. 4 shows a scatter beyond 200 days, maximum amplitude seems to occur at about that period, just where the stars of highest luminosity fall in Fig. 2. The relation between amplitude and maximum luminosity is shown in Fig. 5 and corroborates this finding; the longer periods are marked for convenient identification. Baum (1952) and Walker (1955) have found that the reddest bright giants in NGC 5272 and 6341 are slightly variable with a long period. By extrapolation of our data, their small amplitudes may mean periods between 50 and 100 days. Their luminosities are all comparable and average $M_{pg}=-1.6$; this finding is completely consistent with Fig. 5.

However, amplitude is inversely correlated with *mean* magnitude (Fig. 6). This relation shows up the extraordinarily large amplitudes of the variables with period greater than about 175 days. In fact, the long-period variables in the field are often divided into two groups on the basis of light amplitude: the variables with $A_{pg}>2.5$ mag. are called Mira stars, and those with smaller A_{pg} are called ordinary long-period variables (Ledoux and Walraven 1958) or SRa variables (GCVS 1958). On this basis, we see from Fig. 4 that all the stars with periods greater than about 175 days are Mira types, whereas those with smaller periods belong to the ordinary long-period group.



FIGS. 5 and 6. Light amplitude is plotted against magnitude for LP variables (solid circles) and long-period SR variables (open circles). Arrows indicate minimum amplitudes. Numbers refer to the period in days.

V. SEMIREGULAR VARIABLES

Unlike the long-period variables, the semiregular variables are not infrequent in globular clusters. In 1890 Packer discovered the first two, located in NGC 5904 (M5). These two objects are actually RV Tauri stars, which we define as a subclass of the yellow semiregulars (F, G, or K spectral class) and find characterized by alternating shallow and deep minima, which occasionally interchange (Payne-Gaposchkin, Brenton, and Gaposchkin 1943). The yellow and red semiregular variables are fainter than the RV Tauri stars, but unless their spectra are observed, the red ones cannot always be distinguished from the RV Tauri stars by means of their light curve alone. Since red semiregulars seem to be associated with type I populations, their true frequency in globular clusters must be small or even nil, except for four of spectral class M2.5 in the abnormal cluster NGC 104. However, NGC 104 is unusually strong in metals, and although it is unquestionably globular, many of its characteristics closely resemble those of old galactic clusters like M67 (Wildey 1961). The red semiregulars are very similar to the red giants in these clusters.

Since few spectra of the purely semiregular and irregular variables are yet available, we shall confine our attention primarily to the RV Tauri stars. Nine of these are known in globular clusters and have double periods ranging from 51 to 105 days. We have plotted the customary double period between deep minima against maximum magnitude in Fig. 7. The purely semiregular variables (open circles) show a great scatter in magnitude except for a clustering around 100 days, as is found for the field (Ledoux and Walraven 1958). On the other hand, the RV Tauri stars (dotted circles) define a fairly clear relation, dropping to lower magnitudes at greater periods. The highest luminosity

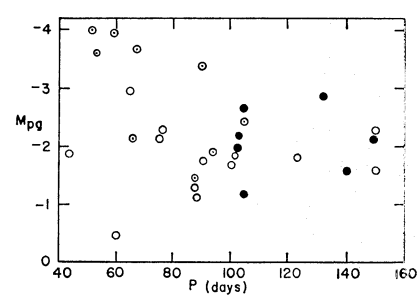


FIG. 7. Period is plotted against maximum magnitude for RV (dotted circles), SR (open circles), and short-period LP (solid circles) variables.

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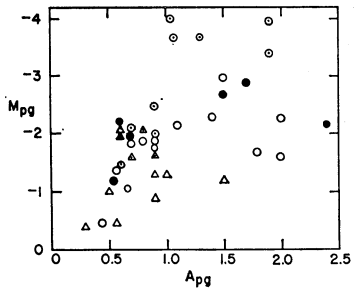

FIG. 8. Light amplitude is plotted against maximum magnitude for RV (dotted circles), SR (open circles), short-period LP (solid circles), and I (triangles) variables. Possible RV variables are denoted by dotted triangles.

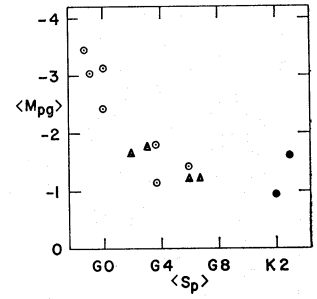

FIG. 10. Mean spectrum is plotted against mean magnitude for RV variables. Same symbols as in Fig. 8.

occurs at 50 days. At present, the comparison of these stars with the score of RV Tauri stars found in the field is impossible, although the rough data available (Rosino 1951b) suggest that the field stars do fall slightly in luminosity class with increasing period. However, it should be remembered that the RV Tauri population in the field probably represents a broader spread of ages, chemical compositions, and hence luminosities.

Plotting period against mean magnitude yields a line of slightly lesser slope displaced about half a magnitude fainter than the line suggested by Fig. 7. Therefore the amplitudes are almost constant with period. Moreover, they are all less than 2 mag., in agreement with data found for field variables (Ledoux and Walraven 1958). The correlation of amplitude with luminosity is shown in Fig. 8. The symbols of Fig. 7 pertain here, but in addition the open triangles represent irregular variables and the dotted triangles possible RV Tauri stars (according to sources in Table I). Some of the irregular variables listed in Table I have been omitted for the sake of clarity. Since amplitude is apparently not closely related to period, such a scatter in the amplitude–luminosity plot is not surprising. Nevertheless, the variables with greater regularity seem to show a better correlation than those with less regularity.

For the RV Tauri stars, amplitude increases with earlier spectrum, as shown by Fig. 9. A division of the RV Tauri stars into two groups based on spectrum, luminosity and amplitude is apparent from Figs. 8 and 9; we shall comment on this division below. Since plots using mean values of spectrum and luminosity gave results very similar to those using maximum values, they are not presented, except for Fig. 10.

We now note on the H-R diagram for the RV Tauri stars (Fig. 10) that the long periods (100 days) and small amplitudes occur at later spectral classes, where the variables that we have denoted as possible RV Tauri stars also fall. Small-amplitude irregulars appear to belong to the red-giant branch of globular clusters, as do the incipient long-period variables of small amplitude and probably short period (Baum 1952; Walker 1955). Both these irregular and long-period variables occur in the same magnitude range as the small-amplitude RV Tauri stars (−1 to −2 photographic; see also Fig. 3). Thus there is an apparent offshoot from the giant branch at this point. It rises to higher magnitudes and earlier spectral types, where the RV Tauri stars with shorter periods and slightly greater amplitudes are situated.

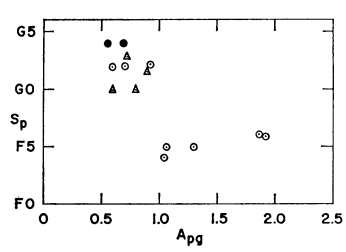

FIG. 9. Light amplitude is plotted against spectrum at maximum for RV variables. Same symbols as in Fig. 8.

VI. CONCLUSION

Period–Luminosity Law

Rosino (1951b) found that in the globular clusters the W Virginis stars, RV Tauri stars, and yellow semiregulars like AG Aur form a sequence on a period–luminosity plot, if the single period of the RV Tauri stars is used. He found a peak in the luminosity for the RV Tauri stars of single period about 35 days; our data show the peak at a period closer to 25 days. We have also extended the sequence into the long-period domain. This whole system of curves is shown in Fig. 11, where we have included Petit's data (1958) for 25 Cepheids with periods ranging from 1 to 20 days, and marked the position of the RR Lyrae stars. Arp (1955b)

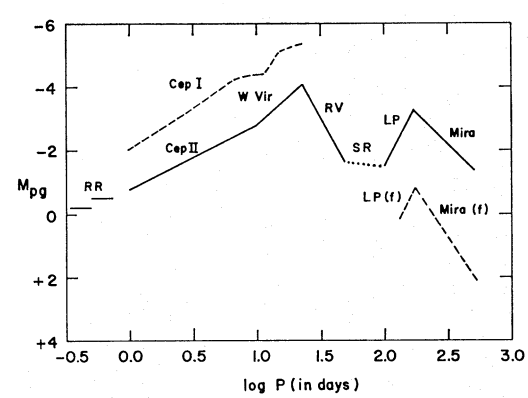

FIG. 11. Period–luminosity relation for the variables in globular clusters (solid lines) and in the field (dashed lines).

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TABLE IV. Correlations between the variables with $P>20$ days and the RR Lyrae population in globular clusters.

RR Lyrae poor clusters	RR Lyrae rich clusters
LP present	LP relatively uncommon
long-period LP	100-day LP
long-period RV	short-period RV
long-period RR a, b?	···
NGC 104, (121), (1978), Abell 4, 4833, 6205, 6553, 6712, 6779, 6838	NGC 3201, 4590, 5024, 5272, 5904, 6121, 6715, 6981, 7006, 7078
Intermediate: NGC 362, 5139, 6656, 7089	

studied thirteen of these Cepheids and showed that five with $1<P<12$ days are type II Cepheids, ordinarily characterized by large light amplitudes. The W Virginis stars reach their peak characteristic of broad maxima at 15 days, whereas those with periods around 20 days often show incipient RV Tauri behavior.

For comparison with field stars, we have used data given by Ledoux and Walraven (1958) for the classical Cepheids and by Osvalds and Risley (1960) for the long-period variables. Just as in Fig. 2, the latter data have been transformed by assuming $C.I.=+2.0$. The field stars are indicated by dashed lines in Fig. 11. As mentioned before, the data do not warrant plotting a relation for the RV Tauri stars and semiregulars in the field; all we can say is that from Joy's work (1952) they are probably members of a mixed population.

It must be emphasized that all these variables have characteristic and not necessarily correlated positions on the color–magnitude diagram, since they occur at different evolutionary phases. For example, the RV Tauri stars and Cepheids lie close together at high magnitudes, whereas the long-period variables are displaced to fainter magnitudes and farther to the red along the giant branch.

Correlations with Integrated Properties of Globular Clusters

Apparently the frequency of long-period variables is correlated with the frequency of RR Lyrae stars in a given cluster, in the sense that more long-period variables occur in the absence or with the paucity of RR Lyrae stars. Moreover, in this case the periods of the long-period variables and RV Tauri stars are relatively long; even the few RR Lyrae stars that occur are usually, although not always, of the long-period Bailey types *a* and *b*. Significantly, clusters rich in RR Lyrae stars tend to have long-period variables of shorter period (100 days) or none at all; the only exception appears to be NGC 7006. The correlations are indicated in Table IV, along with the cluster NGC numbers. Clusters NGC 362, 5139, 6656, 7089 seem to be intermediate between the two groups. Since the RV Tauri stars and long-period variables at maximum light are relatively bright in globular clusters, it is probable that in the better observed clusters, all large-amplitude variables with periods greater than 20 days have already been picked out.

Although little or no correlation appears to exist with diameter and integrated magnitude, RR Lyrae poor clusters tend to have later spectra and less concentration than the RR Lyrae rich clusters (see Table V). Kinman (1959), among others, has shown that later spectra indicate higher metal content. Therefore, if the cluster is open and has a late-type giant branch, the greater is the chance of finding the longer period long-period variables, which usually have a Population I composition in the field. But, in short, these are criteria for the old galactic clusters. In this connection, we note the presence of the 477-day variable in the transition cluster NGC 7789.

Even clearer are the following correlations with integrated spectrum and concentration class: F3–5 and A–B (RV clusters, omitting NGC 6712 with the 105-day

TABLE V. Correlations between the integrated properties and the RR Lyrae population of globular clusters.[a]

RR Lyrae poor			RR Lyrae rich			Intermediate		
NGC	Sp	Conc	NGC	Sp	Conc	NGC	Sp	Conc
104	G3	III ···	3201	···	X ···	362	F8	III ···
(121)	F7.5[b]	··· (B)	4590	F0[d]	X B	5139	F7	VIII ···
(1978)	···	··· A	5024	F4	V A,B	6656	F5	VII B
Abell 4	···	XII ···	5272	F7	VI B	7089	F3	II A
4833	···	VIII ···	5904	F5	V B			
6205	F5	V A	6121	(G0)[c]	IX A, B			
6553	(G5)[c]	XI A	6715	F7	III C			
6712	G4	X: B	6981	G0–1	IX A			
6779	F5	X B	7006	F3–4	I B			
6838	G6[c]	X (A)	7078	F3	IV C			
Mean	G1	IX A, B	Mean	F6	VI B	Mean	F6	V A, B

[a] Unless otherwise noted, spectra are due to Kinman (1959), who used $CH/H\gamma$ ratio. Concentration classes are due to Hogg (1959a) and Kron and Mayall (1960), respectively.
[b] Thackeray (1959).
[c] Kron and Mayall (1960), where a value in parentheses is assumed.
[d] Mean of values given by Kinman and by Kron and Mayall.

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TABLE VI. Correlated characteristics of globular clusters.

Characteristic	Group I	Group II	Group III
spectra (individual)	almost normal (A)	more metal-weak (A,B)	very metal-weak (B,C)
spectrum (integrated)	late (G4)	intermediate (F6)	early (F2)
giant branch	faint (2.2)	intermediate (2.5)	bright (3.0)
		redder	bluer
		asymptotic limit	less bending
horizontal branch	red	equal	blue
RR Lyrae stars	few	...	...
	...	rapid a, b (0.55 day)	slow a, b (0.64 day)
LP variables	present	uncommon	absent
	slow	rapid (100 days)	...
Cepheids	classical light-curves		W Vir type
NGC	104, 6356, 6838	4147, 5272, 5904 6205, 6656	5024, 5053, 5466, 5897, 6254, 6341 7078, 7089

variable), F5–8 and B–C (100-day LP clusters), F7–G6 (long-period LP clusters, omitting the anomalous NGC 7006). These correlations are consistent with Tables IV and V, if we note that the RR Lyrae poor clusters with RV Tauri stars (NGC 6205 and 6779) do not have late-type spectra.

Arp (1955b), in an examination of globular clusters, found that many of the parameters characterizing the clusters were correlated. He divided the clusters into two groups on the basis of the mean period of the RR Lyrae stars. Group 1 included NGC 5272, 5904, 6121, 6205, 6981 and Group 2 included NGC 5024, 5139, 6254, 6341, 6656, 7078, 7089. Sandage and Wallerstein (1960) classified a third group of very red clusters, sequentially before Arp's Group 1. The summarized data are collected in Table VI, where we append data on the long-period variables. The individual spectra are on Deutsch's system, and the number in parentheses that indicates the strength of the giant branch is the difference in visual magnitude between the horizontal branch and the giant branch at $(B-V)_0=+1.4$. The clusters used by Sandage and Wallerstein are listed at the bottom; Arp's Group 1 is the same as Group II and Group 2 is the same as Group III, except for NGC 5139, which is not listed, and NGC 6656, which should fall into Group III on the basis of mean period of the RR Lyrae stars. NGC 6205 is also anomalous, judged by the blueness of the horizontal branch, but these two clusters are very young, so that mass as well as chemical composition differences may have affected the evolution (Sandage and Wallerstein 1960). The color–magnitude diagrams of NGC 6205 and 6656 are very similar (Arp 1958c). Age differences among all the clusters might also introduce some scatter in Table VI.

Comparison of the clusters listed in Tables IV, V, and VI (with particular reference to the spectra, RR Lyrae population, and long-period variables) shows that the RR Lyrae poor clusters comprise an expanded Group I; perhaps they include part of Group II. The RR Lyrae rich clusters comprise most of Group II and Group III. Thus the two groups based on richness in RR Lyrae stars overlap the three groups based on mean period.

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